LOGIC

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LOGIC

OR THE MORPHOLOGY OF KNOWLEDGE

BY

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PREFACE TO THE SECOND EDITION

In preparing the second edition of this work, I have endeavoured to put myself in the position of a reader of the first edition, aware of recent logical discussions, and interested to know how the book he is reading would respond to them. I have therefore offered explanations which I hoped might be of service to such a reader, by enabling him to judge of the attitude which my theory would naturally assume towards a number of problems which have recently been emphasised.

I have made few alterations in the text,[a] but have added a good many footnotes, referred to by letters of the alphabet, while the old ones are as before referred to by numbers. I subjoin at the end of the Preface a list of substantive additions other than footnotes.

I am gratified by Mr. Keynes's appreciative reference to some of my views, in the fourth edition of his Formal Logic. I note that while favourable on the whole to my treatment of the concept as in close union with the judgment, he remains cold to my interest in the reciprocity of propositions and in the exclusiveness of members under a disjunction. I only mention these points, because they seem to me to be involved in the ideal of complete expression by adequate conditioning of connections, which I take as defining the essential cognitive interest, on which, as the central idea of my work, I especially desire to insist.

It is, in my view, from the operation of this ideal, that there arises the development of substantial knowledge within forms of sentence framed for all sorts of varied emergencies, and diversely one-sided in their emphasis. This is why the dispute as to the meaning of propositional forms, on the field of mere usage, is really unending, but also why it is worth while to take pains in eliciting, from the full spirit and purpose implied in sentences, an import in the way of judgment relative to the ultimate interest which is continuously operative within them. The true meaning of propositions lies always ahead of fully conscious usage, as the real reality lies ahead of actual experience (ii. 303–4 below). To maintain the reality of a central cognitive interest as such, and the doctrine that it expresses its nature, though roughly, through forms of sentence[a] which cover phases of judgment progressive towards its satisfaction, is the double motive which inspires my logical theory.

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[a] The addition on i. 57, concerning Mr. Keynes's view of Extension, has involved some alteration of the text.

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[a] I may venture to cite from Knowledge and Reality, p. 57, a few words which express the problem as I stated it to myself *ab initio*, and still envisage it. 'Affirmation about the unanalysed present perception may be considered as one extreme; supposal, having its subject indicated by a mere idea, definite but fictitious, as the opposite extreme. The forms of proposition from which the kinds of judgment have respectively drawn their names, are most precisely adapted for the expression of these two contrasted activities. For the true region of human knowledge, which for the most lies between the two, neither form of sentence, neither class of proposition, or even of judgment,

The addition 'On a Defective Formulation of the Inductive Principle' is based on a paper published in the Proceedings of the Aristotelian Society of London; that 'On the Limits of a Genetic Theory of Logic' on a paper published in the Psychological Review. I have to thank the proprietors of both these journals for their permission to use the papers in question.

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is, if pressed home, absolutely and unrestrictedly appropriate,' and the passage proceeds to describe the transition between the two extremes.

I may here refer very briefly to Miss Jones's interesting treatise on 'A New Law of Thought', with Professor Stout's Preface, both of which contain much that would repay a fuller discussion.

The substitution of *S* is *P*, as the general formula of thinking, for *A* is *A*, which, taken as such a formula, is meaningless, will I hope be adopted by logical theory and practice. By dealing with a difficulty which so great a logician as Lotze could not overcome, the suggestion shows itself to possess a considerable value.

The form *S* is *P* is certainly appropriate to the expression of the judgment. But the precise analysis by which Miss Jones interprets it raises, I think, a very fundamental difficulty, to which I propose merely to call attention. (The same analysis has been repeatedly urged *totidem verbis* by Mr. Bradley and others, but with restrictions which point to the difficulty in question. See Bradley's Logic, pp. 29, 167, and the author's Knowledge and Reality, 273, and Essentials, 140, where the story from Thackeray has the same moral as Professor Stout arrived at in his Preface, though the point is less clearly stated).

That a judgment asserts difference of Intension along with Identity of Extension is an interpretation which holds good *prima facie* over the area of what commonly passes as the Categorical Judgment. But (1) beyond this area it is not applicable, at least in any natural sense. And (2) we are thus forced to see that within this area it is not perfectly applicable.

(1) When what the judgment affirms is explicitly a law of connection or implication between intensions—when, say, you have before you a true hypothetical judgment, the analysis in question does not really apply. You cannot reduce the meaning of such a judgment to the identification of cases. Systematic continuity of any kind, causality relation of antecedent to consequent, of condition to conditioned—these seem quite irreducible to an identity of subjects plus difference of attributes. But that one intension can 'be'—involve or imply—another is a possibility which, as I understand, Miss Jones absolutely and in principle denies (cp., however, p. 46 of her work). I should have thought that this denial brings her view into sharp collision with usage,

e. g. on p. 41, where, if I understand right, it is contended that admitting the judgment 'All politicians are statesmen'. we are yet entitled to deny that the quality of statesmanship can be inferred from that of being a politician. And so elsewhere, pp. 45–6, and 73.

Here is the root of the difference between us referred to on p. 7 of 'A New Law of Thought'. Between the letters *A* and *B* I recognise no such continuity as I look for, say, between a cause and its effect. And therefore I call them unfit to symbolise the continuous identity expressed in a judgment which states a law. '*A* is *B*' would not, in fact, be self-explaining as a formula for the new law of thought. '*S* is *P*' is rightly chosen, because it obviously refers to the structure of the judgment. But what this is, still needs explanation.

(2) 'Un giudizio è sempre la formulazione d'una legge,' Varisco, 'I massimi Problemi,' p. 89. The point here is in the word 'sempre'. At bottom, according to the view which I have adopted, *all* judgment, and even association, contains in it the principle of universal connection, of a law, however imperfectly formulated. From the very beginning, conjunction covers connection. The doctrine is well known, and it would be idle to re-argue it here. I need only point out that for good or evil, what stands between us is the conception that identity must be reduced to numerical identity. The judgment is split up into numerical identity plus intensional difference. This answers *prima facie* for apparent categoricals, but I doubt whether it is sound in principle even for them.

It occurred to me, indeed, that the author might intend to restrict the explanation to these judgments, which play so large a part in the operations of Formal Logic. But I believe that any such restriction would be disowned. (See analysis of a geometrical truth on p. 46, and note the treatment of the inseparability of the *S* and *P* intensions as merely *compatible* with the meaning).

I may add that I welcome the criticism of Jevons's 'Simple and Obvious Law of Thought', p. 36, for which, so far as I know, the author is right in claiming priority. And I do not deny that in so far as a universal connection presupposes a world or system within which it falls, a further analysis of identity in difference might bring out features bearing a superficial likeness to those of Miss Jones's conception. But I cannot think that any analysis can finally acquiesce in a view which restricts itself to irrelevant conjunctions.

PREFACE TO THE FIRST EDITION

THE conception of Logical Science, which has been my guide in the present work, is that of an unprejudiced study of the forms of knowledge in their development, their inter-connection, and their comparative value as embodiments of truth. If an attempt founded on such a conception appears to err by over-ambitiousness, I can only plead that an honest effort in the right direction does not depend for its value solely on its intrinsic success. It is at any rate a heavy wager laid by the author on his judgment of the true aim and future of his science, and may attain results as a suggestion which it misses as an achievement.

In the present centrifugal state of logical research, no undertaking, perhaps, can be entirely valueless which aims at reintroducing some sort of unity into the enquiry. Such an aim is necessarily involved in the idea of a single connected science of logical processes and products. It has therefore been my object to maintain the central identity of judgment and reasoning throughout, and in no case to permit the variety of applications due to diversity of matter to interrupt the connection and subordination demanded by unity of principle. Although in periods of transition logical science has been most effectually advanced by detached discussion, in separate chapters, 'books,' or volumes, of Inductive or experimental method, of equational reasoning, and of the Logic of Chance, yet it seems plain that a time must come when the various cases and species of inference shall fall into their respective places as organic members within the intellectual whole. But when this transformation is effected it is necessary that they should lose something of their interesting peculiarity and novelty; and it is natural, from the difficulty of the task, that they should lose more of these qualities than is necessary. For this reason, as well as from incompetence in the field of exact science, I claim no discoveries in scientific method or in mathematical reasoning, but shall be content if my attempts to represent them have not many more errors than are either necessary or natural.

There is in England a healthy objection to system-making, and a preference for free criticism, against which I should be sorry to offend. I think however that systematic form is essential to clear exposition and to really effective criticism, and I have not supposed that my work will be considered as a system in any other sense than that thus implied. In particular, I do not hope or even desire that the specific names which I have invented for some kinds of Judgment and argument should come into general use. I have endeavoured in every case to retain the accepted usage of the generic names, which are all that are recognised in ordinary logical discussion, and have only used strange titles for subdivisions which have no accepted place in existing logic, and which merely serve to insist upon certain views of logical evolution.

It is almost superfluous to acknowledge here what I owe to other writers, as the Index bears copious testimony to the amount of my borrowings. In bringing together 'Inductive' and 'Deductive' Logic I have followed more particularly Sigwart and Jevons; in the arrangement and analysis of Judgment-forms and forms of Inference I have gone to a great extent in the track of Lotze, and also of Hegel, to whom, so far as I know, the idea of this organic treatment of Logic is primarily due; and in fundamental theory of judgment as in many details I owe even more, probably, than I have acknowledged, to Mr. Bradley, whose work, 'Principles of Logic,' appears to me no less valuable now than it did three years ago.[1] On one particular point, relating to the simplest process of equation (colour-equation, for example), I am especially indebted to Mr. Spencer's Psychology.

Every student will understand that my obligation to former writers is frequently as great when I diverge from them as when I assent to their opinion. As I have often been led to express disagreement with Lotze, portions of whose views certainly appear to me strange in so eminent a philosopher, I ought to say that but for his great work on Logic the larger part of what I have written would never have come into my head. I may also express my strong conviction that the reform of Logic *in this country* dates from the work of Stuart Mill, whose genius placed him, in spite of all philosophical short-comings, on the right side as against the degenerate representatives of Aristotle. A glance at the Index will suffice to show how constantly I have referred to his treatise.

I may venture, finally, to discharge an obligation of older standing, and at the same time to emphasise the guiding idea of my work, by observing that the first germ of unprejudiced interest in the forms of knowledge was implanted in my mind, when wholly innocent of Hegel or Lotze, by some remarks made in a course of Logic lectures which I had the good fortune to hear about twenty years ago, the lecturer being Mr. Alfred Robinson, of New College, Oxford. A comparison, which he threw out, between the study and analysis of judgment-forms and the study and analysis of the forms of flowers or plants has never left me since, and I have never seemed to myself able to exhaust its suggestiveness. If I have at all reproduced for others the spectacle of continuity and unity in the intellectual life, combined with the most varied and precise adaptation of its fundamentally identical function to manifold conditions and purposes, which this comparison never fails to present to my own mind, I shall so far have succeeded in the object of my work.

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[1] See the author's 'Knowledge and Reality, a Criticism of Mr. Bradley's "Principles of Logic"'. Kegan Paul, Trench & Co. 1885.

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ABBREVIATIONS

Bradley's Appearance = Bradley's Appearance and Reality, ed. 2. Sonnenschein, 1897.

Bradley's Logic = Bradley's Principles of Logic. Kegan Paul, Trench & Co., 1883.

Keynes = Keynes's Formal Logic, ed. 4. Macmillan, 1906.

Prichard = H. A. Prichard's Kant's Theory of Knowledge. Clarendon Press, 1909.

Essentials = Bosanquet's Essentials of Logic. Macmillan, 1895.

Sigwart = Logik, von Dr. Christoph Sigwart. 2nd edition. Freiburg, 1889.

BOOK I

OF JUDGMENT

CHAPTER I

OF JUDGMENT AND JUDGMENT-FORMS IN GENERAL

I. The Nature of Judgment as such.

I. JUDGMENT is coextensive with affirmation and denial, or, which is the same thing, with truth and falsehood. True and false are not indeed terms applied exclusively to judgments; but yet in all their applications their essential meaning depends upon judgment. The sensations of a diseased organ may be abnormal, but cannot possibly be false unless, on the strength of them, we judge erroneously. A false note is a real sound, a false man is an actual individual. It is not their existence, but a judgment implied in their nature, that gives meaning to the censure of falseness. The musical note is not what its place demands; the man is not what he pretends or aspires to be; it is the demand or pretension, ascribed actually or by metaphor to thing or person, which condemns them as false in as far as it is unrealised.

Thus truth and falsehood are coextensive with judgment, and depend on the fact which is its primary condition; the fact that a thing may have an ideal relation to reality over and above its own particular existence; so that its existence, though in itself real and actual, is empty and valueless in the absence of the further reality that such a relation demands. Truth must belong to something whose unreality is not simply non-existence; or how could falsehood exist? The essence of falsehood or fiction is that there should be an actual something that pretends to be something else.

Thus if we describe Judgment as the act of thought which is capable of truth and falsehood, the description, although tautologous, is not unsuggestive. It tells us that we are to look for the differentia of judgment not in a mere mental fact, but in some further value with which the mere mental fact is invested.

Symbolic ideas.

i. This primary condition of judgment recalls us to the subject of the Introduction. In judging, we use ideas, but the ideas which we use are not mere particular mental images, the perishing existences which pass through consciousness, and which, *qua* particular psychical states on a level with mere sensations, never recur. Ideas,[1] according to Locke, though particular in their existence, are general in their signification. In judgment, ideas are employed solely for the sake of their general signification, and without reference to their particular existence. An idea, considered as a general signification, is what was described in the Introduction as the meaning of a word. It is not without effort, as we all know, that we can find in our consciousness any intermediary between the word on the one hand, and the reality on the other ; and when it is brought to our notice that a reality cannot as such be a state of our individual consciousness, we are sometimes tempted to deny that it has any representative there beyond the name. We see from this how utterly the symbolic and secondary employment of psychical images obliterates all consideration of their particular existence as mental occurrences. We no more take note of them than, on meeting a welcome friend, we give ourselves a detailed account of the peculiarities by which we recognise him. The word and its reference—a reference to some continued identity in the world of meanings [2]—are inextricably welded together. It is only by reflective analysis that we discover, within and auxiliary to the meaning of a word, the particular psychical images by help of which we symbolise it.

And the meaning tyrannises over the psychical image in another respect. Besides crushing out of sight its particular and exclusive existence, it also crushes out part of its content. The psychical images that pass through our minds might be compared to a store of signal flags. Not only is it indifferent whether your signal flag of to-day is the same bit of cloth that you hoisted yesterday, but also, no one knows or cares whether it is clean or dirty, thick or thin, frayed or smooth, so long as it is distinctly legible as an element of the signal-code. Part of its content,[a] of its attributes and relations, is a fixed index which carries a distinct reference; all the rest is nothing to us, and, except in a moment of idle curiosity, we are unaware that it exists. The well-known difficulty of detecting misprints arises from the same despotism of the meaning. Let the main index-letters of a word be correct, or even the main index-words of a sentence, and we are off at once in thought to the word or sentence which is indicated, and remain unaware of minor variations in the content employed as index.

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[1] Essay on Human Understanding, Book III, chap. 3, sect. 11.

[2] See Introduction, sect. 7.

Thus the idea, as used in judgment, is a general signification, or in other words, a fixed reference. And because fixed, it is limited; limited to portions of content which serve as indices of the reference, and are compatible with psychical accompaniments that vary with the series of images. I will give another instance. Some one speaks to me of the Aegean Sea, which I have never seen. He tells me that it is a deep-blue sea under a cloudless sky, studded with rocky islands. The meanings of these words are a problem set to my thought. I have to meet him in the world of objective references, which as intelligent beings we have in common. How I do this is my own affair, and the precise images at my command will vary from day to day, and from minute to minute. It sounds simple to say that I combine my recollections of sea and sky at Torbay with those of the island-studded waters of Orkney or the Hebrides. Even so, there is much to adjust and to neglect; the red cliffs of Torbay, and the cloudy skies of the north. But then again, my recollections are already themselves symbolic ideas; the reference to Torbay or the Hebrides is itself a problem set to thought, and puts me upon the selection of index-elements in fugitive images that are never twice the same. I have *first* to symbolise the colour of Torbay, using for the purpose any blue that I can call to mind, and fixing, correcting, subtracting from, the colour so re-called, till I reduce it to a mere index-quality; and *then* I have to deal in the same way with the meaning or significant idea so obtained, clipping and adjusting the qualities of Torbay till it seems to serve as a symbol of the Aegean."[a]

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[a] I can see no difficulty in this conception, if we bring in the acquired content, e.g. of a word. Critics might say that this equals meaning, and that the text explains meaning by meaning. But here, perhaps, they supply the answer. For a word, they point out, has a psychical fringe, which, they again point out, is not meaning (cp. Stout, Ar. Proc., 1903, p. 13, and Hoernlé, Mind, 61, p. 81 ff.). For my own part (see p. 40 above, and ii, p 298), what I take to matter is the use. In an original or acquired content (though already symbolic, as in my view, all ideas are; see p. 70 below), when not being used in definite judgment, the aspect of mental existence becomes prominent. The use in judgment I take to be a question of degree (p. 5 above). I may add that I am aware of the passage about Mr. Bradley's phrase 'acquired content' in Professor Stout's paper above referred to. But I continue to think that the term is explained by the words 'The meaning may be part of the original content, or it may have been discovered and even added by a further extension' (Bradley's Logic, p. 3). And it appears to me clear that with this explanation it is true that the content of a sign, though not, of course, the whole content, always qualifies for us the thing signified. See Nettleship, Remains, i. 23.

Here then we have the first essential of judgment. The ideas used in judging are not particular existences but general significations, or objective references. No mere mental occurrences as such, no series or combination of particular images, can by any possibility be a judgment. It is the essence of judgment to claim a value which is beyond the mere mental act itself, and which is therefore liable to be divorced from the mental act; and this divorce, as we have seen, is what falsehood means. That is false, which *is*, but like a false coin, has not the significance which it claims. In judgment, then, all ideas are symbolic, that is to say, have a constant reference.

Can ideas be symbolic apart from judgment?[b] If no ideas in a human consciousness *are* apart from judgment (see Introd. 2. ii) this question falls to the ground so far as concerns that consciousness. But the discussion referred to made it clear that *apart from ultimate analysis* we do entertain ideas without judging them true, as in the question and in the negation, and that these ideas are symbolic. The further problem may then be pressed upon us: 'Are there at all ideas which are not symbolic?' In identifying the *human* intelligence with a continuous judgment we seem to have denied in advance that non-symbolic ideas *are*, for that consciousness. The answer is that α. In judgment itself the idea can be distinguished *qua* particular in time or psychical fact, and *so far* is not symbolic, and β. In all those human experiences from which we draw our conjectures as to the animal intelligence, when in languor or in ignorance image succeeds image without conscious judgment, we feel what it is to have ideas as facts and not as symbols.

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[a] Then, it has been said, my test of truth is correspondence, i.e. how far my idea is like the original. I reply, to me this is meaningless. There is no given original. Sense-perception gives no original. How can it, when its main task is self-amendment? A man may have seen the Aegean, and yet have a worse idea of it than a man who has not. The ultimate 'original', got by amending our perception till it could be amended no more, would involve the Absolute, and would be like nothing in our world of percepts. But the working test for us is what idea does most justice to all the experiences concerned.

[b] For the significance of this paragraph see p. 39 above.

Reference to reality

ii. Granting that symbolic ideas cannot ultimately be entertained without judging, it does not follow that to judge is merely to entertain ideas. In what does the act of judgment consist? An act it undoubtedly is; an act which is as certainly present, and which we find as hard to describe, as the much disputed act of volition itself.

I shall attempt in the first instance to make the essentials of the matter quite clear in a simple case, with which we shall afterwards find that all more elaborate instances agree in fundamental structure.

If I say, pointing to a particular house, 'That is my home,' it is clear that in this act of judgment the reference conveyed by the demonstrative is indispensable. The significant idea, 'my home,' is affirmed, not of any other general significant idea in my mind, but of something which is rendered unique by being present to me in perception. In making the judgment, 'That is my home,' I extend the present sense-perception of a house in a certain landscape by attaching to it the ideal content or meaning of 'home'; and moreover, in doing this, I pronounce the ideal content to be, so to speak, of one and the same tissue with what I have before me in my actual perception. That is to say, I affirm the meaning of the idea, or the idea considered as a meaning, to be a real quality of that which I perceive in my perception.

The same account holds good of every perceptive judgment; when I see a white substance on a plate and judge that 'it is bread', I affirm the reference or general meaning which constitutes the symbolic idea 'bread' in my mind, to be a real quality of the spot or point in present perception which I attempt to designate by the demonstrative 'this'. The act defines the given but indefinite real by affirmation of a quality, and affirms reality of the definite quality by attaching it to the previously undefined real. Reality is given for me *in* present sensuous perception, and *in* the immediate feeling of my own sentient existence that goes with it. The real world, as a definite organised system, is *for me* an extension of this present sensation and self feeling by means of judgment, and it is the essence of judgment to effect and sustain such an extension. It makes no essential difference whether the ideas whose content is pronounced to be an attribute of reality appear to fall within what is given in perception, or not. We shall find hereafter that it is vain to attempt to lay down boundaries between the given and its extension. The moment we try to do this we are on the wrong track. The given and its extension differ not absolutely but relatively; they are continuous with each other, and the metaphor by which we speak of an extension conceals from us that the so-called 'given' is no less artificial than that by which it is extended. It is the character and quality of being directly in contact with sense-perception,[a] not any fixed datum of content, that forms the constantly shifting centre of the individual's real world, and spreads from that centre over every extension which the system of reality receives from judgment.

Waiving then this distinction, though as a matter of degree it may find a place in the enumeration of judgments, we find the same general features in all judgments of perception. There is a presence of a something in contact with our sensitive self, which, as being so in contact, has the character of reality; and there is the qualification of this reality by the reference to it of some meaning *such as can be* symbolised by a name. It cannot be alleged in theory that a name is essential to judgment. At least for 'name' it would be necessary to substitute in such an allegation 'some symbol'. The spatial order of things which we see whenever we open our eyes, is, *qua order of things,* the content of a perceptive judgment, in which universal ideas are presented through sensuous symbols.

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[a] This has sometimes been remarked on as if it implied a sensationalist view of Reality. But the point is merely that in the 'this' of sense Reality appears in a quasi-satisfactory form, having a sort of self-completeness. When we begin to explain and define it, of course it breaks down, and demands the effort to complete it by way of thought.

The subject in every judgment of Perception is some given spot or point in sensuous contact with the percipient self. But, as all reality is continuous, the subject is not *merely* this given spot or point.[1] It is impossible to confine the real world within this or that presentation. Every definition or qualification of a point in present perception is affirmed of the real world which is continuous with present perception. The ultimate [2] subject of the perceptive judgment is the real world as a whole, and it is of this that, in judging, we affirm the qualities or characteristics.

The claim to be true, which as we saw belongs primarily to judgment, indicates the same relation. In every judgment, as Mill incisively contends, we profess to speak about the real world and real things. 'The Sun' means 'the Sun'; and whatever that may be, it is not anything *merely* in my mind; not relative purely to me as a conscious organism; not a psychical fact in my individual history. Every judgment, perceptive or universal, might without altering its meaning be introduced by some such phrase as 'Reality is such that — —', 'The real world is characterised by ——.'

Thus in the Perceptive Judgment at least we find the meaning or objective reference of an idea—such a content as is indicated by a name—affirmed to characterise some reality present in sense-perception, and through it, reality as a whole. We shall find that all Judgments of every kind share the main elements of this description; only that the reference to an indeterminate element of present sense-perception is gradually displaced by the introduction of explicit ideas to describe the immediate subject. Such ideas disguise but do not remove the reference to Reality as the ultimate subject in every judgment; they have, however, important effects in modifying both the act of affirmation, and the nature of what is affirmed. When I come to examine the chief types of judgment, I shall have to consider the nature of these effects. But I intend in the first place to say something of the proposition, from the analysis of which many current views about judgment are derived.

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[1] See Introd. 2. i.

[2] See Introd. 2. i. Analysis and cross-examination readily verify this as a fact. After admitting any judgment to be true, you cannot deny its modifying effect on any portion whatever of your real world; i.e. it has been admitted of the real world as a whole.

Judgment and Proposition.

iii. The enunciative sentence—the unit of language which represents a judgment—is called a proposition. Language, as we saw, supplies the fixed symbols which stand for ideas. It would be rash to say that there can be thought without language—if language includes every possible system of recognisable signs—and wholly perverse to imagine that the ideal of intelligence lies at all in the direction of a severance of thought from words. The Introduction, in dealing with Names, showed us the absurdity of any such conception. But yet the spoken or written proposition differs fundamentally from the judgment.

I do not think that it is convenient to rank the narrative or temporal affirmation as a 'proposition' (German 'Satz'), and reserve the name of judgment for an act of thought which has some purpose in the way of classification or definition. To do so is in English terminology at least to confuse a distinction of degree with one of kind; but it is worth noticing that such a nomenclature has been proposed,[1] and that according to it judgment proper would begin at the point where inference and necessity become explicit. For against any *doubt*, judgment maintains itself as an inference,[2] and this is exactly the test that has been held to distinguish a judgment from a proposition. To affirm that a carriage is passing the house, Hegel says,[1] is not a judgment unless there is a question, e.g. whether it is a carriage or a cart; i.e., I suppose, unless some general connection of attributes is intentionally affirmed. Now a general connection involves a ground, and so an inference. Thus the classification in question would have the merit of suggesting that judgment begins with inference. But the point of commencement taken is really arbitrary; though judgment and inference begin together, yet both begin before this point.

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[1] Hegel's Logic, Wallace's Translation, pp. 258–9.

[2] Bradley, Principles of Logic, p. 404.

I prefer then to take the proposition all through as the actual spoken or written enunciative sentence; while the judgment is the intellectual act which depends in various degrees upon words or other symbols, but is different from any mere combination of words or symbols whether heard, read, or remembered.

The essential differences between judgment and proposition may be arranged under two heads, which cannot however be wholly separated from each other. I shall first speak of the so-called *parts* of the judgment, the current conceptions of which are derived from grammatical analysis of the proposition; and then pass on to consider how far the idea of a transition in time, which is inseparable from the apprehension of a sentence, is applicable to the judgment as such.

The parts of the judgment.

a. The division of the Judgment into Subject Copula and Predicate is obviously derived from the analysis of the enunciative sentence. The finite verb, which is a proposition in miniature, contains all these elements within itself; and the history of their being distinguished within the sentence is the history partly of linguistic evolution and partly of grammatical or quasi-logical analysis. Even the separation of the substantive from or within the verb, is, I suppose, an early analytic development of language; and it is the tendency of modern speech, no less than a supposed convenience of thought, that has finally transmuted Nominative and Verb into Subject Copula and Predicate. The Copula in the modern sense was unknown to Aristotle, although the use of the predicative Verb 'to be' attracted his attention and drew from him a somewhat inadequate explanation. If we return, however, to Aristotle's main position, and, in agreement with students of English grammar,[1] regard the Judgment as made up of Subject and Predication (ὄνομα and ῥῆμα), we have got rid of one fiction in the separate Copula, but the distinction which we retain may still be challenged. It is plain that the judgment, however complex, is a single idea. The relations within it are not relations between ideas, but are themselves a part of the idea which is predicated. In other words, the subject must be outside the judgment in order that the content of the judgment may be predicated of it. If not, we fall back into 'my idea of the earth goes round my idea of the sun', and this, as we have seen, is never the meaning of 'The earth goes round the sun'. What we want is 'The real world has in it as a fact what I mean by earth-going-round-sun'.

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[1] Cp. Mason's English Grammar, Jones's Analysis of English Sentences, and Wrightson's Functional Elements of the Sentence.

This view, stated thus extremely, would not only annihilate the copula with separate content, but also the whole distinction of Subject and Predication, and it is an argument in its favour, that, in doing so, it would seem only to lay bare in all judgments the elementary type which forces itself on our observation in the simplest perceptive apprehension; at a stage, that is to say, before the grammatical subject, which creates our present difficulty, appears explicitly in the proposition. But we shall see in tracing the evolution of judgment, that it is impossible to dispense with the distinction of Subject and Predication, and that the appearance of contrast between propositions which have and which have not grammatical subjects, is caused by the necessity of representing immature thought in developed language; so that the thought in which distinctions are rudimentary must either be mutilated by the omission of an element, or transformed by explicit articulation. It is impossible to *represent* a judgment by a single noun belonging to a modern language, though such a noun is often all that we utter. Such a judgment should really be represented either by a rudimentary sentence, that is, by some element of language not yet reduced to the position of a part of speech, *or* by a miniature sentence, i.e. by a verb.

In other words, although the ultimate Subject extends beyond the content of the judgment, yet in every judgment there is a starting-point or point of contact with the ultimate subject; and the starting-point or point of contact with reality is present in a rudimentary form in the simplest perceptive judgment, as it is explicitly in the later and more elaborate types.

Then it would come to this. Subject and Predicate in the actual judgment are really distinct, as a real identity from or in its differences. The relation of their contents *is* itself ideal, and not a relation between ideas; but nevertheless the judgment demands this relation; for the judgment is my consciousness *qua judging*, and my consciousness in judging identifies the ideal or symbolised reference which constitutes the predication with its own construction of the Real world.[1]

The difficulty is that you cannot affirm without introducing a distinction or reference into the content of the affirmation; and yet such distinction or reference, being part of what is affirmed, and not a relation between what is affirmed and something else, cannot, it would seem, be the essence of the affirmation. What is the connection between the two things; between the reference of 'is-building-a-wall' to 'Balbus', and the affirmation that the *whole* idea 'Balbus-building-a-wall' is true of reality? What has the action of Balbus to do with my affirmation that Balbus acts? The latter seems wholly unconnected with the former, and yet they are inseparable.

And the answer is that the real world is primarily and emphatically *my* world. I take it to be real in virtue of its contact with me. Therefore though the ideal relation within a judgment is not a psychical fact in my mind but a fact affirmed objectively of the real world, yet, the real world for me being the world that hangs from my present perception, I identify my assertion about it with its assertion of itself. In every judgment the ultimate subject Reality is represented by a *selective* perception, or idea, which designates a something accepted as real. This something, taken as standing for reality, is the actual subject of the judgment, and is qualified by the ideal content which forms the predication. No judgment can be found in which Subject and Predicate are not apparent. Reality is one, but its presentation varies; and it is impossible to judge without explaining where and how Reality accepts the qualification which we attach to it. The presentation of Reality, qualified by an ideal content, is one aspect of Subject and Predication; and my individual percipient consciousness determining itself by a symbolic idea, is the other. That the latter is identified with the former follows from the fact that in entering upon the world of thought my consciousness enters upon the experience of logical necessity—the *nisus* towards complete expression.

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[1] Cp. Introd. 2. i.

Thus I am of opinion that Subject and Predication are essential elements in the Judgment. But whereas in the judgment they are differences within an identity, in the proposition they are isolated parts of an extended whole; and the copula, which in Judgment is merely the reference that marks predication, and has no separate content, becomes in the proposition an isolated part of speech. When therefore the analysis of the proposition controls the interpretation of the judgment, each of these parts of the sentence is treated as a separable content, and perhaps as a separate psychical existence; and we are told of two ideas or two ideal contents, and a variable copula, itself also an ideal content, which indicates the varying relations[1] between them. In this sense, Subject and Copula and Predicate are mere fictions. The judgment is not a relation between ideas, nor a transition from one idea to another, nor does it contain a third idea which indicates a particular kind of connection between two other ideal contents.

The real nature of the copula we have seen already. It is the mere sign of affirmation, and, though usually conveyed by a finite verb in languages which possess one, does *not* depend on tense. Aristotle, who was inclined to include 'indication of time' in the differentia of affirmation, was nevertheless aware that judgment could take place 'absolutely' (ἁπλῶς) as well as with note of time (κατὰ χρόνον). Moreover a verb can exist without definite tense, and predication can exist apart from a distinctly formed verb.

The reason why the verb, where there is a verb, is appropriated to the act of predication, is not that the verb signifies time, change, or action, but that it is, as has constantly been repeated, a miniature sentence. Not merely does the verb 'agree' with its subject—the adjective also agrees with its substantive—but by convention, or explicitly in the person-ending, it includes within itself a reference to given reality, and can therefore stand alone as an enunciation, which no other part of speech can do.[a] In other words, the verb is *prima facie* a content referred to a real individual subject, and though the subject may be by the help of additional phrases defined, set down as imaginary, or even denied, the verb has always in itself the force of this demonstrative reference. An adjective implies a reference to something else, but the something may be a mere idea; it is only the verb that professes to select an element directly related to the speaker's apprehension, and to attach a significant content to that element.

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[1] Lotze, Logik, sect. 52. It is very doubtful whether in this passage Lotze escapes the error which he imputes to others, of 'reducing a logical operation to a mere psychical occurrence'.

It is in the demonstrative force of the verb that we must look for its fundamental predicative force. I suppose that the collocations which in Greek, and more or less in many languages, have power to turn the epithet into a predicate, owe their significance to a quasi-demonstrative emphasis. In 'the white horse' (ὁ λευκὸς ἵππος) there is nothing that can be taken as a reference to a special point in reality; no indication of a real existence, either in the ideal content or out of it, which we propose to qualify by its meaning. In 'the horse (is) white' (ὁ ἵππος λευκὸς) there is an indication of a line between *α* an individual that may be real and *β* a content that may be attached to him or it, and therefore the instinct of reason which sees a judgment wherever a judgment is possible, takes the individual named as if it were an appeal to perception, i.e. a demonstrative reference to reality, and the content as a quality ascribed to the real subject so obtained. In the universal judgment this demonstrative reference becomes merely formal; but it continues in all language to supply the symbol of judgment.

Judgment in relation to Time.

β. It has been proposed to distinguish Subject and Predicate simply as earlier and later in time, and the above instances of demonstrative reference appear to support this notion. But rather than admit it to be correct, I should surrender the distinction altogether and adopt the view that there is no subject in the judgment as such. For it is absolutely impossible that priority in time should subsist between the parts of a completed judgment. But if not, the priority of the subject would exist merely in memory; and an act of thought cannot be characterised by a mere recollection of the process that generated it.

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[a] See, however, p. 99 below.

In what sense is it true that the Judgment is in time, and in what sense not? It may be convenient to distinguish between arriving at the judgment, and subsequently modifying the judgment, although the two processes are, as we shall see, really continuous.

In what follows I do not identify the aspect of Judgment as in time with the series of images *qua* psychical occurrences that pass through the mind while we judge. It is probable that the view which defines Judgment as a change is influenced by the particularity of ideas *qua* events of consciousness as well as by the constant transition from judgment to judgment. But the former element ought to have been eliminated by what has been said above.

Arriving at Judgment.

1. In arriving at a Judgment, as when we hear a sentence and 'wait for the verb', or scrutinise an approaching person until his name comes into the mind, we undoubtedly appear to begin with a ready-made Subject, to which a Predicate is added by a subsequent transition. But closer attention will show us that this is not the case. We have always some anticipation of the meaning of a sentence, and this anticipation takes the shape of a provisional judgment or judgment in outline, very probably disjunctive in type, the shape of which becomes more definite as we follow the sentence, until the final clause determines its ultimate content. In the first place, consciousness, when any ideal content whatever is presented to it, absolutely refuses to abstain from judging; and in the second place, what comes first could not have (as it undoubtedly has) the significance of a subject, unless with reference to something already referred to it in the way of predication.

The case of perception leads to the same conclusion. You can come to no judgment by help of perception unless you interrogate perception; and you cannot interrogate perception unless you have in the mind some general idea as a basis for further specification.

Thus, in reaching a particular apprehension or perception, there is a transition that occupies time. But the transition is not from Subject to Predicate, which we will call S and P respectively, but from s–py (where y indicates superfluous detail, which is omitted when the perception becomes clear) through Σ–Π, to S–P. To speak of a transition from S to P is wholly false. We never have an S first, and then tack a P on to it; we have always an inchoate judgment or a choice of judgments. The process is not like adding one piece in a mosaic to another; it is more like enlarging a hole, which has centre and circumference from the beginning.

The complete Judgment.

2. What has been said of the transition by which we arrive at a judgment cannot but apply to the judgment when arrived at. It is clear indeed that we are thus led to regard the completion of the judgment as an arbitrary distinction, dependent solely upon our momentary interest. The completed judgment, like the process by which it is obtained, obviously possesses duration. It is absurd to suppose that a judgment cannot be dwelt upon, and only exists as a momentary transition from S to P. Such a conception arises from the confusion of two points of view, either of which may be taken as a presupposition, and reconciled with the other by a mistaken compromise. It may be assumed that the judgment, as such, is not in time, and then this assumption has to be reconciled with the obvious fact that judgment as an intellectual process is a transition that occupies duration; or it may be taken as certain that the judgment is a transition in time, and then we have to face the experience that its essential parts do not fall outside each other in succession. To treat it as an instantaneous transition is a ridiculous attempt to combine the character of occupying duration with that of not being in succession. It recognises both principles, and satisfies neither.

As we have seen in the process of arriving at the judgment, the act of judging as an occurrence in consciousness presents itself in the aspect of an interval of consciousness extended in time, and therefore including successive differences within it. But it does not include succession because the nature of the judgment is to be successive, but because the flux of sensations and ideas is always pressing new material upon consciousness, and a perception, once attained, satisfies no interest by being further dwelt upon unless it gains in content from moment to moment. Thus the duration of the judgment as a transition in time is, so to speak, its external aspect, the aspect which, as a whole, it presents when compared with other occurrences in consciousness ; and this duration is theoretically capable of any degree of extension. On the other hand, as between Subject and Predication, that is to say, within the judgment, there is no transition at all. S and P are modified *pari passu*, and so, as a relation between them, the judgment is not in time. This relation is a continued identity S–P which includes within it the differences s–p, Σ–Π, and so on. The transition is not from S to P, but from s–p to Σ–Π within the general signification S–P. The idea of mere momentary existence has therefore thus much truth, that if you cut across the interval of consciousness occupied by a single judgment at any point whatever, you will always find in the plane so laid bare both S and P in one or other of their forms. They are in every minute part, but they are not confined to such a minute part. Judgment breaks up into judgments as rhomboidal spar into rhomboids, but nevertheless it is one through its whole extension.

But if a judgment can be thus extended, what do we mean by *a* judgment, and how do we know when we enter upon a new one ? The question is in each case a material one, being in fact the question of continued identity, and it is impossible to give it a formal answer. As a first approximation we might say that a single judgment is any extent of judging activity that can be summed up in a single proposition. But as the proposition takes its value from the judgment, and not *vice versa*, this is no more than an appeal to the fact that we succeed in distinguishing single judgments. The question is one of continued identity, and therefore must be dealt with as concerning organised wholes or systems. A mere extension of a system, or a mere omission within a system, does not bring us to a new and different system. The clearest cases of transition from judgment to judgment are those in which language uses a mere conjunction. When, on the other hand, we have propositions united by the inferential particle, it is a matter of degree how far they stand for separate judgments. Ultimately perhaps every inference may or should be represented as a single judgment, as being a mere extension of an existing whole of thought, and not a transition to a different one. Such an idea conflicts with the traditional differentia of inference, that it *should* lead to a new judgment ; but this only means a *bona fide* extension of the previous whole, such that, *if taken in abstraction from the process that generates it*, it *would* appear a perfectly new judgment. At least in elementary cases it is easy to see how inevitably inference shrinks up into single judgments, if we look at the actual life of thought.

Take such an every-day judgment of mixed perception and inference as 'He is coming downstairs and going into the street'. It is the merest chance whether I break up the process thus, into two judgments as united by a mere conjunction, or, knowing the man's habits, say, when I hear him half-way downstairs, 'He is going out.' In the latter case I summarise a more various set of observations and inferences in a single judgment ; but the judgment is as truly single as each of the two which were before separated by a conjunction ; for each of them was also a summary of a set of perceptions, which might, had I chosen, have been subdivided into distinct propositions expressing separate judgments ; e.g. 'He has opened his door, and is going towards the staircase, and is half-way down, and is in the passage,' &c. If I simply say 'He is going out' I am not a whit the less conscious that I judge all these different relations, but I then include them all in the single systematic content 'going out'. 'Cromwell Road runs east, and the Brompton Road north-east,' are two judgments ; but if the road happens to be thought of *qua* continuous, one would say, 'Cromwell Road turns from east to north-east, where it becomes the Brompton Road.' Again, 'Knightsbridge and Kensington Gore run east' may be generalised as 'The street from Kensington Church to Knightsbridge Barracks runs east'.

Thus a judgment is one in respect of the continued identity of its Subject and Predication, and this identity cannot be defeated by the inclusion of difference, but only by the failure to recognise continuity. It is obvious that the generalised forms in the above instances presuppose a work amounting to colligation of facts, if not to elementary induction and analogy. If the included judgments were never separately made, the inferential work of colligation has not been explicitly done; but it would be found absolutely impossible to draw the line between cases where it has been done and those where it has not. Every judgment would on scrutiny reveal differences which had more or less been absorbed into its formation. It follows inevitably that every systematic inference considered as a judgment is single and not multiple. However this may be, it is clear that extension in time is no bar to the unity of judgment.

Scheme of arrangement of Judgments.

II. Judgment, as we have seen, is primarily the intellectual act which extends a given perception by attaching the content of an idea to the fact presented in the perception. The whole of consciousness, in as far as it is the consciousness of a single world that shares the reality of our waking self, may be regarded as a continuous judgment, which qualifies our present feelings and surroundings by the knowledge of what is more remote in space and in time. From the point of view of common logic, that is of individual knowledge, the intellect sustains its world by continued effort, as Atlas held up the sky.[a] Every judgment is an effort of this kind, affirming on the one hand that the same reality which we touch in the present is rightly described by such and such an idea, and on the other hand that such and such an idea is real with the same reality as that which we touch in the present. The 'idea' of which I am speaking is, of course, not the particular existence or single occurrence of a psychical image; it is the general signification for the sake of which we use the psychical image.

The object of this Book is to analyse judgment into its principal kinds, and, as a necessary consequence, to trace their affiliation. We shall find that no linear arrangement will represent these affinities. Judgment, as the effort of thought to define reality, must vary with the kinds of reality to be defined no less than with the degree of its success in defining them. An equation is to one kind of whole what a definition by genus and species is to another, or an appreciation of aesthetic value to a third ; the function of judgment is present in each of these activities, and the difference between them is the difference of the wholes which they respectively analyse. They are divergent developments of the same relation, in each of which an aspect has become predominant that remains subordinate in the others.

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[a] This seems to me *prima facie* a plain and obvious fact. Every one's world is relative to his capacity and energy of mind at a given moment. If you ask 'What, then, becomes of the real world?' the answer is that that is a metaphysical question, but by no means specially hard to answer from this point of view as compared with any other. The real world lies, I suppose, or rather lives, in the efforts of all spiritual beings to sustain a unity of experience, taken in their completeness as successful. Of course it does not live merely in judgment.

But we shall have to deal with convergence no less than with divergence. The unity of the judgment excludes no complexity of synthesis, and in determining the species of a plant or the character of a man we may be obliged to employ, among others, accurate determinations of number, time, and space. That is to say, the treatment of a content by abstraction as a spatial or numerical whole may be re-absorbed in a more concrete treatment of it as an organic, aesthetic, or moral whole.

It might indeed be urged, from the point of view of metaphysics, that every kind of judgment must have its value and no more as a contribution to the whole of Reality, and that therefore the series of judgments, arranged according to the degrees of their significance for knowledge, must after all be linear. In other words, if a whole in number or a whole in space is not final in itself, but demands something further to complete its significance, this might be enough to show that we ought not to represent it as the goal of an independent series, but rather as a stage or phase of construction, subordinate to the more concrete forms of knowledge.[1] I should not greatly object to such a view, and shall endeavour in some degree to meet its requirements by exhibiting the more complete and concrete syntheses as involving the reunion of aspects which have been developed in the abstract. But though the forms of space and time are involved as a fact in the perceptive construction of the world of individual things, yet the mathematical analysis of these forms is an effort of the same scientific spirit which recognises the principles of order in the world of things itself. The two aspects of constructive science are co-ordinate and complementary activities of reason, and it would be ridiculous to treat geometrical analysis as prior to the perception of characteristic size or proportion. Intelligence is many-sided, in spite of its unity; and its aspects, which are correlative to each other, lose their true interdependence by being drawn out into a linear series.

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[1] Cp. Plato's arrangement, in the Republic, of the mathematical sciences in an order proceeding from abstract to concrete.

SCHEME ILLUSTRATING AFFILIATION OF JUDGMENTS AS DESCRIBED IN BOOK I.

RUDIMENTARY OR INTERMEDIATE SERIES OF JUDGMENTS
(Judgments of Quality.)
Impersonal Judgment;
Demonstrative Judgment;
[beginning with 'this', 'here', &c.]
(Judgments of Quantity.)
Comparative Judgments, &c.
(Judgment of Proportion.)
Measurement, &c.

Concrete or Categorical Series.	*Abstract or Hypothetical Series.*
(Singular Judgments.)	(Enumerative Judgments.)
Individual Judgment, &c.	Plural or Particular Judgments.
(Universal Judgments.)	Collective Judgments.
Quasi-collective Judgment.	Equation.
True Generic Judgment.	Abstract relations of Space and Time.

Hypothetical Judgment.

Disjunctive Judgment.	Infinity in Space and Time.

Purpose of scheme

i. I subjoin a scheme of the arrangement which I propose to follow in the remainder of the present Book. It is simply intended to assist the reader in apprehending the views which I submit, and is not meant to be a bed of Procrustes for the facts of logic. I take it that variations of arrangement and nomenclature are as inevitable in logic as in botany, and are not undesirable in either science; for they force upon our minds the truth that species are but sections of evolution, and that their arrangement is merely subsidiary to a correct apprehension of the process which we divide into such intervals.

Explanations of scheme.

ii. I will begin with a few words in explanation of the scheme which I have adopted.

There is no need to apologise for describing some types of judgment by appellations which are not to be found in Mill, Whately, or Hamilton. Recent attempts to restore to logic its hold on living concrete thought, a direction in which Mill was himself an able and adventurous pioneer, have made us familiar with a whole chaos of psychological, grammatical, and quasi-mathematical titles applied to phases of the judging activity. All that I have done has been to concentrate in a single review the best estimate that I could make of the typical character and true affiliation of such phases, attempting to give each judgment its appropriate place in relation to *all* the principles employed in the classification, and eschewing the too common habit of adducing various groups and appellations in two or three successive chapters, without any distinct reference of the one grouping to the other.

But besides adding to the traditional scheme of judgments, the arrangement suggested involves a dislocation of its parts, and the omission of one familiar antithesis. These innovations, though by no means original,[1] may conveniently be indicated and justified in a few introductory remarks.

Categorical and Hypothetical.

a. 'Categorical' and 'Hypothetical' are taken to designate fundamental characters of knowledge, and not mere grammatical appearance. It follows that the natural series of judgments commonly known as singular, particular, and universal, cannot wholly fall within the genus categorical, but at a certain point and in a certain degree assumes a hypothetical character. More especially, the formal equivalence of the singular to the universal judgment is replaced by a profound distinction of kind between the two. The disjunctive judgment again has a place and value of its own, and is not a mere proposition or grammatical form.

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[1] The treatment of the universal judgment as fundamentally different from the singular in categorical character was really initiated by Mill in so far as he analysed the content of affirmation into coexistence of attributes; and has been adopted and accentuated by Lotze, Sigwart, and Bradley.

Divergent species.

β. The true quantitative judgment—not the 'universal' judgment of ordinary thought, which derives only its name from quantity—reveals itself as a divergence from the central development of the judgment by reason of its extreme abstraction, in which one element of the relation essential to judgment almost disappears.

Analytic and Synthetic.

γ. The familiar terms 'analytic' and 'synthetic' are not made use of in the classification, because they belong to the theory in general and are not distinctive of any particular types.

I subjoin a short explanation on each of the above subjects.

Categorical and Hypothetical.

α. A categorical judgment asserts an actual fact absolutely. A hypothetical judgment asserts only the consequence that follows on a supposition. The distinction between the two seems clear. It is the difference between 'There is a bad smell in the house' and 'If there is an escape of gas there will be a bad smell'. But when we come to the 'Universal' Judgment the line of demarcation is at once blurred. Hamilton gives 'Rainy weather is wet weather' as an instance of a categorical proposition, and 'If it rains, it will be wet' as an instance of a hypothetical. In the former, according to him, 'rainy weather' is 'unconditionally thought to exist'. But is it? *Prima facie* the two propositions represent *the same* judgment, that is to say, their difference is grammatical only, and their meanings are identical. It may be that the categorical shape conveys a presupposition which is absent from the hypothetical enunciation—the presupposition that rainy weather exists *in rerum natura*—and into the question thus raised we shall have to enter at length later on. But it is clear at all events that the 'categorical' form conveys in this case a meaning which is in a large measure, if not entirely, hypothetical.

By referring to our account of judgment as such we shall see that the distinction before us involves a fundamental difficulty. Every judgment affirms an idea of reality, and therefore asserts the reality of an idea. Now an idea is necessarily abstract, because determinate; and therefore all judgment involves abstraction. And abstraction is the essential element of hypothesis; it consists in taking up into an idea some elements out of the content of experience, for the sake of consequences which attach to the elements so taken up. Therefore it seems, as the real world *for us* is maintained and extended by Judgment only, that in all extension and even maintenance of the given reality there is involved an element of abstraction, which is the same as to say that in all categorical judging there is an element of hypothesis. The relation of these characters to each other throughout the history of the judgment will be the main principle of the evolution which I shall attempt to describe. But we must accept as the usage of thought, which we are to explain, that the assertion of actual fact coincides as a rule with the individual or singular judgment, and that the universal affirmative of formal logic, on the other hand, *may* in every case be taken as purely hypothetical.

The categorical character of a judgment in the above simple sense may be tested by the possibility of expressing its meaning by an impersonal proposition, however awkward may be the necessary grammatical transformation. For 'Caesar was crossing the Rubicon' we may substitute 'There was Caesar crossing the Rubicon'; but for 'All men are mortal' it is impossible to frame such an equivalent, for the reference to given reality in the impersonal expression would be at once contradicted by the abstract 'all', which = 'any' or 'if a–', and so points, in the absence of any more effective assertion of the limited unity of the race, to an infinite series. If it is possible to say 'These are all the men who, &c., &c.', the 'all' cannot be the true generalising 'all', but must indicate a sum of known individuals. It will be necessary however to point out hereafter that the distinction between these two senses of 'all', or in other words the limit of individuality, is not absolute, but is a matter of degree.

These instances suggest the principle to which we shall adhere, viz. that every assertion is as absolutely categorical as the nature of its elements will permit; and that demonstrative or individual judgments are in the plainest sense categorical, because the realities indicated by their subjects are of a nature that can be given, in a way in which the realities indicated by more definite abstractions cannot. In every case the *real* subject is the reality indicated; in every case this subject is alleged to exist; but the question is how and in what way it is capable of existing; in other words, what is the kind and degree of its individuality. For only what is individual can have actual existence as a whole. An infinite series cannot have such existence,[a] and therefore cannot be taken to have it. The 'all' in this latter case remains a demand with which we cannot comply.

Divergent species.

β. The content of a judgment is always a significant idea, that is to say, a recognised[1] identity in differences. The varieties of judgment correspond to the forms which identity in difference is capable of assuming.

An identity in relation to its differences may always be regarded as a whole in which they are parts. An expanse of the same colour includes the changing lights and several points of space through which the one identical colour extends; the policy of a government includes the details into which its principles are developed; the moral character of a man is a whole in which his several acts of volition are the variously dependent parts. In this wide sense, as a synthesis of differences, not as a sum of units, the relation of whole and parts is a fundamental relation of all judgment. It is only when the differences or parts assume the maximum of homogeneity, and conceal, so far as is possible, the individuality of their relations to the whole, that the parts become *units*, and the whole a *total* or a *sum*. The relation of unit to sum total, that is, of quantitative part to quantitative whole, is thus obtained by abstraction. It is not the complete natural relation of concrete identity and differences, but is a device of knowledge which by sinking all other aspects of a given content is enabled to regard it as a sum of units, that is to say, as a whole of magnitude.

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[a] See p. 163 below.

[1] 'Recognised' is necessary to restrict the description to *significant* ideas. For a sequence of images in elementary reproduction, such as we ascribe to the animal mind, *is* an identity in difference, though we do not suppose it to be an objective reference, i.e. a *recognised* identity. See Book II, chap. i, sect. 3, The Reproduction of Ideas.

For this reason it seems right to consider the judgments of number, with the kindred judgments of magnitude in space and duration in time, as belonging to an outgrowth of thought which diverges from the complete evolution of judgment. All these judgments begin as qualitative, but become quantitative by intentional abstraction, and end in the creation of ideal totalities (abstract number, abstract space, abstract time[1]) which we are unable to think of as complete, and therefore are debarred from treating as actual totalities. This, I may point out, is a case of the connection between individuality and actual existence, which I spoke of under the last head.

Analytic and Synthetic.

γ. Every judgment is both analytic and synthetic. This would not by itself be a sufficient ground for refusing to employ these terms as heads of classification, for it is more or less the case through the whole of Logic that terms must be employed to mark predominant aspects rather than exclusive characters. Nor do I find a sufficient ground of objection in the psychological comment that the judgment which adds a fresh predicate to a subject to-day must become tautologous or analytic if repeated to-morrow, and that therefore it merely depends on individual knowledge and memory whether a given judgment is synthetic or analytic. Any conception of dominant quality, function, or essence, is enough to make this comment futile, and without such a conception it would seem that science is impossible. It is a superstition to suppose that the progress of theoretical explanation in terms of general law threatens the doctrine of essence, form, or function. However clearly an individual thing may be explained as a section of evolution or a meeting-point of forces, there will always be a definite continued identity conferred by characteristic form or function. No explanation can destroy the actual relations of whole and parts which form the essence of everything that is real. Knowledge has quite enough fixity to give meaning to the contrast of analytic and synthetic judgments wholly apart from the progress of individual minds.

The reason why I no longer care to lay emphasis on the antithesis in question is not that it is purely 'subjective'—for this is not the case—nor even that it is only a distinction of degree—for that is the character of most distinctions in Logic; but simply that it is not a sufficiently specific antithesis to be of practical value in classification. I suppose that if the terms were to be employed, we should call those judgments analytic which attain to an adequate explanation or appreciation of a complex whole. The best instances might be the definition or the disjunction, the equation, and judgments passed upon moral and aesthetic value. In all these cases we have the whole completely given in its parts, the identity in its differences, and therefore we are entitled to consider not so much the nature of the whole reconstructed, as the exhaustiveness of the reconstruction. But, as the above instances show, adequacy or exhaustiveness exhibits itself in contents whose nature is wide apart, and therefore it has no convenient place as a general character in a classification.

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[1] Cp. Locke's Essay, II. xiv. 21, on 'Duration'.

On the other hand, as terms belonging to the general theory of judgment, analytic and synthetic are of profound significance. I said at the beginning of this section that every judgment is both analytic and synthetic. This assertion demands no explanation, if we remember our account of judgment as always involving identity in difference. But I will attempt to illustrate its meaning more fully.

If I say 'Caesar crossed the Rubicon', I start with an individual Caesar, whose continued identity extended through a certain space of time and revealed itself in a variety of acts, and I exhibit his identity in one of the acts and moments—its differences—through which it persisted. What I mean by the affirmation is that he, *the* Caesar[1] who had before conquered Gaul, and who was afterwards murdered on the Ides of March, displayed his character and enacted part of his history by crossing the Rubicon. This is a clear case of exhibiting an identity in difference. But the process has inevitably two aspects. On the one hand, I analyse the individual whole that is called Caesar by specifying one of the differences that may be considered as a part within it;

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[1] Contrast Lotze, Logik, sect. 58.

on the other hand, I construct or make synthesis of the individual whole in question, by exhibiting it as a whole that pervades, and absorbs in itself, each or all of its differences. It is only an *individual* whole that is obviously present in *each* as well as in *all* of its differences, as the individual Caesar in the act of crossing the Rubicon. A totality whose unity is incomplete, such as 'all men', is only *implied* in *each* of its differences, and is not *given as* a whole in anything short of *all*. But this does not alter the fundamental nature of assertion. Every judgment exhibits a whole in its parts, and parts as contributory to a whole.

Much has lately been said of Kant's celebrated instance, the equation $7 + 5 = 12$. We have here a total, twelve, which can be compounded in an immense variety of ways, and we display this total as identical, whether expressed by its place in the series of numbers (which implies one and the simplest mode of its formation) or treated as the sum of two other totals, each of which is expressed in the same simple way. It is obvious that if analytic and synthetic were reciprocally exclusive characters, the question 'Is this equation an analytic or a synthetic judgment ?' would be wholly unanswerable. If 12 were not the same number as $7 + 5$, the judgment would not be true ; if $7 + 5$ gained nothing by being defined as 12, the judgment would cease to be a judgment at all. $7 + 5$ is one of the differences which constitute the nature of the total 12, and by *constructing* 12 in this way we ipso facto *analyse* it.

The relation of these two processes, or rather two aspects of the same process, is so fundamental in all knowledge, being in fact *the* relation which especially characterises knowledge as such, that I may be pardoned for continuing to insist on it by help of another set of considerations. The notion of a plain difference between taking to pieces and putting together arises from actual operation on material things. This origin of the metaphor involved in 'analysis' and 'synthesis' has reacted and still reacts injuriously on our conceptions of intellectual processes. In mechanical operations we cannot pull to pieces and put together the same thing by the same act, and which of the two we can do is determined by the material handed to us. If a thing is complete already, we cannot put it together any further; the only alternative then open to us, as between these two processes, is to pull it to pieces; and so *vice versa*. But this feature of material operation cannot be transferred to thought, and for this reason, that the essence of thought is to show the process in the result, and exhibit each as necessary to the other. Therefore, if we construct in thought, the materials out of which we construct have not lost their separateness when the fabric is finished; the fabric as it is still issues from them as they were; if not, we have dropped a link, and our construction is unwarranted. The synthesis, one might say, is based on the analysis; but this would ascribe a false priority, because the fragments supplied to us only *become* an analysis as the synthesis, which relates them to a whole, progresses. Apart from the synthesis they are mere fragments, and therefore are not an analysis of anything. The workman who puts together the parts of a watch has first the handful of wheels and springs, and then the completed watch; he cannot have both at once, and in as far as he has one he has not the other. Moreover, when he has made the watch the wheels and springs are together and are not separate, nor are they separable consistently with the existence of the watch. Synthesis in this sphere is incompatible with analysis, and *vice versa*. But a man who wishes in thought or calculation to construct any instrument out of parts has a very different task. Every element of the handful of parts must have its place and functions clearly retained in the intelligence which constructs the whole; for the whole, as a whole of intellectual synthesis, exists no longer than its parts are clearly apprehended in their relations. 'Yes,' it may be said, 'but the distinction must remain that even in thought you may *either* begin by considering detached wheels, &c., and finding out how they must act in the watch, *or* by looking at a watch and detecting, within its completed system, the separate parts and their relations. The former process is synthesis, the latter analysis.'

This is true so far as judgment or inference is an activity in time, and includes within itself a transition in time. In so far as it has this character, the process of thought can simulate or share the characteristics of material operation. But this does not affect the internal nature of judgment, as I have pointed out in discussing its temporal character. The question is not whether you begin with the whole or with the parts, but merely what sort of whole and what sort of parts you begin with. Given an escapement wheel, I may chance even to be ignorant that it belongs to a watch at all; but none the less I judge of it as a part in a whole, which whole I can at first only think of, perhaps, as 'some piece of mechanism that depends on a catch playing into a delicately toothed wheel'. The further intellectual construction of this mechanism and the ultimate definition of it as a watch, is, according to the views of the passage just referred to, not a transition from *S* to *P*, but a transition from the judgment *s–p* to the judgment *S–P*. We therefore find analysis no less than synthesis to be the internal essence of every minutest section of the judgment or inference in question. In the same way, if a watch is put into my hand with instructions to find out what makes it go as it does, I have primarily a thing in space as the given whole, and indefinite wheels, springs, &c. (which as yet I cannot distinguish by position or characteristic shape) as given parts. No doubt in space all the parts which I shall need to learn are given in position within the whole, and so we tend to describe the problem as one of analysis, in contrast to the other (in which I had to find out or imagine the position of the parts in the whole) as synthesis; and these titles serve well enough as superficial descriptions of certain cases to which judgment and inference are applied—not of *any* judgment or inference as such. But the whole is not, in the latter case any more than in the former, given as an intelligible machine, nor are the parts given within the whole of knowledge because they are within the whole of space. In other words, to see the escapement wheel lying inside the watch does *not* 'give' me this wheel as a part of a mechanical arrangement; to know it as a part of *such* a whole I must understand it; and in understanding it, i. e. in my analysis, perform the synthesis of the watch as a definite mechanical contrivance.

Therefore not only is every judgment both analytic and synthetic, but it is analytic only as far as it is synthetic. It can only be called analytic or synthetic *par excellence* if, by the same confusion that causes the judgment to be regarded as a transition from *S* to *P*, we consider the joint analysis and synthesis of one whole as the analysis or synthesis of another; because in that case we seem to have a fixed and given whole, and to predicate of it nothing but parts, or *vice versa*. In this confusion there is an element of truth. Though *s* must become *S* when *p* becomes *P*, yet *s* has continued identity with *S* and *p* with *P*, and therefore the transition in time from *s–p* to *S–P* does all that could be done by the unreal transition from *s* to *P*. Present me with a pattern *s* which is a tissue of intersecting curves *p*, and when I have analysed it into the thistle design *P*, the pattern *s* is transformed to my eye into a distinct and beautiful design *S*; but *S* is the same that was *s*, and in that sense we have connected *s* with *P*, and we may represent *P* as the analysis of *s*, only not forgetting that it is the synthesis of *S* which is the same as *s*, and that therefore in predicating *P* of *s* we *ipso facto* transform *s* into *S*. And thus the complete understanding of a watch as a mechanical system, expressed in the joint analysis and synthesis *S–P*, may be accepted as happening to involve, *par excellence*, *either* the intellectual analysis of the watch as a given whole in space *s*, *or* the intellectual synthesis of the watch out of given separate parts in space s_1, s_2, s_3, &c. The given whole, or given parts, can be thus allowed to pass as merely whole, or merely parts, because they are not respectively whole and parts in the sense contemplated by the judgment *S–P*, and therefore it does not press home their relation to one another. The watch seems to be from the first a ready-made whole, a round thing *s* in space, which can only be analysed, and not constructed, by the judgment *S–P*. But it *is* further constructed, not as a round thing in space, but as a mechanical system, by means of that judgment.

CHAPTER II

QUALITY AND COMPARISON

I NOW turn to examine specific types of judgment; but in doing so, I must beg leave to remind the reader of the principle which I have laid down as governing any enquiry into a continuous development.[1] What we wish to master is the nature of a process, the scientific history of a function. To do this, we must of course study and arrange its detailed manifestations; mere generalities are valueless. But we need not be disheartened if our subdivisions and specific names are different here and there from those adopted by better authorities, nor even by the possibility (which can hardly be absent from a highly detailed treatment) that we may misinterpret some phase of evolution, or lay down some inconvenient demarcation. If the main problem is thoroughly faced, and the analysis of some chief typical forms accurately conducted, the reader will be in a position to correct blunders and to supply omissions by the light of the knowledge so gained. If we can help him to sound principles and practice of morphology, he will make short work of particular varieties of type.

Judgment, we have seen, is, speaking generally, the intellectual function which defines reality by significant ideas, and in so doing affirms the reality of those ideas. I use the term 'define', because to define implies something given which is to be defined; and it is an essential of the act of Judgment that it always refers to a Reality which goes beyond and is independent of[2] the act itself.

The Judgment of Quality.

1. We will now look at the judgment in its simplest form, which I have ventured to call the Judgment of Quality, and which, with its immediate sequel, the Judgment of Comparison, finds linguistic expression in the Interjectional or Impersonal Proposition.

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[1] See Introduction, p. 15.

[2] On a criticism which this expression has attracted, see vol. ii, ch. ix.

Meaning of Quality.

i. By Quality I understand, not all attributes without distinction, but the unanalysed content of any idea, when treated, in its unanalysed simplicity, as a feature of reality. It may be that all qualities are capable of being analysed into relations; but for our present purpose the question is not whether a quality *can be*, but whether it *is* so analysed. Even the diagrams familiar to us in Euclid, which exist for the very purpose of being analysed, have each its peculiar look or effect, lopsided or symmetrical, solid or slender, circular or bristling with angles. Qualities of shape, however, are as a rule quickly analysed into relations of proportion, so that the commonly cited phenomena of colour and sound give better instances for our purpose, while perceptions which are named, like softness and sweetness, with some reference to pleasure and pain, are the best instances of all. It might indeed be suggested as a definition of quality that it is that aspect of any perception or idea in which it gives rise to pleasure or pain.

Judgment of Quality proper.

ii. The Qualitative Judgment proper affirms a nearly simple content directly of present Reality. An absolutely simple content is indeed an impossibility; every 'red' or 'sweet' or 'pleasant' belongs to some context and includes some differences. But, as I have attempted to explain in the last paragraph, a qualitative content is very nearly simple. The context which makes its difference is the context for the sake of which we affirm it, and is thus presupposed, and not itself affirmed as a further complication. If I exclaim 'How hot!' I do so because the weather or the room is hot, or perhaps if I am feverish, because I am hot for no obvious reason. In all these, no doubt, the content 'hot' belongs to something; it is not isolated from the varied surroundings of my position, but exists in and extends over some of them. But I affirm it without specification, or rather as a first effort to make specification, of my position and surroundings in general, and neglect to analyse its relations, or only analyse by implying that it has some relation or other.

Thus much for the Predication. As for the subject of a pure Qualitative Judgment, there is *prima facie* no assignable subject within the judgment. No ideas are employed to limit the aspect of reality to which the predication refers. The whole of what is perceived at the moment, or more probably some unspecified aspect of it or element within it, is the subject, and it is of this that the content is affirmed. By ' unspecified ', I do not merely mean unspecified in words spoken aloud, but undefined by any such act of consciousness as employs symbolic ideas and tends to call up words. But if thus unspecified, how can the subject be indicated, limited, or selected ? I answer, simply by the concentration upon it of perception or attention, the exclusive gaze which might be represented by pointing with the finger, and which, though it *has* limits, is definitely aware of none. In this sense, confining what is 'present to perception' within the limits of what more especially arrests attention and is taken as 'This' *par excellence*, we might say that the subject of the Qualitative Judgment is always the whole of what is present to perception.

The best illustrations of this rudimentary Judgment are drawn from significant Interjections, or from significant phrases used as Interjections. A distinction must be observed between the true Interjection,[1] the outcry which relieves the urgency of feeling, and the affirmation which refers the content of feeling as a quality to the surroundings that evoke it. The cry of an animal is often significant for us ; it tells us what the animal feels, and why ; but we do not therefore reckon it as the expression of a judgment. What makes the judgment is the idea that exists in our minds and yet that only has truth as referred to Reality. And therefore if we doubt whether we have a judgment before us, we should ask if it is anything that could intelligibly be denied. It is impossible to *deny* the animal's mere expression of its feeling ; and although we may see a reason for its feeling which we know to be an illusion (e. g. if a dog barks joyfully in expectation of being taken out, when he is not going to be), yet there is nothing which we can deny unless we can suppose that this illusion exists in the animal's mind as an idea distinguished from and referred to reality. *If* we could believe this to be so, we should have to admit that the animal judges. But short of this, there is nothing to deny. The dog sees me take my hat and stick, he has a set of mental images connected with going out, and he expresses pleasure. I cannot deny that he has the images or that they give him pleasure. It would be only if he could take these images as standing for something other than themselves, and so distinguish their meaning, a future event in reality, from their existence, as present images in the mind, that anything could possibly arise which I could intelligibly deny. The ideas used in judgment must exist before they can be denied, and therefore their existence cannot be denied, but only the affirmation of their meaning.[1]

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[1] Cp. Lotze, Logik, sect. 48.

Coming to human interjections, we might think that 'Alas!' characterises the present as grievous; and it is certainly sometimes *answered*, though hardly denied. But the answer, even if it takes the form of a denial, is usually rather prohibitive (imperative) than negative; though if self-deception or hypocrisy are suspected, 'alas!' may be interpreted as 'I am sorry', and in that sense denied. The distinction to be kept in mind is here one degree more subtle than in the last case, for the objective content of the judgment is the fact of a feeling in the mind of the person who judges. Thus we have, if 'alas!' is a true interjection, the emotion of grief present in the mind of the person, which merely forces him to utterance by way of relief. But if it is to be taken as a judgment, then we have in any case the symbolic idea of sorrow, *existing* in psychical images, but having its meaning beyond them, and a reference of this meaning to the present perceived being of the person in question. And above all this, if the judgment is true, there must be, as before, the actual felt emotion of sorrow, though if it is false, this element of the complication is absent. It is possible that this peculiar complication, of the idea with the actual feeling behind it, is responsible for the curious duplication of personality which is sometimes experienced in protracted pain or anxiety. The person whom we analyse and judge seems other than the person who all through the process has the feelings which are being analysed, or rather are forcing their disagreeable peculiarities on our attention. 'If *I* were suffering so, how horrible it would be,' we repeat to ourselves. Just as the complementary image comes between our eyes and the sun, so the idea of our feeling comes as the object of knowledge between us and the feeling itself, which remains in the background and resists our successive efforts to include it in an adequate idea. The sufferer remains to us distinct from the person whose suffering we conceive and affirm.

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[1] These remarks are made purely for the sake of illustration. I have not the least prejudice against admitting that animals can judge, if the admission can be warranted by fact, and does not involve truncating the theory of judgment. Domestic animals certainly *seem* to use the imperative, i.e. to insist on the realisation of their ideas. Cp. Bradley, Principles of Logic, p. 33. On an idea without forethought or image, see Bradley, Appearance and Reality, p. 607.

However this may be, when we come to such quasi-interjectional expressions as 'Bad!' 'How ugly!' 'Such pain!' 'Oh horrible!' we are unquestionably dealing with judgments.

The Impersonal Proposition is also a suggestive counterpart of the judgments in question, which have even been treated as Impersonal Judgments. But the impersonal form of sentence has become in developed language so purely a grammatical fiction, that it no longer illustrates with special appropriateness any one type of assertion; although it exhibits a certain coincidence with the range of the concrete existential or singular categorical judgment.

A few instances however occur to us at once—and philology might be able to furnish more—of impersonal sentences that really seem to have stood for judgments whose subjects were not especially designated by means of ideas, but were accepted as merely the given in perception. Such may be 'Methought', 'Him list', 'Mir traümt's',[1] 'Es trieb mich', 'Taedet', 'piget', &c., 'It rains', 'es macht heiss', ὕει. As, in the plastic Greek imagination, Athene may suggest a man's thought, and the actual Dream stand over him in sleep, so, it would seem, in the beaten track of language, the thought and the dream are simply referred to 'it' or to 'something'; to the present reality or to an indefinite element within it. Though the formed verb presupposes a distinct reference of ideal contents to real subjects, yet the habitual use of expressions in which this reference is blunted and neutralised, testifies to a survival, in certain preeminently obscure relations, of a rudimentary type of judgment. This, I think, is the only connection that we can safely assume between the Judgment of Quality and the Impersonal Proposition.

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[1] In Greek, a conservative though flexible language, 'I dreamt,' is never, so far as I know, impersonally rendered: but on the other hand, in Homer the dream is personified. The principle is the same in so far as my dream is not referred to me as my act. Indeed the coincidence is curious with the view of those who have held that the subject of an impersonal proposition is the content of the verb itself as in 'the rain it raineth every day'. It is much the same to say 'Es traumte mir' and to say 'A dream came to me'.

The Qualitative Judgment is the germ and simplest case of the Perceptive Judgment. Perception is a wide word, including, as frequently used, any so-called immediate apprehension; even that, for instance, by which we are supposed to see the necessary truth of one of Euclid's axioms. But if we are to give the term a distinctive logical meaning, we should do well to restrict it to so-called immediate apprehension when dealing with the portion of reality which is in contact with the individual through the senses. Perception thus defined deals primarily with what is present, but extends it by ideas which go beyond the present. When we recognise a man and call him by name, we are said to 'see who he is', *i.e.* to perceive. In this case our perception is expressed by an idea that goes a long way beyond the present and brings in the man's entire personality.

The Qualitative Judgment does *not* differ from the judgment which recognises an individual, by being shut up within a minute interval of present space or time. It is not minuteness of extension or of duration that distinguishes the reference of this simplest case of the perceptive judgment; the qualitative affirmation may deal with what *is* really a considerable area of space or interval of time. The distinction is not one of magnitude, but of definiteness. There is always a risk of construing the absence of quantitative determinateness into determinate minuteness of quantity. This is just what we want to avoid. The qualitative judgment knows nothing at all of duration or of extension, and can have no specified individual for its subject. It is thus confined within the given presentation in so far as the universality, whether abstract or concrete, is absent that alone could extend it beyond. The Reality which is the subject is the given as given, not as a universal that reaches before and after; the content of the predication includes no negative element, summarises in itself no diverse manifestations, and thus neither refers to anything beyond the present, nor in any specific way to the present itself. It is attached to the present, by the mere fact of its actual reference to presentation, not by anything within its explicit content. The first specification, the first establishment of an identity that can be called by a name, is the work of this judgment, and is not presupposed by it. We must take it, I think, after the discussions of the Introduction, that the establishment of a name—a permanent identical symbol of a meaning—must *on the whole* have coincided with the establishment of meanings as such, distinct from psychical occurrences, and capable of being referred to reality. I have attempted in the discussions in question to qualify the rashness of this unverifiable[1] historical allegation, by pointing out that the two co-ordinate processes—the constitution of symbolic ideas and of linguistic symbols—must be regarded as processes of gradual and unconscious adaptation, widely differing from the methodical extension of nomenclature according to modern ideas. But the logical track is the same, whether the historical evolution is quick or slow, conscious or unconscious.

Perception as above defined would include two species which have been called respectively the 'Analytic' and the 'Synthetic' judgments of sense. If these distinctions are to be seriously treated, and we are to speak of any judgments as merely analysing a presentation of sense, without going beyond what is given within it, then, I think, we must identify the Analytic Judgment of sense with the Judgment of Quality. It appears to me quite idle to treat a description of an ordinary scene, such as 'The blacksmith is at his forge mending the ploughshare', as a case of a judgment confined within *present* perception. Every element of the description is a concrete individual, including innumerable differences, involving elaborate categories, and extending indefinitely into past and future. If ever there was a constructive or synthetic judgment, this is one. It would surely be more appropriate to treat these common perceptions, which we deal with lower down, as Synthetic judgments of sense, because they interpret what is given by ideal contents that go beyond it. And then we might reserve the title 'Analytic Judgments of sense' or 'Judgments of Quality' for the activities represented by the Interjectional and Impersonal propositions of which I have spoken, and the true Demonstrative propositions of which I shall speak below.

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[1] Of course there can be verification by analogy; and it is hard to draw the line between this and actual verification. The characteristic cries of animals and children have significance for us, and we cannot suppose that they do not react upon the intelligence of those who utter them. It would be far easier to understand how animals should acquire language, if, like children, they did so in fact, than it is to understand how they stop short of it.

The difficulty of identifying simple forms of judgment is intensified for modern reflection by the definite and diverse articulation of the elements of modern speech. The languages which embody the reflective thought of Europe, both ancient and modern, resist the expression of elementary perception in two ways. First, for a germinal thought we need a germinal word. But the languages which mould our ideas have no germinal words. Every word, in the languages of European culture, is a particular 'part of speech'. That is to say, it is adapted to fulfil some one function in a sentence, whether substantive, verb, pronoun, or conjunction, and, if used alone, has an air of incompleteness which forces us to 'understand' supplementary words. The existence of the Judgment of Quality is but slightly corroborated by the fact that a single word often conveys a judgment. Many such single words are conventional symbols for quite definite sentences. But the judgment of which we have been speaking corresponds to a whole sentence in the bud, with its differences unevolved. How far philology could furnish authentic representations of the sentence in such a stage—words which are not '*parts* of speech', but entire though undifferentiated *units* of speech—it is beyond the limits of the present work to enquire. Even the verb of ancient Greek or Latin, which required no supplementary pronoun to represent its subject, is one degree more appropriate for the purpose than any element of modern speech.

And the second difficulty is really a case of the first. Most judgments are expressed by help of a verb, and if we employ a substantive or adjective alone, it urges us to 'understand' a verb. But a verb is in our languages above all things a *tense*; and for a rudimentary judgment like the judgment of Quality, a tense is exactly what we do not want, least of all the elaborate duration-tense of the present (τρέχει, amat, 'he is acting'). We want to affirm neither duration nor yet point of time; we simply want to qualify the given by a content, without specific limitation or extension. It is true that the logical present, the absolute present of the universal judgment, marks no limitation of time, and it depends on the nature of the content involved whether universality of time as an infinite series is asserted in such a case. But whether it claims universality in time or by negation of time, such a judgment implies the conception of time as an abstract whole and is posterior to this conception, while the reference of which we are speaking is prior to the origin of the systematic idea of time.

These properties of developed language,—and prior to developed language it would scarcely be possible to have analytical reflection,—may be compensated, but cannot be cancelled. The fact that sometimes thought is behind language, and at other times struggles to pass it (it would be hazardous to complete the antithesis by saying that thought can really outstrip speech) is a fruitful source of misinterpretation. Children learning to speak, or savages learning a European tongue, are like the wizard's apprentice uttering a spell; they are incapable of grasping the significance or controlling the effect of their words. And all human beings perpetually oscillate between limits, different in every case, on the scale of intelligence; for not only is the student's or politician's world of thought very different from that of an illiterate man, but every man varies in the level of his intelligence according to momentary conditions of interest and capacity. The same difficulties of interpretation which are found by a student in the speech of a child or Anglicised savage, subsist in a less degree as between every man and every other, and as between every man and himself.

Up to this point the constituent elements of the Judgment have been naked before us. There was the Subject,—the actual contact in which reality pressed upon our sense-perception,—and there was the significant idea by which we defined it. It is this case that affords the strongest support to the view which denies all meaning to the distinction of Subject and Predicate, as a distinction of elements within a judgment. The Reality to which we ascribe the predicate is undoubtedly self-existent; it is not *merely* in my mind or in my act of judgment; if it were, the judgment would only be a game with my ideas. It is well to make this clear in the case before us, for in the later forms of the judgment it will be much disguised. Still the reality which attracts my concentrated attention is also within my act of judgment; it is not even the whole reality present to my perception; still less of course the whole self-existent Reality which I dimly presuppose. The immediate subject of the judgment is a mere aspect, too indefinite to be described by explicit ideas except in as far as the qualitative predication imposes a first specification upon it. *This* Reality *is* in my judgment; it is the point at which the actual world impinges upon my consciousness as real, and it is only by judging with reference to this point that I can refer the ideal content before my mind to the whole of reality which I at once believe to exist, and am attempting to construct. The Subject is both in and out of the Judgment, as Reality is both in and out of my consciousness.

The demonstrative Judgment.

iii. We have now to consider a slightly more definite type of judgment, which we may still rank among the judgments of quality, although we can detect in it the beginning of a further growth.

When we say, '*This* is hot,' '*Now* it is raining,' '*Here* it is dark,' the demonstrative pronoun or particle designates the point in given Reality to which the affirmed content is to refer. The point is designated, but seems, *prima facie*, not to be described. The demonstrative has a meaning, no doubt, but its meaning seems to consist of a mere reference to what is presented before perception, and therefore does not seem to introduce any abstract limitation that qualifies the given subject. Compare, for instance, 'This is hot' with 'This metal is hot'. The latter judgment may possibly be met with 'This is not metal at all', and by such a reply the judgment is cut in two, and the more significant half becomes a conditional assertion whose condition does not apply to the 'this' in question. But '*This*' *alone is on* a different footing. You cannot say, 'There is no *this* at all.' There is always a *this*, as there is always a *that*; and the same applies to *here*, *there*, *now*, and *then*.

One of these demonstratives indeed appears at once to take us over a boundary. 'Then' requires a past or future tense in the predication; and in referring to the past or future we get beyond analysis of the present, which is the province of the analytic judgment of sense. The problem is one of real importance, but its point is not where we are most likely to look for it. It is not that the Judgment of quality refers to a point of time, and that 'then' takes us outside this point; it is that the Judgment of quality is prior to the idea of duration, and that we have now introduced the idea of duration definitely into the subject. The effect on tense, which happens to be the vehicle of predication, produced by taking 'then' as the subject, calls our attention to the fact that every present includes a past. The contrast of 'here' and 'there', not happening to affect the verb, did not force on our notice the equally real universality of the present in space. Every 'here' is made up of 'there's' as every 'now' is made up of 'then's'. And thus in designating a given subject as 'now' or 'here', we have unawares included in the subject a 'then' and there', and by introducing universals of space and time have set our faces to leave the region of the qualitative judgment. The demonstratives stand for ideas, and it is therefore *through ideas* that in the judgments now in question we refer to reality. But the demonstratives have the peculiarity that *their application* cannot be denied, as can that of a determinate idea such as metal, and therefore, though they characterise the given

Reality as appearing in space or time, yet they can specify no nexus, introduce no condition, which may be absent in fact, and through its absence may save the judgment from falsity by rendering it inapplicable. If heat is not present in the 'this' to which it is ascribed, the judgment 'this is hot' is false without reserve; but if to 'this' we add 'metal' then the absence in fact of the condition 'metal' makes the whole judgment ambiguous and inapplicable. Thus we have in the demonstrative judgment of quality, as in the simple or pure judgment of quality, a perfectly categorical judgment; the Subject must, in its nature, exist, and the Predication must therefore be alleged to hold good of actual existence. It is noteworthy that of this perfectly categorical judgment we cannot say whether the existence of the Subject is affirmed or presupposed. Where we are dealing with the given *qua* given, the difference between affirmation and presupposition has not emerged.

The Judgment of Comparison.

2. The contrast between 'now' and 'then' suggests to us the consideration of such judgments as are expressed by 'Now it hurts less than then', 'This is redder than that', 'Here it is hotter than there'. The form of these judgments indicates as their appropriate title the Comparative judgment, or the Judgment of Comparison. They arise naturally out of the Demonstrative judgment of quality, because, as we saw in the case of 'now' and 'then', it is impossible to prevent the present subject from revealing differences within itself. 'This,' as more clearly defined, will display itself as a part, 'not-that' within 'this and that', 'now' as a part 'not-then' within 'now and then,' 'here' as a part 'not-there' within 'here and there'. The whole, when thus resolved, displays differences of quality between its parts, or rather the given reality reveals itself as a whole for the first time when it breaks up into parts united by an identical but varying quality. Even if we forget that the 'this' and 'that' ever entered into a single whole, yet the identical quality because of which we compare them contains in itself the essential of the comparative judgment, viz. the *explicit recognition* of difference in identity. 'Redder,' 'hotter,' 'less painful,' are terms that go beyond mere

quality by introducing the conception of more and less, that is to say, the beginning of quantity. I give instances, bracketing the explanatory words which would be superfluous in presence of direct perception, and which belong to a higher level of judgment than that which we are discussing. 'This [paper] is green, and this [part of it] is lighter green than that.' 'Now [all to-day] it[1] hurts less than it did [yesterday]; but [just] now it hurts more than it did [a moment ago].' But in cases like these we are apt not to notice that we are predicating differences within a single identity, the green paper, or the whole of to-day; though we must be aware that we imply them in the comparison of quality. And therefore, having pointed out the underlying character of such a simple analysis as the above, I will pass at once to a case which is one degree more complex, but which displays the essence of comparison beyond possibility of mistake. I refer to the case in which a given whole of perception designated by one demonstrative has parts distinguished within it by means of the others, and differences assigned to it conditionally upon these distinctions. There is no difference of principle between defining 'this', within 'this and that', and defining it within 'here and there'. The only advantage is that a demonstrative of another kind is more readily taken as a condition, while one of the same kind is apt to be understood as a jump to a wholly new subject. We will therefore merely change the *parts*, which might be new and substantive wholes, into *conditions*. '*This* is redder now than it was then.' 'This is hotter in this part than in that.' 'This [green paper] is lighter here [in this part of it] than there [in that].' These instances clearly show the primary datum revealing itself as a whole with parts distinct yet bound together by a common quality.

Quantitative Comparison.

i. Let us now examine the essential nature of the comparative judgment in one of the above instances. It is not my intention to enter upon the niceties of quantitative comparison at the present stage.

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[1] The 'it' in this instance is on the verge of introducing an identical subject. I did not, however, mean it to stand as a definite subject, but merely as the grammatical complement of the impersonal verbs.

We will take the instance, 'This is redder now than it was then;' or in the simpler form, 'This is redder than it was.' We will take the Predication first, and then consider its reaction upon the subject.

That which is redder is also red. The red and redder are both red, and yet differ from each other, not, or not merely, in other ways, as in time or place, but in respect of their redness. It has been sufficiently insisted on that there cannot be difference without some identity, as for instance a red and green surface are identical in respect of reflecting light. But these, though the same in as far as they reflect light, are not the same in the light which they reflect. There is a break between the two colours, considered as colours,[1] which nothing can bridge, and the immediate perception of their discontinuity supplies the terms which indicate the difference between them. The surfaces which are both red, but one redder than the other, are separated by no such break. If one changes into the other, it does not cease to display the same quality that it displayed before. But a quality that changes, and yet remains the same quality, has passed into quantity, which might be defined as difference, not merely *in* identity as its meeting point, but consisting *of* identity as its material.

Thus the fundamental identity and difference of judgment are specified by the comparative judgment as whole and parts in the simplest form of that relation: viz. the form in which the whole differs from any part by an interval which consists of other homogeneous parts. Parts in this sense differ from units only by lack of precise comparison; but *precise* comparison is posterior to the conception of a whole, of which we are just considering the first establishment. We do not measure or count until we know of some totality that requires definition by these processes.

The reaction of comparison upon a simple subject indicated by a demonstrative, that is, on a mere spot or point upon which perception is concentrated, is an essential step towards the recognition of an individual totality. The present of space or time is as we saw in its nature continuous. Therefore the spot or point on which perception is fixed, and which we indicate perhaps by 'this', will undoubtedly exhibit differences under analysis. Such analysis is brought to bear by the judgment of Comparison. The spot or point[1] in which a change of degree is observed forces itself on by that fact from being a mere spot on which the eye is fixed to the first stage of individuality as a synthesis of differences. Change is not necessary to this result, though negation in some form is. The observation of parts differing in degree within the spot which we have in view is as effective for the purpose as the detection of succession within the time which we call 'now'. The mere spot fixed by perception begins under such analysis to assume the character of a Thing; and by a parallel process, the distinctions of Time and Space begin to emerge as parts within homogeneous systems.

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[1] It is possible that, considered as amounts of light, red or green may share a continuous element and so have quantitative relation. If so, this is another case in point. For the difference is then in respect of the characteristic which forms the identity.

It is obvious that such a point in the evolution of thought would correspond to the first distinction of Noun, Finite Verb with Tense system, and the more elaborate spatial and temporal adverbs and prepositions. But it would not be fair to test the correspondence strictly by negative instances. Language fits thought as a very loose glove; and if it were the case that we could find several languages in which our familiar parts of speech, more especially the tense system, do not exist, we should still no doubt find that the distinctions, whose origin we are examining, are represented in some other way than by linguistic signs, or are thought even if not represented. It would be ridiculous to contend that the Chinese do not think of self-identical and independent things, even if it is true that their language has no special class of nouns substantive. Nevertheless, the contrast of development between different languages has or has had its meaning; and it appears to me absolutely impossible that a people whose sole language was Hebrew could have had the accurate consciousness of time as a system which came easily to the Greeks and Romans of the classical age. I believe, indeed, that the origin of the Aryan tense-system is not beyond the ken of philology, and that its probable history reveals an evolution much like that which has been here suggested—a transition from simple unspecified reference, to reference differentiated by a temporal system.

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[1] It is hard to escape the dangerous pitfall of speaking as though there were no perception but sight, and therefore as if the germ of the judgment were always fixation of the look in space. The focus of attention may operate through any sense, and is characterised at any moment by that identity which the judgment makes explicit. But the identity is referred in rudimentary judgment not to a special content as subject but to what could only be paraphrased as 'That which engrosses my attention', the present feeling which the judgment determines into thought.

'This,' then, as the subject of a continuous Quality including differences tends to acquire an individual name. I have pointed out in the Introduction that the process of Naming in a world distinctly organised by knowledge cannot be that which belongs to the unreflective epochs of thought. A natural name must be a petrified description. The linguistic element which stands for the content of the Judgment of Quality is already a name. And some such element, in the simplest case perhaps that element itself, will emerge in later forms as a description of the newly distinguished individual, which in the Judgment of Quality is only known as 'this'. 'This red (leaf) is redder than it was.' And when the individual is once revealed as a whole with parts by this judgment of Comparison or synthetic judgment of sense, the ascription of other differences to it cannot but follow. It seems obvious that an adjectival appellation, or at least an appellation of unspecified grammatical class, would come first, and the hardening into a substantive be a later process. I incline to think that the hardening of a description by usage, and the isolation of its elements by employment in different judgments, must have been the real and natural process of naming.

Comparison in Space and Time.

ii. It is evident that judgments which assert distinctions of Space and Time, without proceeding to measurement by units, must be ranked among comparative judgments, or, as these might otherwise be called, judgments of continuous quality. It is not the business of Logic to analyse the means by which the consciousness of extension or succession is obtained. Logic only deals with the nature of such a consciousness, and not with its psychical genesis. But we cannot entertain a doubt that position in Time or Space *can only* be indicated to consciousness by qualitative marks that fall outside the content which is perceived as in Time and Space. Our inability, in many or most cases, to detect these marks by immediate observation (I have never been able to analyse my seemingly direct perception of the quarter from which a sound comes), cannot, I think, outweigh the possibility of showing other means by which the eye can judge distance, the ear direction, and the memory recall a series in its serial order, and no other.

The only logical importance of this psychological analysis lies in its confirmation of the idea, suggested by the facts of language and the very nature of quantity, that Space and Time must imply qualitative discrimination as an element of quantitative comparison. Nearer and further must be qualitatively distinct spatial perceptions, as red and redder are qualitatively distinct chromatic perceptions. Thus, the abstract totalities of Space and Time have their germ in comparisons effected by perception co-ordinate with the perception of continuous quality and of its differences.

But the demonstrative judgments have forced upon our notice a further and peculiar distinction within space and time as continuous qualities, which is known in space as difference of direction, and in time as difference of past and future. 'Nearer' and 'further' are different spatial perceptions; and it is possible that 'this' and 'that' may be naturally equivalent to 'nearer' and 'further', as 'here' and 'there' must be, or must soon have become.[1] But besides 'this' and 'that' we have in space the distinction of 'that' and 'that', 'there' and 'thero', just as in time besides 'now' and 'then' we have 'then' and 'then'—a distinction which may apply to points equally removed in past and future, and therefore cannot be reduced to quantity, i. e. degree of remoteness. 'That' and 'that', 'there' and 'there', imply no difference or remoteness, nor indeed do 'this' and 'that' necessarily do so.

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[1] Even if, prior to a spatial distinction recognised as such, there may have been a less defined distinction as between 'by me' and 'not by me'.

All these may just as well mean on the right hand and on the left as nearer and further. They must indeed *be* comparable in distance, but it need not be distance that furnishes the distinction between them. With other qualities the case is different. There cannot be two different reds that match; all reds that match, i. e. that are 'equal', are the same. And if there can be two or more different musical sounds that have the same pitch, this is because the distinction between them is one of kind, not of simple quality, because, that is, they are composite perceptions which are estimated with reference to one element within them taken as dominant or essential.

Thus it seems that (1) Space and Time appear in the germ as mere qualities whose continuity is displayed in the judgment of comparison like that of any other qualities. If apprehension of Space and Time really depends on 'local' and 'temporal signs', we must suppose that the peculiar definite externality which characterises extension could only appear by degrees, and that perception must have been transformed from perception of contents *plus* local signs, to perception of contents *arrayed in extension by means of local* signs, that the *logical* character of spatial perception, its continuity and homogeneous differences, must have been present from the first in any system of apprehension which could develope into our spatial world. The judgment of continuous quality admits of this.[a] And (2) the distinctions which first present themselves within a given spatial or temporal perception are not simply and solely differences of quantity, though in time more so than in space, and in both capable of quantitative expression. They are more analogous to differences of kind. We must therefore take the spatial and temporal demonstratives, apart from explicit quantitative comparison in space and time, as in themselves comparative contents involving continuous quality.

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[a] See Professor Stout, Mind, 77. 7, 'The essential fact is that colour presentations and touch presentations, both as actual sensations and as images retained or revived, are diffused in a continuous quantum within which it is possible to distinguish parts outside of each other, beside each other, and between each other.' I am sure this is right. Then the question is whether the local sign theory is consistent with it. All that concerns us in logic is to maintain that the quantitative differences have a qualitative basis, both of identity and of distinction.

The divergence from the main progression of the judgment, by which space and time are erected into totalities having a special structure of their own and a peculiar mode of existence, must be taken as beginning with the resolution of a given 'now' and 'here' in the judgment of comparison. In this resolution we have the two grades, corresponding to distinctions of direction, or to the distinction of past and future, and to quantity respectively; first that in which 'this' and 'that' may be e.g., to left and right (not nearer and further), where the spatial comparison is implied rather than expressed, corresponding to the judgment, 'This is red, and that is green;' and, secondly, that in which a true spatial comparison is introduced into the content, as in 'This is nearer than that', which is analogous to 'This is redder than that'.

So-called Qualitative Comparison.

iii. Under the head of Comparison it is usual to treat of like and different in kind, as co-ordinate with more and less. But it is to be observed that such a co-ordination is not accurate. Mere qualities, as such, are disparate, incomparable with each other. The judgment of quality pure and simple, as we have seen, excludes comparison. This is red, this is black, this is golden, this is sweet, this is sour. These are successive and isolated judgments of quality; and the semblance of comparison which they now bear is due exclusively to the advanced point of thought at which language places us to begin with.

Comparison of degree, as we have seen, includes difference or elementary negation within the limits of a single quality, but the differences themselves, in regard to the aspect which makes them distinct, remain disparate or incomparable. In other words, every part of a quantitative whole is distinguished by a peculiar quality as well as united with the rest by an identity of quality. Every shade of red, besides being a degree of red in general, is also a particular hue and produces a distinct impression. Every perception of warmth is qualitatively peculiar, and often it is not without an effort that we can recognise the character in respect of which it can increase or decrease. Every inch in a yard measure, every cannon ball in a heap, in thus distinguished; and if it were not so, the parts would have no stability and the quantitative whole would cease to exist.

We therefore are driven to conclude that quantitative comparison is not *prima facie* co-ordinate with qualitative, but rather stands in its place as the *effect of comparison on quality*, which so far as comparable *becomes quantity*, and so far as incomparable furnishes the distinction of parts essential to the quantitative whole. It is with this latter aspect in which qualities are incomparable that qualitative comparison as such must be connected. It is thus, *prima facie*, comparison of the incomparable. Any two shades of red, regarded as shades of red, are respectively more and less. But they must also be, as we have insisted, *different reds*, and if regarded simply thus are pronounced incomparable as a result of comparison. If we pass, as we can, by slow transitions, along the complete solar spectrum, comparing each colour with that which formed our starting-point, we shall arrive first at differences which may, and then at differences which must, be thus regarded. The difference between red and green, for instance, is not to ordinary perception a difference in the same quality; and if it can become measurable, can only become so by reference to an identical quality, such as brightness of illumination, which falls outside the peculiarities of red and green as such.

I believe that it is futile to attempt the measurement of difference except in respect of a continuous quality. And the mere affirmation of difference, without the attempt to measure, appears to me absolutely devoid of meaning. The mere judgment, 'These two colours,—these two sounds,—or, these two perceptions (a colour and a sound), are different,' is an imperfect and unreal judgment, which in this form, and apart from a meaning which I shall explain below, is as I believe to be found nowhere but in logical text-books. It may best be considered as an incomplete quantitative comparison, in which the parts are distinguished, but their place in a continuous whole has proved impossible to determine. In-this sense, the mere judgment of difference would mark the initial effort of quantitative comparison. Such a relation is illustrated by the well-known fact that qualitative difference, e. g. between two musical notes, is perceptible before its quantitative nature (their relative pitch) is ascertained. *Such* a judgment of qualitative difference may be regarded as a first determination of quantity; for its point is merely to deny identity of quality, and in matters of simple quality to be identical is to be equal and *vice versa*. In this sense the judgment of qualitative difference, such a judgment as we make when we see that two colours do not match, is an aspect of the initial stage of quantitative comparison.

But another judgment of sameness or difference, which it is almost impossible to avoid confusing with the above, has its true place in classification and analogical inference, and, if explained as mere qualitative comparison, is an unreal fiction. Such judgments are: —'These two instruments are not in tune;' 'Gladstone and Chamberlain are very different men;' 'That victory is uncommonly like a defeat;' 'The globe-flower is just like a hellebore, only it is yellow.' Here we are not speaking of an immediate qualitative identity and difference, but of essential and dominant qualities or rather attributes, in other words of differences valued by a presupposed standard or purpose. The idea of a standard involves the idea of kind, and kind goes beyond quality. Therefore, we come to a conclusion which I think frees us from much sham accuracy and pretended precision. The judgment of difference is never made apart from a standard of difference. The apparent exception, when such a judgment denies identity of quality, is simply the first step in quantitative comparison, and it is by quantitative comparison that precision must in such cases be obtained. But the class of judgments from which our later instances are drawn do not refer to identity of *quality*, but identity of *kind*. They presuppose classification, and affirm difference or likeness with reference to this classification. All attempts therefore to introduce a quantitative estimate into these generic judgments of difference are founded on a confusion between judgments of quality and judgments of kind, on an attempt, that is, to reduce the latter to the former. This does not deny that the latter may imply the former in addition.

It is a futile introduction of psychology into logic to speak of measuring difference by the difficulty or duration of a psychical transition; the measure of the difference is what we mean by the difference, and what we mean by it depends on the series or classification within which we affirm it. Apart from such a standard the judgment of difference is nonsense; it becomes like 'The soul is not square'. How idle to inform us that Gladstone and Chamberlain are different! How superfluous to affirm that one plant is like another! Any assertion like these, if it is not referred to a ground of distinction, in these cases to political and botanical classifications respectively, is as destitute of content as a bare negation. The instance of two instruments pronounced out of tune with each other may seem not to be in place under this head, and to be a judgment of true qualitative comparison. I inserted it expressly to indicate the line of demarcation. Different colours are such as do not match, i.e. are not discernible in simple quality. But in comparing musical notes we have not to do with simple quality,[1] but with dominant quality,[1] i.e. kind. 'A note' is identified by its pitch, and different notes are sounds differing *in pitch*. Therefore in pronouncing notes to be different we do not merely deny that they are indiscernible; we deny something further than this, we deny that they are indiscernible *in their dominant quality*, viz. their pitch. That the pitch is itself a quantitative attribute makes no difference; for it is a quantitative attribute, which has become characteristic, and therefore stands logically in the same position as any other basis of classification.

In short, then, we must not confuse quality and kind. Kind is dominant or characteristic quality and involves a series of ideas which we have not yet discussed. And whereas comparison in respect of simple quality is prior to and absorbs itself in quantitative comparison, comparison in respect of kind is subsequent to quantity and involves other ideas. An *isolated* judgment of difference can have no meaning except as the first stage of quantitative comparison, the negation of identity (= this and that are unequal). The attempt to assign gradations to the mere judgment of difference rests on a confusion between quality and kind, each of which has in itself an adequate and objective principle of measurement independent of psychical transition, and in the case of kind, incapable of reduction to quantity.

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[1] I use quality in the logical sense, in which it includes timbre (musical quality), pitch, and loudness.

CHAPTER III

MEASUREMENT—QUANTITY AND PROPORTION

Measurement and Individuality.

1. MEASUREMENT is the equation of any whole, by comparison, to a numerical aggregate of determinate parts. The parts may be determinate through reference either outside or within the whole to be measured; but if the reference falls within it (as when we say a man's whole height is so many times the length of his head) the whole must be complex and contain subordinate systems. The reference may also take the shape of relations which are not purely quantitative (as a tone or semitone in music, apart from its physical cause, is simply a difference between two peculiar sounds); but in that case we are passing out of the region of pure measurement. Some reference, however, there must be in measurement beyond that to the *simple* whole which is to be measured. It is no measurement of a line to divide it into 100 or 1000 equal parts. We must know *what* else they are parts of, besides being parts of the line to be measured. The length of a line is measured when it is equated to feet and inches—to the length, that is, of some actual piece of metal agreed upon as a standard—the pitch of a note is measured when we have determined its place in the scale or the number of vibrations per second that enter into it; the specific gravity of a substance is measured when we have stated the ratio of its weight to that of an equal volume of distilled water at a certain temperature. Here a verbal difficulty may be cleared away. If the weight of the substance before us is twelve times that of water, our definition of measurement applies straightforwardly. We equate the whole substance in respect of its weight to a numerical aggregate of twelve parts, each of which is determined by equation to a known volume of water.[1] But if the substance is $\frac{1}{12}$ of the

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[1] Practically of course we do not heap up volumes of water in one scale till they balance a substance in the other scale. The process

weight of water, we seem not to be breaking up the whole which is being measured into an aggregate of parts, but to be representing it as a part within another aggregate. This is a mere matter of practical convenience. The equation of a whole to a numerical aggregate is as much involved in the expression $\frac{1}{12}$ (a twelfth part), as in the expression 'twelve times'. In measuring, we bring two terms into precise equation, and the entire relation of whole and part is involved in each. The numbers in which the same part enters into two or more wholes are in every case the organon of measurement.

Simple measurement. Quantity.

i. Measurement is to begin with Simple Measurement, resulting in pure Quantity.

Simple Measurement consists in judging of a perceived object that it is a whole containing a certain number (one or more) of a determinate unit. Measurement is thus a development of comparison, which is the first revelation of the unit, or equal part—the result of successful equation—within continuous quality. Simple qualitative identity, for instance, such as that of colours which match, may be set down to comparison or to measurement according as it is or is not ideally referred to a scale of degrees. In pure qualitative identification we have sometimes no idea of possible degrees, and such identification must be regarded as the earliest germ at once of measurement and of comparison. Thus 'The taste of this is the same as the taste of that' is mere identification or comparison if it only means that the tastes of the two things are indiscernible, but is measurement if we are considering whether the one taste is sweeter than the other. We have seen that qualitative identification *is* only the germ of quantitative comparison, and that the two are not co-ordinate.[1]

How is the unit fixed? It is fixed, as we have seen, by equation or identification of it as the same throughout the various wholes or aggregates into which it enters as a part.

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employed is equivalent to weighing first the substance, and then the water which it displaces, against known standard weights, and taking the two results as a ratio; i.e. by help of the balance we state the two things to be compared in terms of an aggregate of already known and determinate parts; one thing will = 12 oz. and the other 1 oz.

[1] p. 116 ff.

This process of equation tends to repeat itself *ad infinitum.* Pure quantity is an essentially relative attribute. Hence in Simple Measurement the paradox of knowledge takes an extreme form, for every measurement presupposes and provokes others *ad infinitum*. The tables of weights and measures of our arithmetic books are enough to illustrate this. They are long lists of equation after equation, by means of which all objects that are measured or weighed are ultimately equated with some single portion of matter or relation in nature,[1] arbitrarily selected as a basis of the division and multiplication that facilitate comparison. The substitution of a determinate physical motion supposed to be constant for a particular portion of matter makes no change of principle so long as it is taken *qua* a term in a fixed ratio or mere ratio, and not *qua* a term in a generalised ratio or proportion. But in fact the two ideas are at bottom inseparable. Everything fixed is *qua* fixed, potentially generalised. The wave-length of what is a particular red to a normal eye does not vary (so far as I know), but this is so to speak an accident; and if we take in non-normal eyes, all that is certain is that this wave-length preserves its place in the colour-series above some wave-lengths and below others. Its fixity for red judged by the healthy eye has however caused it to be suggested for an unit of length. Weight—the relation of a portion of matter said to weigh 1 lb. to all other portions of matter in respect of their gravity—becomes a generalised relation or proportion the moment we consider distance from the earth's centre. '1 lb. at earth's surface : 2 lbs. at earth's surface : : 1 lb. five miles up : 2 lbs. five miles up.' When a thing is described generically by the number of its own parts this is more obvious, for the ratio is then *ipso facto* generalised by the mere recurrence of individual things. But simple measurement of perceived objects gives simple ratios expressed in singular judgments with external reference. As the judgment becomes general the ratio becomes first formally proportional, because the ratio is generalised as against instances in which it occurs, and then really proportional, because this generalisation comes to apply to cases in which the corresponding terms are *different magnitudes*. The ratio between measures[1] is *formally* proportional; that between weights, explained as above, is *really* proportional.

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[1] Such as a fraction of the earth's circumference. But any such relation is likely to be variously determined at different times, while it is not convenient to alter the basis of a system of measures. I believe that practically all systems of measures depend upon the actual material standard, which as a mere piece of metal, not capable of being tested by any general relation in nature, must be reckoned as a purely arbitrary standard.

Complex or mediate measurement. Proportion.

ii. Thus measurement necessarily becomes complex, ideal, or mediate; i. e. in short generalised. In this aspect it first appears within the singular judgment, and then breaks loose from it.

Every relation established by measurement is a ratio, or relation between magnitudes; and as incommensurable magnitudes are *for logic* a contradiction in terms, every ratio can be expressed in so far as it is a true ratio (in so far as its terms are magnitudes) by a relation between two numbers. Number refers the relation to an abstract whole of quantity, and therefore determines the identity of the relation by its place in a self-identical articulate system. But the effect of such expression is to generalise, while the results of *simple* measurement can warrant no generalisation, and therefore are not spoken of as a ratio, and often not reduced to their simplest numerical expression. Nothing would be gained by saying that a given plant 5 ft. high had a height which was to 1 ft. as 5 : 1, nor by judging that *this* piece of lead is to *this* piece of gold in weight as 11 to 19. The ratios so affirmed would still be destitute of general significance, would be mere facts, alleged of a particular reality present in perception, and would therefore gain nothing, but might lose their truth, by abstract expression.

But thought always tends to coherence and necessity, and we cannot even employ a determinate idea to assist us in pointing out an object of perception without creating the impression that the ideal content which we use will be characteristically connected with the content of our predication. When this occurs in the judgment of measurement, ratio passes into proportion ; that is to say, the ratio enunciated as true of the particular given subject becomes a universal rule applicable to all variations compatible with the determinate idea which conditions the subject. Such measurement is complex, because the unit on which it is based is no longer single and fixed, but variable in absolute magnitude, though determined by a condition. It is ideal, because no longer a mere fact of sense-perception, but enunciated as flowing from a content intellectually defined. It is mediate, because the reference to reality which constitutes this as every judgment, is not direct, but has to pass through a condition before it can attach to reality.

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[1] By ultimate refinement, this too is *really* proportional. Suppose all measurable things to expand and contract preserving their ratios, we could never know it. Given a foot-rule, we could still construct a yard-measure, however the absolute length of the foot rule might vary.

Therefore if we take one of the above simple measurements, and even without removing the demonstrative 'this', insert a determinate condition into the content, we shall find that the whole affirmation is greatly modified in its nature. Let us judge that 'This piece of lead and this piece of gold, *being of the same volume*, are found to be in weight as 11 : 19'. Then we at least suggest the erection of the ratio 11 : 19 into a law of proportion : 'The weights of equal volumes of lead and of gold are as 11 to 19.'

So again, we may have simply counted the leaves on a plant-stem, going round the stem in the same direction, till we find a leaf immediately over or under that from which we began. If we do not know that these ratios are characteristic, or if we suspect the plant to have been injured so as to make its ratio undiscoverable, we may simply judge, 'On this plant I find five leaves in going twice round the axis.' Whereas if we insert the name of the plant or tree, and use an abbreviated expression for the ratio of divergence, we at least suggest the idea of a characteristic law of the leaf spiral. 'This oak shows a divergence of $\frac{2}{5}$' (i. e. the sixth leaf is directly over the first, and in counting from 1 to 6 you have gone twice round the stem). It is obvious that here generalisation and characteristic attribution have begun. Granting that we have not yet any assertion about the *genus* oak, or even about the species in question, for the 'This' hinders such an interpretation, yet we unquestionably are awakening to the expectation that the tree before us will present the ratio in question in all its different parts and from time to time, and to the problem whether and how far we may drop the 'This' which indicates particularity. When the 'This' is dropped the judgment ceases to be singular, unless it is attached to a proper or in some way individual name. Whether and how far, failing these refuges, it ceases to be categorical and becomes hypothetical, or how far individuality continues or even revives after the loss of particularity, as a factor in the evolution of thought, is a question which will frequently occupy us in the sequel.

Characteristic ratio, or proportion, may refer to standards external to the whole which it qualifies, or to relations within that whole. In the former case the qualitative quantity remains subject to relativity and eked out by equations *ad infinitum*, hardly less than in the case of simple measurement. In any table of specific gravities, for instance, we have a number of substances each severally characterised by the ratio of its density to that of distilled water at a temperature of about 39.1° Fahrenheit. Now in the first place, each of these several ratios may obviously be regarded as a proportion in so far as it applies without variation to any volume of the same substance, in the sense that the weight of *any* volume of any substance is to the weight of the same volume of water[1] in the ratio of the specific gravity of the substance. Proportion is defined as 'equality of ratios'; and equality of ratios obviously is identity of *the ratio*, and exists between every ratio and all cases in which it applies.

But in the second place, although in these cases we have proportion, yet we have also relativity *ad infinitum*. Select the specific gravity of some one body, and suppose that of all the rest except water itself to be erased from knowledge; the significance of our one fragment of information would then be all but gone, were it not for the accessory idea, which we cannot now get rid of, that the rest could easily be recovered.

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[1] I omit for the sake of brevity in this and parallel cases to repeat in every sentence the precise determinations by which the standard unit is made a standard. But it is all-important to remember that there are such determinations, and that they in turn need re-determination *ad infinitum*. There is no ultimate unit.

Apart from this accessory idea, the supposition makes it plain that the ratio between the density of silver or flint glass and that of water is not sought out as valuable *per se*, but is valued as a means of equation with all measurable densities. In such instances as these we have the first grade of characteristic quantity or proportion, still subject to an external relativity which extends into an infinite series.

Now though this relativity never disappears as an aspect of human knowledge, yet characteristic quantity can assume a more self-sufficing position than that which has just been described. Instead of developing the *this* and *that* of the comparative judgment into separate units connected only by an abstract identity of quality, we may consider them as structural elements within a concrete whole. As before, we shall find ourselves at first in the stage of simple measurement and pure quantity. But the tendency to advance to proportion is in cases of this type much more pronounced, because the parts of an individual whole are more likely to vary in connected ways and therefore lend themselves to proportion, than the elements of wholes external to one another. Still we begin with pure quantity. '*This* plant has petals exceeding the calyx segments,' or 'has radical leaves half the height of the stem', or 'has twenty-one carpels and forty-two stamens', or, as we said above, 'has a divergence of $\frac{2}{5}$ between its stem-leaves.' In the first instance such judgments as these are mere judgments of perception, or at the outside of direct historical fact, and the ratio which forms the content of predication is therefore not a proportion, because it has no extent of application. In proportion, the ratio is to the cases of the ratio as Intension to Extension.

But the moment thought has seized a significant[a] idea, it is committed and must go wherever the idea carries it, in despite of the demonstrative 'this'. And the moment that any such internal ratio as those which I have suggested is taken to be characteristic, i.e. to be involved in the significant idea, it becomes a *proportion*, i.e. a law of structure which holds in spite of varieties of size, shape, and number, although, at least in natural objects, always subject to limits which as regards the proportion itself are arbitrary and external.

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[a] I have here altered the term 'designative' into 'significant', because of the contrast between designation and ideas. See ii, pp. 260 ff.

Proportion is the simplest expression of individuality.[a] All intelligent recognition of individual objects depends either on proportion or on some principle which involves proportion. It is in this that the truth lies of the well-known Pythagorean doctrine that all things are embodiments of number. All things have aspects and effects which find generalised expression in number. Shorten a snipe's beak, take one from the divisions of the horse-chestnut leaf, or misplace the accent (a variation of loudness and duration) in an English word, and recognition falters or fails. Even a human character or an artistic inspiration, though not in itself susceptible of numerical expression, leaves traces in all its acts and products of an individuality that takes shape in proportion or qualitative quantity. An exhaustive statistical treatment of a man's life in all its tangible aspects would give, by the graphical method, not indeed his character, but a set of proportions penetratingly significant of his character.

It may seem indeed that in common hurried thought recognition simply attaches to some pre-eminent quality, a bright colour, a marked outline, a peculiar movement; and that such elements as these, without extension into proportional systems, furnish the practical meaning of words in ordinary life. But in the first place, this is perhaps a superficial analysis of perception. I very strongly doubt if the element of proportion, both external as in size compared with surroundings, and internal as in shape, symmetry, or harmony of sound or colour, is ever absent in a recognitive perception of an individual thing. A really abstract quality would hardly mean anything; we should be able to place it nowhere in our world; and if we even recognised its degree of intensity, that would at once constitute a quantitative element. It has indeed been observed that a familiar scent (one of the least articulate of qualities) has a notable power of stirring associated memories. But this seems so noteworthy just because the scent does *not* recall any individual thing, but rather brings back a general state of feeling connected perhaps with entire scenes and incidents, but especially with emotions.

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[a] That is to say, when once we have abandoned or transcended the unanalysed 'this', which has individuality in a different sense.

And in the second place, if an abstract or mere quality were used designatively [a] in judgment, it would not grasp or enter into the nature of a real individual; it would simply be a falling back towards the demonstrative affirmation with its 'This', which may on occasion be eked out by any element that draws attention.

But all ordinary recognition of individuals undoubtedly depends on the judgment of proportion. We cannot indeed tell the specific gravity of a metal by the mere sense of pressure or of resistance, but we know how a sovereign ought to feel when we lift it on the palm of the hand; and though we may call this effect on the hand a quality, it is plainly a quality pervading differences and so quantitative, and moreover taken as characteristic and so proportional. Consider once more the effect of altering an accent in English (I do not speak of languages in which accent depends on pitch) as exemplified in the change of 'conquer' into 'concur' by transferring the stress from the first to the second syllable, or the utter unrecognisability of such a term as 'sleeping-car' when pronounced with a heavy stress on the second syllable,[1] and a light stress on the first. Here it is the internal proportion that is modified, with the result of destroying the peculiar rhythm by which in a great degree the ear instantaneously recognises a word. The more marked an individuality is the more it depends on internal proportion. Every instrument fitted for a purpose has internal proportions dictated by its purpose; a knife is sharp at the edge and blunt at the back; the thickness of the blade in its transverse section depends on the requirement of strength on the one hand and on that of dividing without displacement on the other, and these requirements together dictate a certain set of proportions characteristic of a blade suited for some particular purpose. The length of the blade compared with its width depends on such another

[a] See notes on pp. 126–7. The argument here amounts to saying that a quality cannot be used designatively.

[1] The author once heard the words thus mauled at Calais, and could not imagine what was being said, though no elements of the sound were omitted.

set, and its temper on a third. All of these being on the one hand relative to each other and on the other hand relative to a purpose are internal proportions subject to limits prescribed by external proportions. It is by acquaintance with the perceptible character impressed by such proportions as these that we readily pronounce on the use of objects made by the hand of man, and that we detect, somewhat less readily, the *actual* purpose served by adaptations in the organic world. Such attributes as are expressed in these proportions form, for perception, the content of individualities.

It follows from these considerations that the question of individuality in contents the main attributes of which arise from external proportions is not an easy one. Such are nearly all inorganic substances, except where adapted to a purpose by man. External[a] proportions *per se* produce no effect of individual unity, and though it is true that all substances occur in particular fragments which have definite characteristic forms, yet the ratios which would express these forms are not absolutely typical or essential (though perhaps all substances prefer some shapes to others, and have typical fractures, &c.), nor do the particular masses of substances or volumes of gases demand or receive individual names. Professor Jevons has called attention to this curious fact, which goes deep into the nature of individuality. Below the level of organic form, or form given by human interference, what do we mean by a *thing*? Of course we may take a lump of metal or an ounce of water, a handful of sand or a jarful of chlorine, and speak of it as a thing; but we shall be puzzled to find any name that recognises its separate identity as 'lion' or 'spade' or 'house' recognise that of the contents that form their meaning. Is gold a class-name, say a specific name, and are the actual pieces of gold individuals under it? This does not seem to be right; a class-name is true, without further determination, of the individuals under it. 'This is a lion,' 'This is a spade,' &c. But you cannot say 'This is a gold': you can only say 'this is gold', where 'gold' almost = 'made of gold', i.e. is adjectival, and has no plural, or if it has one, uses it for different kinds, not for different pieces, of gold.

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[a] There seems no meaning in this word except in contrast with the inner proportions of an organism. It must be understood of a difference of degree between organic and inorganic structure.

It is enough at present to call attention to this difficulty as illustrating the place of structure in individuality. It should be noticed that a structure however complex which repeats itself homogeneously throughout all atoms of a certain substance tends to confer individuality, if at all, on the minute units in which the complex structure exists, but neither on the substance as such nor on its larger fragments; the supposed minute structure is not the structure *of it* or *of them*, but only a structure repeated *within* it or them. A heap of corn is, *qua* heap, no more organic than a heap of sand. But it may be for instance that, in virtue of a common structure, all the iron that anywhere exists is united by reciprocal reaction in a common magnetic world; if so, it is then up to a certain point single and individual. The further consideration of this difficult subject belongs to the discussion of the Individual and Generic judgment.

Qualitative unity of Individuals.

iii. The relation of Quantity to Quality results at this point in a further problem. Assuming that a thing which has marked individuality has always a number of pervading qualities, each of which contains gradations and a distribution expressible by a ratio or proportion, what are we to say of the interconnection of these various systems of proportion with one another? Is it necessary that there should be a general proportion of proportions which, whether our actual apprehension of it be rough or exact, must be taken as capable of expressing the various systems of attributes as gradations within one and the same totality? Is there any sense in talking of the proportion not only of length to length and of colour to colour, but of length to colour; or of the proportion not only between rhythm and rhythm or pitch and pitch in a melody, but between rhythm and pitch as elements of the musical effect?

The view which we took above of the effect of comparison upon quality forces this suggestion upon us. *If*, in other words, a single qualitative effect pervades any and every totality which we apprehend, and *if* within such a quality there are parts perceived as differences *of it*, then these differences must in respect of that quality be regarded as gradations. It is not necessary to press this conception home at present. It is possible that there may be individuals whose unity lies in an idea only and not also in a quality, and that an idea may hold together without crushing them into gradations, antitheses which a quality could only admit as quantitative. But it is worth while to bear in mind[a] that there may be a quality of 'effect', or secondary quality, within which even form and colour or pitch and rhythm may take their place as degrees, just as the repetition of analogous though different forms in a picture or design gives the impression of a pervading character which is more and less intense in different parts of the work. Take for instance a picture about which there is a question whether Turner painted it, or a song which is ascribed without certainty to Shakespeare. In such cases we point to this and that characteristic as more or less Turneresque or Shakespearean; and the elements so designated need not be in actual sensuous quality comparable with each other. It might even be suggested that the exhibition of such a pervading quality was a condition of æsthetic though not necessarily of actual individuality; a suggestion which would raise the fundamental problem whether all actual individuality has, for those who have eyes to see, a thorough characteristic unity. It would seem not improbable that *true* individuality is attained by *actual* individuals in very different degrees.

Change and motion in reference to Individuality.

iv. In explaining the apprehension of individual things,[b] which I have set down to the sense of proportion, it is usual to lay stress on the fact of change, including motion. Change in time and place is no doubt a primary instrument in revealing the fact of individual existence. The moveable and modifiable Thing proclaims itself unmistakeably as distinct and permanent. Nevertheless, for logical purposes change is only a case of difference or negation; or if in fact the two are coextensive, yet change is not the essence of all difference. It may be that every apprehension of difference requires lapse of time, but this is only because our activity is in succession, and does not mean that the differences themselves must be (though they may be) successive.

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[a] I have here cancelled a sentence which implied that zero of a quality was a degree of that quality. See Bradley, Mind, 74, 184.

[b] All the 'individuals' here discussed are of course only *prima facie* individuals. See ii, p. 253.

What the whole matter comes to is this. Difference is the principle which when generalised is known as Negation ; in as far as contents merely differ, they are merely not each other. Every continuous quality includes negative determination, i.e. differences, elements which are not each other. Among other continuous qualities, duration and extension characterise a large part [1] of the world of sense-perception, and duration characterises in one aspect everything that comes before the human intelligence. It is not, therefore, surprising, that not only the parts or elements of individuals, but the individuals themselves as parts or elements of our perceived world, should bear a negative relation to each other and to themselves in time, in space, or in time and space together.

Now if we contrast change and motion on the one hand with mere perceived difference on the other, as influences bearing upon the apprehension of individuality, the distinction between the two cases amounts simply to this, that in the latter we have a single set of differences, which can go but a short way, as a rule, to exhaust any identity, while in the former we have at once a summary of innumerable differences or negative relations. These differences, in virtue not of their mere spatial or temporal distinctness, but of their positive content, are read off in judgments which may [2] coalesce into one or may be distinguished into several determinations of the individual. These judgments may have a negative or positive form, but must always express a partly negative element of apprehension, viz. that ' what is not B is not A '.

When we see a moving animal against a variously coloured background, different elements of its outline and colouring are successively thrown into relief by successive contrasts, and perception traces its form with increasing completeness by the negations which these contrasts furnish, i. e. by the correction which is effected when a line or colour, which the moment before ran into the background, is sharply defined by a change of contrast. The perceptive judgment resulting from this change will take the shape 'That (the dubious border of colour) was not B (a particular part of the animal's outline) and therefore is not A (does not belong to the animal).' More elaborate interpretations than this may of course attach themselves to the motion of a separate material thing; thus, for instance, that which moves all together must in all probability have mechanical cohesion, for it is unlikely that in presence of varying friction at its different points the parts of a moving appearance should keep together apart from such a condition. But cohesion is for our present purpose merely a definite content assigned to unity, and the negative relation of the moving *thing* to what does not partake of its cohesion is established by just the same logical process as its relation to that which does not partake of its distinctive shape or colour. Change in time without motion has the same effect; nothing is more readily detached from its background and apprehended as individual than an object whose transformations take place before our eyes in surroundings which do not share them. The judgment or judgments[1] 'What is not a_1, a_2, a_3, &c. is not A' seems to form itself in such a case almost without an effort of mind. We shall be told indeed that 'What is not a_1, a_2, &c. is not A' is a mere inference from 'A is a_1, a_2, &c.' This objection would raise questions which cannot be dealt with till we treat of negative and of inductive inference. Here we need only insist that however we may elect to describe the process of the negative instance, it is easily seen to be the most effective instrument of definition. When we inject a system of vessels with coloured fluid, in order to observe them under the microscope, it is not the particular colour, red or blue, that we look for, but the contrast between the artificial colour and the dull grey or yellow of the background. Even granting that we start from 'The vessels which are to be traced (A) are the red lines (a_1, a_2, &c.),' still this judgment cannot have scientific precision apart from the determination of the detailed not a_1, not a_2, &c., and when the not-*a*'s assume a positive character, the negation ceases to be an inference from the affirmation A is *a*. And *change* is an infinite succession of such contrasts, that gives every element of the complex individual before perception its chance of being sharply defined, and by successive negations defines each of them both against its own permanent elements and against the background. Instead of the simple change of colour by injection, let us think of the effect produced by rotating the polariser or analyser while observing an object that modifies polarised light. The successive but gradual changes of colour, illumination, and background which are thus obtained bring out the details of a structure as clearly as if we could handle it and move it freely in space. Change and motion merely do for a single individual identity what a comparison of instances does for an abstract identity. That is to say, they show through what contrasts the individual can pass, to what negations it can be subjected whether within its content, or between its content and the background from which it is distinguished, without losing characteristic identity. Change and motion have their logical value simply as embodiments of difference.

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[1] I cannot think that sonorous bodies appear to be clothed with sound as luminous bodies are with colour. To me sound is an unextended perception, though referred to a cause in space.

[2] See the general theory of judgment in time, chap. 1, above.

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[1] See the account of judgment as an act in time, p. 79 ff.

Abstraction and Necessity.

v. In the Judgment of Measurement we find ourselves face to face with the element of abstraction and necessity, the medium in which exact science moves, and the occasion of the most fundamental *crux* for logic as for ethics. We are no longer, as in the Judgment of Quality, simply ascribing the meaning of an idea to an unspecified reality given in perception. We are indeed, as always in judgment, defining the reality which perception presents to us; but we find that in trying to define any special feature or element within it, we are under constraint, not merely as always from the pressure of perception, but from the inability to select and connect at pleasure within the presented content. Judgment—so far as it escapes from the distractions of mere association—proceeds in grooves or along threads which are always leading it across and out of the picture. It cannot, in the present stage at all events, simply characterise a given identity by differences related to it and to nothing else. Such differences, as we have seen, would have no stability, and could characterise nothing; although the more highly organised and individual the identity the more capable it is of prescribing a necessity to subordinate wholes which appear as differences within it. The course of judgment within the present whole of perception is determined by connections which refer beyond that accidental whole, to other more comprehensive totalities, and ultimately, in every case, to the system of the known world. The connections thus prescribed between part and part within some systematic whole are *necessary* connections, and judgment, in so far as it is controlled by them, is abstract or hypothetical judgment.

But the appearance of this element in the judgment of perception makes it simply self-contradictory. The specification of a subject by means of an idea, which is only meant to point out a feature in present reality, brings the judgment into a groove of necessity, and all but makes its affirmation conditional. A speaker who has affirmed that 'This execrable ruffian should be hung' will probably, if convinced that the man is not a ruffian at all, consider that the non-existence of the condition precludes the application of the judgment; i.e. in spite of the 'this', he will maintain that his judgment was essentially conditional. We shall not find it easy to decide whether the idea in question was really a condition or a predicated content. In the latter case the judgment is falsified by the non-existence of the fact indicated in the subject, in the former it is not. There can be little doubt that we must follow the analogy of 'this', 'here' and 'now', by treating ideas, which characterise[a] the subject, as presupposed rather than affirmed; but seeing that in this case the idea which forms the presupposition is such as may or may not be realised, and yet is welded together with a presupposition—the 'this'—which cannot but be realised, there is an inevitable ambiguity in the judgment,—the ambiguity between absolute and conditional assertion.

Absolute and Conditional Affirmation.

vi. In logic as in ethics, Individuality or Absoluteness is at first sight opposed to Necessity or Relativity. That which is individual or absolute claims to be self-sufficing; that is to say, to be an Identity which determines and is determined by its own differences, but is not dependent on anything outside itself. Every content partakes of this character in so far, but in so far only, as it has a unity or an interest for its own sake or in itself. A material 'thing', an organism, a work of fine art, possesses such unity in a degree that forces its individuality upon perception and ensures it universal recognition in language. But every content without exception that is exhibited in judgment has such unity or interest in some aspect or to some degree. Even the abstract idea that qualifies 'this' in a perceptive judgment such as 'This cold is intolerable' is taken as the key to the interest of a presentation, as a predominant feature that arrests attention in our momentary surroundings. That the distinctive character which makes the unity of the presentation is abstract and indeterminate follows merely from the judging function being in a rudimentary stage to which a concrete synthesis is still unattainable.

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[a] See note on p. 126.

On the other hand, every judgment may also be regarded under an aspect of relativity or necessity. In so far as a content is necessary it is not self-sufficing, but is a consequence of something else, and in so far as it is relative it fails to explain itself, and refers to something else for explanation. Every content, every identity in human knowledge is on one side wholly of this character. How the two sides, the absoluteness and relativity of the objects of knowledge, can coexist without interfering may be considered if we please, though in my judgment erroneously, as a desperate problem. But that they do coexist we may convince ourselves by the evolutional history of any flower, by the analysis of any ornamental design, by the study, in its genesis and with a view to the influences that conditioned it, of any human mind. And, in one sense, necessity is more universal than what I call absoluteness, or if I may coin a phrase significant of the sense in which I speak of absoluteness, morphological unity. Morphological Unity has degrees, but relativity or necessity has none. The only escape from relativity is in the exhaustion of relations. If, for instance, we can intelligibly speak of the universe as a whole, we must take it, I presume, as the totality of relations, and therefore as bearing no relation to anything outside itself. But this speculation is unprofitable, because what is out of relation is out of knowledge; or it has at most a negative value as against doctrines which extend the relativity which holds within the totality of relations to the ideal totality of relations itself, and so discuss its origin or the possibility of it not having been. This is futile, from the very nature of explanation. All explanation is *within* the universe, not *of* it. Therefore every content that qualifies a subject invites consideration as an antecedent in necessity in a judgment '*if a* is, *then b* is', while it is *not* every content that has morphological unity and so is given as a whole in each and all of its differences. And only such a content as this is adequate to reality, or can stand, without special symbols of reference, for an individual reality. What is not individual can only be a fragment of the real.

The above considerations were touched upon in chap. I, in explaining the pregnant distinction between Categorical and Hypothetical judging, and are to govern, as was there indicated, the application of that distinction in the remainder of the present Book. We start from the principle that all judgment whatever is an attempt to make explicit the nature of Reality, and is directly or indirectly attached to the reality which is presented through perception. The ultimate subject of all Judgment is the Real, and any idea which appears as characterising or even as in lieu of the subject of judgment must be taken as simply indicating or calling attention to some aspect of the real world. That is to say, such an idea must be taken as morphologically corresponding to the 'this', 'here', or 'now' in the demonstrative judgment, to the unnamed direction of perception in the pure judgment of quality, or to the significant ideal content which expands the 'this' or 'here' in the elementary judgment of measurement. It follows from this that in every judgment the immediate subject is *prima facie* taken to be real, and therefore every judgment is *prima facie* taken to be categorical. This does not mean that in the strict sense it *asserts* real existence of the subject, for its real existence is presupposed,[1] but rather that it defines the reality of a real existence presupposed as subject.

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[1] See last section.

But this *prima facie* semblance of the judgment must be qualified. The explicit content which characterises the real subject may be inadequate to the nature of reality. In 'this red thing', 'this metal', even 'this man', the explicit contents 'red', 'metal', 'man' are typical and general, not single and individual; and still more is this the case if we think of such judgments as 'Red is a colour', 'Metal is lustrous', 'Man is mortal'. These contents stand for imperfect and incomplete realities; realities that could only be completed in an infinite series of time and space. The difficulty is not that they go beyond present perception and beyond our knowledge. Caesar is not in present perception and we do not know all that he did, yet in 'Caesar crossed the Rubicon' no one doubts that Caesar is a reality. Caesar is an individual,[a] and his entire identity is present in his every act and attribute. Man is no doubt a definite concept, but its instances or manifestations have not *prima facie* individual identity one with another. The centre of morphological unity is in each separate human being, not in the idea of the race as such. The concept, as we commonly think it, is an abstract idea, and the reality that corresponds to it is a series of individuals, which not merely is *not yet* actual as a whole, but is not in our predication treated as an actual whole. The reciprocal relations which bind together say the English nation or the Roman Empire into a historical individual may be present also more or less in the case of humanity, but when we say 'Man is mortal' this is not the light in which we look at the Subject; we are speaking of individual men whom we designate by help of the concept man, not of humanity or mankind as such, for which it might be maintained that morphological unity is possible. No doubt however, if we push the matter home, even the predicate mortality is affirmed of all individual men in virtue of a oneness of nature running through them all; and therefore we must, as I have said, take the individual unity to be a matter of degree, and to be wholly absent in no content that can be presented to thought as characterising a subject of judgment.

When, in view of cases like these, we qualify the principle laid down above that in every judgment the subject is taken as real, we must do so by the emendation, 'The subject is taken as possessed of that kind of reality of which it is capable, subject to any alteration which the predicated content may declare in such reality'. The first part of this sentence provides for judgments introduced by abstract ideas and not referred to actual individuals, the second for all kinds of judgments which formally affect reality and which are alleged [1] as proofs that content and reality cannot depend upon one another. First among these come the Negative Judgment which sets up an aspect of Reality in order to demolish it, and then all such peculiar cases as 'The chimera is a fiction', 'Nothing is here', 'A wise knave is a contradiction in terms'.

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[a] See note [b], p. 131.

The view to which I have just alluded might raise an objection at this point, which I will only mention in this place, as the real answer to it, if any, can only be found in the whole conception of the judgment which we have adopted. 'Whether an idea stands for a reality or not does not depend on its content, but on that content being recognised as somehow and at some distance or other belonging to the world continuous in quality with the object of present perception—i.e. to the actual world'. My answer to this would be that I have accepted this identification in quality as the abstract logical or rather as the psychological condition of all judgment; but that this identification is empty apart from the specific kind and degree of reality assumed or alleged, and this, as it appears to me, is a matter of content and of content alone.

The Hypothetical Necessary or Relative aspect of judgment is a consequence of the characterisation of the subject by any *determinate* ideal content. It is the universal connection of attributes within systems, as opposed to the morphological unity of individual systems upon which that connection rests. This aspect is undoubtedly perceptible from the moment that the immediate subject is made explicit by help of ideas, but as long as there is a gulf between the ideal content and the latent reality which it designates—the 'this'—the necessary aspect of the judgment is absolutely dissociated from its categorical aspect, and the divergence almost amounts to a duality of the judgment. In some cases indeed the contents employed to designate the subject will have only a partial connection with the predicate, as in 'This flower is a rose'.[1] Such a judgment has been called a double appellative judgment. But I incline to think that affirmation of this type is always on the road to necessity; 'flower' does not indeed tie down the predicate to 'rose', but the thread of botanical classification runs through both. If it were not a flower it could not be a rose. So the two designations are undoubtedly chosen with reference to one another, and the true duality of the judgment is not in christening the rose twice over as rose and as flower, but in the ambiguity between 'This is a rose' and 'This, *if* (or 'in as far as') it is a flower at all, is a rose', which might well be said of the Tudor rose as it appears in some decorative designs.

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[1] Bradley's Principles of Logic, p. 14.

The judgment whose subject is designated by a proper name is at first sight devoid of necessity. For in it determinateness of content is sacrificed to the indication of actual continued individuality, and therefore the relation of necessity or hypothesis, which depends on determinateness of content, is not easily traceable. Yet a name is always capable of acquiring a definite content, which at once brings such a relation into prominence. The indignant denial, 'Gladstone never said that,' 'Socrates never gave immoral advice,' is obviously hovering between the sense of 'A. B. did not go to town to-day' and that of 'An honest man cannot say what is certain to mislead', that is to say between the assertion of fact and that of necessity. And again, if, as explained in the last paragraph, the content which characterises the subject has not the nature of a complete or individual whole, then its reality must be taken as that of which alone it is capable, viz. indefinite presentation in the series of space and time; and this amounts to so little (for the presentation may be as rare as is consistent with occurring at all) that the element of necessity or relativity dominates the element of unity or actuality, and the judgment appears to have as its essential content a necessary sequence or connection about the presence of which in reality little or nothing is affirmed. Such a judgment, and every judgment in as far as it can be thus regarded, is hypothetical, that it to say it runs wholly or partly along a line which may be formulated as 'If *a* is (or in as far as *a* is), then *b* is (or so far *b* is').

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[1] Cp. Sigwart, vol. i, p. 69. E. Trans., I. 57.

On the other hand, even those perceptive or historical judgments, including ordinary assertions about people or places called by proper names, which betray in their content no tendency to enter a groove, i. e. to exhibit a universal connection of attributes, nevertheless must be held as bound by this ideal, which is involved in the employment of contents which have a meaning, and therefore can acquire a determinate meaning. If then, as in such instances we may assume to be the rule, the contents actually employed in judging embody no principle, but merely exhibit irrelevant differences as coexistent in a concrete subject, such judgments, even when true in their first meaning as mere statements of fact, are yet fundamentally false. That is to say, they are true in their categorical aspect but false in their hypothetical or relative aspect, from which, being definite judgments, they cannot escape. They do not express an *a*, upon which, within some real system, the content of predication follows as *b*. But it is important to remember that we are dealing from beginning to end with aspects and not with total differences. I believe that a misapprehension on this head has of late years given rise to an erroneous conception regarding the tendency and goal of knowledge.

Knowledge as both absolute and relative.

vii. It is a great thing to have raised the notions of Logic to a level with the ideas of exact science. This has been done by Mill and subsequent writers,[1] and the work had become necessary, though the views to which it led were not in principle new. The essence of judgment was by them declared to lie in the coexistence and rational connection of attributes, and the ideal of science to consist in the knowledge of the fewest assumptions, from which, if given, the whole course of the world might be deductively derived. As a consequence of these ideas, the universal judgment was, in part by Mill himself and more distinctly by later writers, identified with the hypothetical or abstract affirmation of necessary connections; and, further, by identifying this type of judgment with the extreme case of supposition for the sake of argument, the universal judgment as such was denuded of all affirmation having real content. It was still treated as affirmed of Reality, but was held to be related thereto as a known consequent to an unknown antecedent. It was pointed out that in supposal for the sake of argument no element of the content supposed or of its consequence is affirmed either to be actual or even to be possible, and that nothing more is alleged as fact, in judgment based on supposition, than that Reality, which itself does not appear within the judgment, is such as under the supposed conditions to exhibit the inferred consequent. Truth, it might therefore be concluded, may be taken to illustrate, but it cannot be taken to define, reality. The strong implication of actuality which attaches to the content of many ordinary universal judgments was dismissed as explicable on grounds of habit and confusion. Thus the categorical judgment, in the sense of a judgment which asserts any specific content to be actual, was primarily confined within the limits of affirmation about individuals in space and time, although in disjunctive judgment, and in judgments, if any, dealing with existences beyond time and space, a categorical character was admitted to reassert itself.

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[1] Mill's Logic and Lotze's earlier Logik seem to have appeared in the same year (1843). Lotze, I presume, was largely influenced by Herbart on the point in question. I have in my mind also Sigwart, Wundt (whose doctrine of 'Gegenstandsbegriff' does not, however, seem to me perfectly clear), and Bradley.

In contrast with the conception of logical progress and with the ideal of knowledge which I have just described, it appears to me that a somewhat modified form of the views in question might yield less one-sided conclusions. I should prefer to regard the normal and central evolution of judgment as categorical [a] from beginning to end, and as gaining, not losing, in this characteristic as it passes from perception and history to the more complete forms of science. The implication of real existence which attaches to the content of ordinary generic and universal judgments seems to me to be of the same kind as the implication of existence—for it is no more—which accompanies the demonstrative 'this', 'here', or 'now', or its expansion by a significant idea, or a proper name, or the significant name of any actual, even if not in the full sense individual, totality, such as the English nation, or the Natural Order Rosaceae.

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[a] The truth of this matter cannot really be grasped except by carefully considering such a catena of judgments as forms the basis of Mr. Bradley's Principles of Logic or of the present work. I do think it remains firm that the intention of the Judgment is categorical throughout. It desires to say something true about something real. But the point to master is its twofold nature, in which fact and necessity are in conflict, so that in all its phases there is an unsatisfied implication, and it never expresses either a self-contained fact, or a really perfect

The main function of judgment would then be identified with the exhibition of individual totalities at once in their absoluteness and in their relativity. We should thus not wholly subordinate classification, type, and individuality to the claims of explanatory theory, but endeavour to represent the two as complementary and indispensable aspects of knowledge. Abstract and ideal judgments like those which embody the necessary connections of geometry we should rank as an indispensable divergence, but still as a divergence, from the natural track and tendency of reason, and as attaining their truth most fully when, returning towards that track, they are taken up into the precise determination of typical structure in space, or even of individual realities. We should refuse, in spite of identity in linguistic expression, to take supposition for the sake of argument as the type of universal judgment, and should point out that as supposition passes from selection within reality into free imagination it becomes detached from the real ground of all relations, and ceases even to exhibit a necessary relativity.

It is in accordance with these views that I have treated measurement as involving both the revelation of Individuality[a] (morphological unity) through characteristic ratio, which is the same thing as proportion, and the exhibition of relativity by the reference of the unit to something outside the individual. I now proceed to speak of kindred judgments, which present the essential aspects of measurement in one-sided modifications.

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connection. And so in the end undoubtedly 'Judgments are conditional in this sense, that what they affirm is incomplete'. Bradley, Appearance and Reality, ed. 2, 361.

[a] Cp. note [b], p. 131, above.

CHAPTER IV

MEASUREMENT (*continued*)—ABSTRACT QUANTITY

One-sided forms of Measurement.

2. INDIVIDUALITY as revealed in measurement may be simple or complex, and, if complex, it must involve a variety of simple factors. In a simple individuality, or the simple factor of a complex individuality, the qualitative distinctness of the parts is at a minimum; for any exceptional qualitative difference in any part would challenge measurement and constitute a complication within the unity. When an individual is thus taken in its simplicity, in a single aspect, and yet considered as being a whole complete in itself, it is treated as a whole of *quantity*; that is to say, such a content as is exhibited in the predication of the comparative judgment, but taken as standing in the place of the individual Subject, now that the conception of individuality is attained.

Enumeration.

i. In such instances we find the simple quantitative whole which is thought of as constituted by absolutely homogeneous parts—an idea which we have seen to be never strictly true, for without *some* distinctness of quality the parts would cease to be. Such a whole differs from the normal individual by the lack of anything that can be called dominant, essential, or characteristic within the content itself. There is, for instance, no need to consider its unity in the light of a secondary or aesthetic quality. The unity is already that of a continuous quality, and in the attempt to define it, it lapses almost wholly into relativity, for the determination of the whole depends on the equation of the parts, in an unending series, with other and independent standards. Thus the purely quantitative whole is characterised by being capable of construction by ideal repetition of a unit or fixed part; and such ideal repetition is *enumeration.* Enumeration may seem prior to measurement or identical with it; in measuring we enumerate units, and whenever we enumerate units we measure some totality. When we count the carriages in a railway train we are measuring the train, when we count the sheep in a flock we are measuring the flock, just as when we count the feet and inches in the length of a room we are measuring the room. But usage, as we feel at once, does not bear us out in speaking of the two former instances as measurements, and the reason is plain. In measurement we start from a whole which we characterise by its differences; in enumeration we start from a distinct unit, out of which we desire to construct a collective or aggregate whole—a sum total. The whole in enumeration, which is a predicate, is a weakened form of the individual whole in measurement, which is a subject; and the unit in enumeration, which is a subject (generally distinguished by a natural individuality), is a strengthened form of the distinct unit, ideal part, or constant difference, which forms the predicate in measurement.

Enumeration is therefore in one sense posterior to measurement, because it presupposes, as a naturally distinct unit, the 'thing', the idea of which can only be furnished by a sense of proportion or perception of limit; but, on the other hand, enumeration is an *instrument* of *precise* measurement, which involves the notion of a scale of degrees or aggregate of homogeneous parts. The two processes are constantly concurrent, and only differ in the respective values of the parts and wholes with which they deal. It would be futile to distinguish them from one another, but for the consequences which result from the possibility, first exemplified in abstract enumeration, of systematising the synthesis of parts without relation to a whole. The whole of enumeration is depressed into a mere aggregate, or not even a definite aggregate, and therefore the part into a mere unit, or even into the mere place in which a unit might be. The process with all its corollaries, including the three unreal infinities of Number, Time, and Space, must be regarded as belonging to a form of the judgment-function in which the relation of whole and part is denuded of all structural variety, and therefore the aggregate or sum which is the outcome of that relation lacks predominant unity. This principle is expressed in the saying that in a numerical system the sum of units is the same whatever may be the order in which they are counted, i. e. any part (*qua* part of a total formed by enumeration) can be interchanged with any other part without modifying the whole in which they are parts.

Corollaries.

I proceed to state and illustrate some corollaries which follow from the above idea of enumeration.

Simple counting.

a. Simple counting always consists of a series of singular judgments, and is in this respect on a level with simple measurement.

However abbreviated or abstract each step in counting may be, it can always be expanded into a singular judgment which records its own position in a coherent series. In counting the birds in a covey, or the stitches in a bit of needle-work, we often merely repeat aloud the words ' one ', ' two ', ' three ', and the real nature of the judgment which accompanies them is open to question. In fact, each of these numerals in such a case implies a separate singular judgment, though extreme abbreviation tends to conceal its structure. The essence of counting is in just such a progressive distinction as is conveyed by ' this ', ' that ', and ' the other ' ; ' alter ' and ἕτερος have almost the value of numerals. The exclamation ' One ! ' when we are beginning to count any set of objects means ' *This* unit is a part in the whole which interests me ', e. g. ' One—two—three—*eight* birds in that covey '. The bird is the naturally distinct unit by which in so simple a case we proceed as a matter of course, and the covey is the total up to which we wish to count. If birds get up belonging to another lot, we shall probably desire to keep them distinct, and so count them separately, beginning ' One ', ' two ', &c., over again. It is plain in this or any similar instance that we do not count right on as long as units can be found, but that every step of the enumeration is made with reference to a limit as well as to a unit. This limit, however disguised by our caprices and interests, is simply the common or continuous nature of the unit in so far as it interests us, and in every enumerative judgment without exception the elements of separate unit and common nature may be traced. The unit need not be externally distinct or physically separable ; and the common nature, instead of being hard to trace, may all but obliterate the differences that exist within it. But it is the nature of the unit that furnishes both rule and limit of enumeration, which is a process unmeaning without a limit and impossible without a rule. If we are told to count even all the 'things' in a room, we shall find ourselves obliged to ask what is to be reckoned as a 'thing'. Is a bookcase with 500 volumes in it one thing or 500, or 501? 'In a room', however, is a kind of limit, and assigns a totality to be constructed by synthesis; but if we are asked simply to count, we should reject the request as pure nonsense, because it assigns no totality to be constructed by counting.

What, then, is the meaning of counting 'One'! 'Two'! 'Three'! in starting a race, or of—

> 'And still stood all who saw them fall
> While men might count a score'?

In the latter case, the idea is that of counting the names of the numbers up to a fixed limit at a rate determined by habit or by the time which is required to pronounce the words distinctly. In the former case the object is perhaps not merely to let time elapse, but to set attention in a certain rhythm, so that the tendency of rhythmical anticipation may assist in seeing or hearing the starting signal at the moment it is given. For this purpose the periods should be exactly equal, and in fact at the Oxford boat-races every boat has some one beside it who counts aloud the last few *seconds* before the gun fires. Of course, finally, the names of numerals may be repeated, as any words may, without a meaning. But to do this is not to count.

Thus even simple counting always involves the elements of the judgment—of an identity exhibited in differences and affirmed of reality; which elements present themselves in the shape of a distinct unit within a continuous nature, its relation to which nature is indicated by number. And that the simple enumerative judgment is always singular follows from the nature of the unit, which is theoretically nothing more than the content to which we ascribe distinctness *pro hac vice*; in other words, the unit *is* the difference or part that is taken as distinct by one act of judgment, and it may be said that what we count [a] in enumeration are primarily the acts of judgment, as acts of distinction and relation within a certain continuous quality.

Does it follow from this view that Number arises essentially out of the sense of time or succession? I answer, Not 'essentially'. The connection between number and succession is a psychological and not a logical question. If it is impossible for two related acts of judgment to be simultaneous, as we are naturally inclined to suppose that it is, then two enumerative judgments must always be successive, and in this sense enumeration may depend upon succession. Nor do I think that this connection is necessarily disproved by such observations as that perception can take note of six balls at once being dropped into a box. Such a perceptive judgment, in my opinion, is probably *one*, and applies the result of previous counting, as an attribute resting on experience, to the perceived content, just as we can judge the number of pips on a card from the mere form of the pattern which they make. So if we count by twos, fours, or more, I think that this is complex counting, the unit being the mass of two or four, known to be such by previous experience [1]. A *logical* order among the units, i.e. an order in which the apprehension of each unit has a place in the series conditioned by the separate apprehension of other units, is of the essence of enumeration, although a *material* order among them, i.e. such an order that a unit changes its value by changing its place in the series, is in contradiction with the essence of enumeration. Unless there is what I have called a logical order, we have no security that the unit as such is apprehended at all, and what we take for enumeration may really be an inference like that which detects from the pitch of a note the number of vibrations

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[a] This view has been treated as an assertion that what we count is our own counting, which has no meaning. I understand it as holding number to be the general schema of the relation of parts in any whole, taken as homogeneous so far as the distinctness of parts permits. In counting, as I understand, we apply this general schema to a given whole, and are consequently put in possession, with regard to that whole, of all the relations which the schema involves. Cp. p. 156, below.

[1] See the Author's Knowledge and Reality, p. 92.

per second which generate it. But whether a logical order of succession can only be realised in temporally successive acts of apprehension, is a purely psychological question ; the more so, that, as we have seen, succession and time do not exclude unity of judgment, and the acts of judgment which constitute an enumeration might even, like an inference, be brought under the head of a single continued state of consciousness. In any case, succession in time would be a mere psychical condition of number, following from the unity of the intelligence as forbidding (if it does forbid) two judgments to be made at once. There is no sort of ground for connecting enumeration with the apprehension of *equal* parts in time, unless the equality of such parts be the material purpose of the apprehension in hand.

Discrete and continuous.

β. It follows from the nature of enumeration that *the distinction between discrete and continuous magnitude and the opposition of number as discrete to space, time, and other kinds of quantity as continuous, rests on a confusion.* Number as mere names or mere sounds may be discrete, i. e. disconnected, but then it has nothing to do with magnitude, but is a set of mere words destitute of meaning. On the other hand, number considered as the vehicle of magnitude or quantity is both discrete and continuous ; and the same is true of all quantity, as we saw in examining the comparative judgment, and it is the essence of quantity to be so. The distinctness of natural units such as reciprocally exclusive material things does not make any difference of principle. They, like all units, are numbered in virtue of a continuous quality or identity which pervades them, and every unit, though it may only be suggested by a momentary purpose, is, like them, a distinguishable part, within a whole or aggregate consisting of such parts. The books on a shelf are not merely discrete, and the inches in a yard or the units of weight in a gravitating body are not merely continuous ; in every case the unit is a distinct or discrete part, and the sum is a self-identical or continuous whole. It is nonsense to speak of counting without saying what is to be counted ; and in specifying what is to be counted we specify at once the nature of the continuity and the rule of the discretion.

Judgments affiliated to Enumerative Judgment.

ii. *Judgments affiliated to the Enumerative Judgment.*

The Enumerative Judgment is the root of all quantitative determination; and, as we have seen, all the matter of knowledge above the stage of pure quality is either in itself or in its conditions accessible to quantitative determination. But the judgment in question also contains, that is to say, *is*, though in a depressed form, the universal essence of judgment in the principle of identity and difference; and it is possible for this to be revived by one kind of abstraction into a different relation from that of unit and sum, as it is for it to be further specialised, by an opposite abstraction, into an idealised form of the latter relation. But before indicating the genesis of other types of judgment out of enumeration, we must glance at those which it necessarily generates, and which must be regarded as species of itself. These are, α. the Plural or Particular, and β. the Collective judgment.

The Plural or Particular Judgment.

α. The Plural Judgment, or the Particular of traditional Logic, differs in no essential respect from the singular. It is not however accurately described as a mere aggregate of singular judgments, and indeed this description does not explain itself, for is such an aggregate several judgments or only one? I cannot doubt that the plural judgment is a single act of thought, which determines a certain whole or aggregate, given at the moment, though, it may be, in process of modification, by an attribute or condition such as two, three, or some other number which expresses the reciprocal relations of its homogeneous parts. Thus the plural judgment is not an aggregate of judgments, but a judgment about an aggregate. Therefore the number is to be regarded as a predicated content or determining condition, attributed to a whole consisting of the units which have been counted up to the point at which the plural judgment is taken. That the element of continuity, or designation of the whole whose parts are *to be* counted, must quarrel with a determination by any number short of this whole,[1] and demand a continuance of the enumerative process, is only a case of what happens in every judgment of perception. Such judgments, as we have seen, never embody 'pure cases', i. e. they never fit precisely into a groove of necessity, that is, a sequence of reason and consequent. There is always something relevant omitted, or something irrelevant retained.

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[1] See Knowledge and Reality, p. 65, and above, Introduction, pp. 51 ff.

Thus the particular judgment from the very first *implies* a *ratio*; and the implication may be so strong as to take the judgment out of the category of particular judgments and place it in that of collective judgments, e.g. '335 members of the House of Commons are Liberals.' As every one knows that the House of Commons contains altogether 670 members, this is just the same as to say 'half of the House of Commons are Liberals'. It depends for its meaning on a completed enumeration, and therefore is essentially a collective judgment. It may even be regarded as an instance of simple measurement, i.e. of *ratio* treated as pure historical fact, in as far as its purpose is to measure the voting power of the Liberals against that of other sections of the House. The Liberals and the whole House are equated in respect of numerical strength; the whole House minus half of the whole House = the Liberals. In short, as we have seen, the completed judgment of Enumeration passes into Measurement, though the idea of Measurement is prior to the act of Enumeration. If on the contrary we say that 'Two Cambridge men are coming down to lecture', the number two is *prima facie* non-significant; the judgment tells us nothing of the ratio borne by two to the whole number of Cambridge men, nor even to the whole number of local lecturers. Nevertheless if we think of it, we shall see that either it was wholly superfluous to mention the number, or else some context or latent allusion must imply a ratio. Number has significance only by comparison with number.

The Plural Judgment is equivalent to the Particular of traditional logic in the form 'Some men are mortal' There is no essential difference between 'some men are mortal' and 'four men are mortal'; the two assertions, if interpreted literally, belong to the same logical class. It will appear however below that this literal interpretation does not render the true meaning of the old 'Particular Judgment'.[1]

But we must distinguish from the above a form which has been employed in quasi-numerical arguments, viz. the form 'Most men are mortal', or 'The majority of the House of Commons that has just been elected is Conservative'. These judgments are not merely particular even in immediate appearance. They present on their very surface the relation to a collective judgment which we saw to be at least latent in every particular affirmation. 'Most' or 'The majority' means more than half; and when we speak of half or a quarter or any ratio we assume a completed enumeration of the whole. We must therefore now pass to the Collective Judgment or Judgment of completed Enumeration.

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[1] See on 'Modal Conversion', chap. vii, below.

The Collective Judgment.

β. The Collective Judgment has of late been rightly distinguished from the Generic and from the Hypothetical Judgments, which correspond to the real meaning of the Universal Judgment known to traditional logic. It has been justly pointed out that the 'All' of mere extension or numerical totality does not really express what is intended by such an allegation as 'All men are mortal', or 'All triangles have their three angles equal to two right angles'. In such cases complete Enumeration is inconceivable, and something quite different, viz. the universal connection of attributes which are not results of enumeration (for number is also an attribute) is really the matter affirmed. So far all is clear, and Logic has greatly gained by the distinction.

But when we come to erect a difference of kind, and to treat the collective judgment as purely on a level with the singular or particular judgment, and as in fact a mere aggregate of singular judgments, and as thus separated by an impassable gulf from the universal judgment; and when we further maintain that enumeration cannot warrant its own completeness, then we fall into difficulty and confusion. Even the Plural Judgment, as we saw, is not a mere aggregate of singulars.

The Collective Judgment I understand to be a judgment made about a definite group or limited class of individuals, which individuals are taken to have been exhaustively enumerated, or to be capable of exhaustive enumeration. It is not necessary that the individuals should all be together in space or time; it is not necessary that they should all exist or have existed with any specific degree of reality; but it is necessary that every one of them should be brought before the mind, or should be capable of being brought before the mind. in a distinct and separate enumerative judgment. Such judgments are 'All the books on that shelf are German', 'Every horse that I have bought in the last three years has gone lame', 'All the kings of England since the Conquest but three have died natural deaths'.

When it is said that such judgments as these are on a level with the singular or the particular judgment, I take it that we must exclude from the meaning of singular or particular judgment all dependence, whether latent or explicit, on completed enumeration: otherwise our account of the collective judgment becomes circular. But if so, then we have the 'allness' of the collective judgment staring us in the face as a distinction between it and the particular. And this distinction becomes an absolute severance if we are to insist that the process of enumeration, a process which consists in singular and particular judgments, cannot furnish the warrant of its own completeness.

I have maintained elsewhere,[1] and it follows from my whole conception of the judgment, that enumeration cannot be made intelligible on such a view. It is impossible for enumeration to go on apart from the discriminative control exercised by the pervading nature of the totality under construction upon the successive apprehension of the units. It is this control which takes the form of an inchoate perception of ratio as the counting advances, and of the warrant of exhaustiveness when it is completed. Apart from such an influence of the whole there can be no purpose in enumeration. No doubt therefore complete enumeration is in one sense on a level with the singular and particular judgments, because they present though in an imperfect form (as we have excluded the case of definite ratios such as 'Half A are B') the same relation to totality which the collective judgment completes.

But for the same reason it is impossible to justify an absolute separation between the collective and the generic or universal judgment. The collective judgment, we may say, must emanate from an enumeration of actual individuals, or at least of individuals actually brought before the mind, one by one. But how can we carry a genetic distinction like this into the interpretation of judgments whose actual content is precisely the same? It is the commonest thing in the world for a judgment to be taken as exhausting a group or set of distinct individuals, without resting in any way upon direct enumeration; in other words, a judgment that *might* be obtained by enumeration constantly *is* obtained in some other way. The fact is, that when we have gone beyond sets of individuals present to perception or within the power of the mind to represent at once as distinct individuals (and it would be impossible finally to limit the collective judgment to such sets of individuals), we have entered on a process which is plainly and obviously mediate and hypothetical, and the fact that many judgments thus mediated are taken to refer to a finite group of individuals is a mere instance of our general rule that every content in judgment is taken to have the reality of which it is capable.

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[1] Knowledge and Reality, p. 77.

Is there then no difference between a collective judgment and a true universal? Certainly there is a difference, and it is illustrated though not constituted by the connection of the former with complete enumeration. It is simply this, that a collective judgment deals with a content which can be presented to thought as possessing the character of an aggregate of exclusive units, or finite whole of enumeration. This point of view would involve identification of the collective judgment with the aggregate of singulars only if all enumeration were simple enumeration. But a numerical whole may be obtained by mediate enumeration, and it is to a whole so obtained that the content of many collective judgments is equivalent. It is true that a collective judgment is not a genuine universal, but it is not true that such a judgment must be equivalent to a mere aggregate of singulars. This conception is, in the first place, irreconcilable with the unity of the judgment;[1] and, in the second place, is not warranted by the plain meaning of the propositions in question. It is impossible to characterise a number of individuals by a common name as the subject of a judgment without implying a significance in the characterisation. Even if the predication is true of the different individuals for different reasons, the common interest of the judgment must give it unity of purport. An arbitrary limitation either of number or of time tends, no doubt, to interfere with this significance, and to force an extensional meaning upon the judgment ; but, as we know, the purest extensional meaning is only a minimum of intensional meaning. And there are collective judgments which could not possibly be taken as mere aggregates of singulars. Such are reflective historical judgments. 'All States of the North American Union are prohibited from interfering with the tenure of property.' I need not know this, by simple enumeration ; I may know it mediately, as a provision of the Constitution of the United States. I need not even be actually able to enumerate the States which are included in the Union. But I know that they are numerable, because I know that they are an actual limited group, and so I judge it as a historical fact, though I may know it also as rooted in the nature and tendencies of the Union.

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[1] No doubt there is a difficulty here. 'George and John have gone to school,' 'The two boys have gone to school.' Does the former sentence convey two judgments and the latter only one? The latter

The fact is, that superficial as is the view which makes 'allness' the adequate expression of logical necessity, it is if possible, more superficial to deny their connection. From the first use of the characterising idea, necessity makes itself felt ; and 'allness', or the aspect of a finite totality as an aggregate of exclusive units, is never without a warrant and significance however arbitrarily the totality may be taken. It is by the certainty of complete enumeration that counting becomes, as was said above, the organon of precise measurement. For instance, the series of enumerative judgments, 'One, two, three, four, five, *six* ounces are in the scale balancing this packet,' is convertible, in virtue of their exhaustiveness, into the judgment of measurement, 'This packet weighs 6 oz.'

We saw that the act of counting tends to assume indepen-

only one, certainly ; the former conveys two at first sight ; but if we bear in mind our account of the Judgment as an act in time, we shall see that these two may readily pass into one. See p. 79, supra.

dence, as if it could have a meaning apart from any continuous nature in the units, i. e. in short, apart from an identity presented as a totality. This is not merely owing to the customary abbreviation of the enumerative judgment, as when we seem to count by saying 'One, two, three,' &c.; rather such an abbreviation is made possible by the apparently independent reality of number. This appearance of reality depends on the fact that the numerical series does furnish a generalised scheme of the relation of whole and part when envisaged in the form of total and units.[a] Such a generalised scheme, though meaningless except as applied to a positive content of thought, contains nevertheless definite and necessary relations which are imposed by it on any content to which it is applied; and the presupposition that it is taken as applied to an adequate content fades into forgetfulness that it need be applied to a content at all. But need it? We may surely investigate the numerical series for its own sake. When we say that twice 50 is 100, need we mean that twice 50 of some particular kind of thing are 100 of it? We may surely mean that in the numerical series 100 is separated from 50 by the same number of places as 50 from 0, which relation involves a variety of consequences, all true of the numerical series as such. No doubt this is so, but it will be observed that we have to appeal to the idea of places in the series, and these places are the abstractions of enumerative judgments and imply relation to a content. Such places contain in themselves no reason for stopping at any point of enumeration, are applicable hypothetically to every content, but can yield, in their abstraction, no conclusion about any. It is a well-known fallacy to obtain a concrete estimate by multiplying an amount by the number of times its cause as given has to be repeated. Hardly any concrete quantity is unaffected in the ratio of its increase by a great addition to its absolute magnitude. To say that the stock or trading capital of England is worth so many hundred millions sterling is a graphic expression for the fact that its amount is a million times as great as an amount worth so many hundred pounds. But in the economical sense of 'worth' the conclusion is nonsense. A thing is only worth what it will fetch, and who is going to buy the whole stock of England if thrown on the market at once, at the rate which is commanded by the amounts of stock which change hands in the common way of trade ?

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[a] Cp. p. 148, above.

The numerical series is an ideal scheme of the relations of units within totals, but is itself unreal apart from its applications, not because it is ideal, but because it has in itself no element of limit or totality ; i. e. its units make no choice between belonging to one total and another, and so naturally belong to an infinite series.

This aspect of the enumerative judgment—the system of number—leads to the consideration of complex counting and of numerical infinity. But before pursuing the enquiry in this direction I pause to indicate a reversion or convergence to which the simple enumerative judgment tends in its other aspect.

Enumeration becoming Generic. The Exhaustive Judgment.

γ. Every concrete enumerative or collective judgment bears reference to an identity which controls its selection of units and fixes the limit of its enumeration. This identity is the pervading nature of the units. Now if this nature consists in the characteristic quality of an individual thing, then it is possible that on the one hand it suggests no conceivable limit of enumeration, while on the other hand the characteristic individuality claims completion in respect of its positive connections of content. In such a case there is no true whole of repetition concerned ; no whole, that is to say, which in its nature draws nearer completion by every repetition of an individual. The books in this room are a true whole of repetition ; the human race, to our present knowledge, is not. Thus a problem which is first attacked by enumeration may transform itself unawares. Meeting with a series of individuals in which we perceive some important attribute, we enumerate them as cases of it. But soon their characteristic nature reacts on the function of thought, and we find our successive judgments attempting to grasp a connection of content and not to exhaust a sum total of cases. Now as enumeration is on one side selective analysis, we continue to give our judgments enumerative form, and even couch them by anticipation in the shape of exhaustive totality. Then we have what has been called Induction by simple enumeration, with its results embodied in the Judgment of Allness, which to avoid an un-English expression I shall venture to speak of as the Exhaustive Judgment.

The most varied opinions have prevailed as to the nature and value of this process, obviously because it forms a transition between two distinct lines of thought, marking the revival within mere enumeration of the sense of characteristic individuality which belongs to the judgment of proportional quality and the kindred judgments of individualising thought. We meet, for instance, with the question whether the judgment 'All men are mortal' claims to represent completed enumeration or not. The answer is that its form and its meaning are at variance; in form it does make this claim and in meaning it does not. Such a judgment indicates that the spirit of analogy and of characteristic quality has reawakened within the form of mere enumeration, and is sweeping the line of evolution back towards judgments which predicate individual and generic character. When we come to speak of Induction as a phase of Inference we shall see that there are good reasons for such an awakening. At present we have only to note that the exhaustive judgment 'All men are mortal' is a transitional form between a collective judgment on the one hand, and the generic judgment 'Man as such is mortal', also couched in the form 'All men are mortal', on the other hand; and arises from an incipient reaction of positive content upon ideal schematic form in the process of making number, before an external separation has been effected between the two elements. It is idle to demand the perfection of complete enumeration from the exhaustive judgment; for this latter is a popular and unstable form of thought, and must simply be recognised as such. It is better to treat the collective judgment as inevitably leading up to a connection of attributes and as therefore having its ideal in the spirit of the exhaustive judgment, than to interpret the exhaustive judgment according to the letter, as having its ideal in the collective judgment. We shall have to recur to this subject when we return to the central development of the judging function.

We have now to trace the further abstractions which have their root in enumeration.

Judgment of Mediate Counting.

δ. On the side of the relations between units and total as such, abstraction being made from the positive nature of the contents submitted to enumeration, the judgment of completed enumeration is an ideal or generalised scheme of all possible constructions of such purely numerical totalities. It starts from a measurement or collective judgment of the type 'All the books in this room amount together to a thousand.' The further abstract development of such a predication may be brought about by the most various occasions, but it essentially consists in this, that the positive concrete units of enumeration which stand as subject in a judgment like the above are replaced by generalised relations of ideal units equated to a total which also becomes ideal and generalised. This substitution reveals the fact which alone makes it possible, viz. that in the numerical scheme all units, being abstract, retain the same value in every part of the series; e. g. the units between 50 and 60 count for as many as and no more than those between 30 and 40 or between 0 and 10. Thus a series of units may safely be named by the number of places which it occupies counting from the zero of the whole numerical series, but it is the same wherever and however often it recurs. It is on this characteristic of number that the possibility of mediate or complex counting and of equation, which is implied in these processes, depends. If, for instance, we desire to recompose the sum of books in the room by equating it to component sums or to factors, we do so by conducting a number of enumerations separately from the beginning of the numerical series, and then combining their totals according to the rules of that series, which are known to us simply by experience. Thus, if we count 700 English books, 200 German, and 100 Italian, we find that these sums, considered merely as numbers, are equal to the total 1000 obtained by direct enumeration. 'Considered merely as numbers,' because the rules of the numerical series cannot warrant us against any material influence of the actual individual things upon each other. Some of them may cancel each other, or may produce more by combination ; but this has nothing to do with the properties of number, except for the fact that, when known, it is capable of numerical expression. The component numbers in the subject of a judgment like the above (These 700 English volumes with these 200 German and those 100 Italian make up 1000 volumes) correspond to the characterising ideas in an affirmation like 'This execrable ruffian should be hung'. It is hard to say if they are general conditions, or if they are specifications of fact taken as true merely in the present instance.

But it is clear that in such enumerations we are on the brink of mediate counting, that is to say of the abstract equation $700 + 200 + 100 = 1000$, just as in the individual judgment mentioned above we were on the brink of the generalisation 'Every execrable ruffian ought to be hung'. '*Mediate* counting ! ' it will be said ; 'then we are in the region of inference, and no longer in that of judgment.' It is certain that when we speak of necessary connection between attributes, of hypothetical or mediate judgment, we are in the region of inference ; but it is not the case that we are therefore out of the region of judgment. By 'mediate' in the present connection I only intend to indicate a judgment which has for explicit subject a generalised or abstract attribute, and being free from any demonstrative or sign of perception must be taken as conditioning its predication by that attribute. Such a predication is mediated, i. e. is affirmed of any particular individual only through and in virtue of the attribute expressed in the subject. Mediate counting forms the transition between ratio and proportion just as mediate measurement does. Logically speaking every equation expresses a ratio, and a ratio becomes a proportion directly it is applied—directly its unit is taken as variable. Thus an algebraical equation, which exists with a view to a variety of applications, is *ipso facto* a proportion.

Complex counting is a case of mediate counting ; that case in which we count by units which are themselves sets of numerical places, i. e. by multiplication and division, which may for the purpose be taken to include addition and subtraction. The only difference between multiplication and division on the one hand and addition or subtraction on the other is the equality to each other of the subordinate totals in multiplication and division, which enables them to be counted as units and their number indicated like any other number, by its place in the series counting from 0 upwards. In this sense multiplication is a mere abridgment of addition; it is only a question of form whether we say $2+2+2+2=8$ or $2 \times 4 = 8$. This latter equation represents what is essential as well in addition and subtraction as in multiplication and division, a total analysed into factors and a process. Complex counting, as a case of mediate counting, shares its abstract, hypothetical, and necessary character.

Abstract Counting and Infinite Series.

ϵ. The processes of mediate counting deal with the construction and reconstruction of any given numerical total. Even the quantitative relation of part and whole in its extremest abstraction retains thus much of structural unity, that, given a total of units, it can only be dissolved or reconstructed according to certain rules of combination or analysis. But the quantitative unit *per se*, or rather the one-sided abstraction of the quantitative unit, the mere numerical place which no positive identity links with the other places of the series, has in it no principle of totality or limitation, that is to say no reason for stopping short after one set of such places rather than after another. Enumeration of units as such may be continued at pleasure, and the process of so continuing it without limit is summarised in the conception of numerical infinity.

We have here tracked to its genesis this paradoxical conception, in its right place so powerful for good, and in its wrong place for evil. It would not, perhaps, be beyond the province of logic to comment on its use in its right place, that is to say in mathematical science; nor could a more interesting subject readily be found. But lack of mathematical knowledge deters me from attempting such a comment with any degree of detail; for a logician is aware of the risk incurred by venturing beyond his knowledge, and as he preaches that there is no royal road to truth, must keep clear of the delusion that he himself has found one. But any one may offer a suggestion, and this I propose to do by saying that it seems to me most probable that the scientific use of the conception of infinity rests in every case on a justifiable neglect—justifiable, because that which is neglected has a known nature, and may be set down as insignificant either altogether or from the point of view of a specific purpose.

If, to take a coarse and non-mathematical instance, we set about any task in a way which is demonstrably perverse and inadequate, a looker-on is justified in disregarding our efforts. He will tell us that we shall not get it done in that way if we live to the age of Methuselah. Translated into logical phrase, his comment means that our way of going to work has not the element of totality; the successive efforts which make up the series of our activity, bearing no relation to the nature of the work to be done, do not include in themselves successive parts *of it*, and therefore, *as regards it*, have no tendency to come to an end and will (unless we choose to leave off) go on to infinity. We may even apply this illustration to a simple mathematical idea, say to the case of parallel straight lines. We may consider as the task to be accomplished such a change of direction in either or both of a pair of parallel straight lines that they should cease to be parallel to one another. And we may consider as the means adopted to bring about such a change the production of the two straight lines in their original direction. Then our supposed on-looker would say, 'You might go on for ever at that game;' 'You cannot change the direction of a straight line by producing it in its original direction.' Therefore it would be justifiable to neglect the production of parallel straight lines to infinity, in other words to pronounce that such production cannot alter their character as parallel straight lines, i. e. that even if produced to infinity (which they can never actually be) they do not meet. Probably such a case as this would hardly be recognised as an instance of the mathematical use of infinity, but in as far as it introduces the conception of quantitative infinity as a term in a positive definition it would seem to be at least analogous to such a use.

ζ. I may insert at this point some additional observations upon the question of an actual numerical infinity.[a] New Theory of Infinity.

The previous paragraph expresses the essence of my argument. It still seems to me sound, so far as it goes, and while fully admitting my own want of mathematical knowledge, I cannot see my way to cancel it, or the discussion which depends upon it.

Professor Royce,[b] I think, in his reference to this discussion, has done me less than justice by assuming that I insist, in the vein of Hegel's satirical digression,[c] on the mere tediousness of numerical infinity. But my treatment contains no reference to this topic (unless it were on pp. 191-2 ed. 2 where I reject such a treatment), and in considering the numerical series as the abstract schema of membership in a whole I have implied and stated that it has laws and properties which give it a character and interest of its own. That, taken by itself, it fails, as a consequence of its abstractness, to fulfil the conditions of an individual whole, seems to me obvious; and no less obvious that Professor Royce, in using it as a type of individuality, lays stress on possible incidents of the repetition of positive contents, rather than on the essential nature of repetition to infinity.

With regard to the present state of the question, I will venture on three remarks.

(1) I observed in the first edition of this book that 'relations may be given as actual which only an infinite series could represent on their quantitative side' (p. 168 ed. 2). The same point presents itself to me with regard to the conceptions employed in the recent theory of infinity. I see that certain problems and definitions are given, and that if their requirements could be carried out in actual series an actual infinity would be given; but I am unable to see that this latter condition is fulfilled. The character of givenness does not appear to me to be brought to a point beyond that illustrated by Spinoza in the case of a circle within a circle and not concentric. The figure, which would need an infinite analysis, is given. But it does not follow that the series which would Problem actual.

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[a] Ed. 2, 1911.

[b] World and Individual, i. 508 ff.

[c] Hegel, Wiss. d. Logik (ed. 1841), p. 257.

analyse it is given in and with it. I suggest that this analogy might apply to the conception of self-representative series and of transfinite numbers. I note that the latter cannot be reached by enumeration. I do not know what characters they share with numbers of the natural series; but it is obvious that they can hardly be given in the same sense.

Split in Whole.

(2) In the definition of an infinite whole, it is obvious, and I suppose intentional, that the relation on which the infinity depends, the one-to-one correspondence of a true part with the total of parts forming the whole, expends the material of the whole, if I may speak so grossly, at an immensely more rapid rate than the whole itself progresses. I am thinking of the case in which prime or square numbers are taken as a part of the series of natural numbers. This answers well enough so long as you have an enumeration *ad infinitum* to draw upon. But it seems to me that if you had anything approaching a given totality this spending in advance must produce a contradiction at once. At any point of the consideration, the part is miles and miles outside the whole. While you appeal to infinity as incomplete, this cannot matter. But the moment you seek to apply the character of totality, the arrangement seems to split in two. It is one thing, I repeat, to have a definition which if fulfilled would involve a given infinity, and another thing to show that it can be and actually is fulfilled.

Demand of Logic.

(3) I believe Mr. Russell would agree with me that fundamental principles of Logic could not be overriden by the practice of mathematics. If they really conflict, he would admit, some condition has been tacitly imposed[a] in the transition to mathematical conclusions, though as conclusions in mathematics they may be perfectly sound. But he holds that they do not conflict. His view is that a true Logic is one with the principles of mathematics, and carries the conclusions of the latter without reserve. I cannot see my way to escape from the former conviction. We may be given something else of great interest and value, but unless it fulfils our simple requirements, we cannot accept it as an actual infinity. On the other hand, we can show an actual infinity, in one

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[a] On the nature of mathematical abstraction see pp. 182–3 below.

sense of the word, in the character of wholes with parts such as to embody and reproduce the spirit of the whole. Such parts, in some degree, are the organs of an organism or the institutions of a nation. And in some degree, by what seems to me like special pleading, the character of such wholes is discovered in the mathematical concept of self-representative series.[a] But I cannot think that the two conceptions can really be united, except in the sense that it is probably true that a real infinite, or self-contained individual whole, could not be numerically represented except by an infinite series, which I take to be the same as saying it could not be numerically represented at all.

Infinite number in general.

η. I will now continue the discussion from which this was a digression.

More subtle and interesting are cases in which the continuance of the series makes a difference in the task to be performed, but the whole possible difference can be shown to fall within certain known limits. These cases, which I presume to be of the nature of infinitesimals rather than of infinites (both of which must fall under the head of infinite *enumeration*), may be reduced to the same class as the former if we reflect that in the former also the difference fell within known limits, but these were limits of *kind*, whereas we are now speaking of limits of *quantity*. In cases of this second type we know that a series could be continued to infinity, and that some difference would be made to the problem before us by this being done ; but, on the other hand, we are aware of a limit within which the whole series must fall, and we are therefore able to pronounce that the difference which can be made by its continuance after a certain point is, at least for our immediate purpose, a negligible quantity. Such, I take it, must be the principle of any process which determines e.g. the area of a circle by treating it as between the area of a polygon inscribed in the circle and that of a polygon described about it. So far as I follow the exoteric utterances of mathematicians,[1] the principle of abstraction based on a positive knowledge of the capacities of a series must be at the root of the employment of mathematical infinity.

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[a] A. E. Taylor, Metaphysics, p. 116 ff.

[1] I have in mind more especially Mr. Spottiswoode's Presidential Address to the British Association, which unluckily I cannot refer to.

But our immediate business is with infinite number considered logically, i.e. with a view to its general place in knowledge. And from this point of view we have to notice—

(1) That the idea of infinite number has its genesis in a one-sided abstraction, viz. in the notion of counting without having anything in particular to count,[1] which corresponds to the idea of difference without identity, and of parts without a whole. By such an abstraction the enumerative judgment is destroyed, the essence of judgment as such—the exhibition of identity in difference or of the whole in its parts—being withdrawn from it; and the names of the numbers are turned into a meaningless repetition, the purpose of enumeration having disappeared. We are no longer saying 'One tree, two trees, three trees', &c., but we are merely saying 'One, two, three', and it is for this reason that we need never stop.

(2) Being one-sided, the idea of infinite number is self-contradictory. The essence of number is to construct a finite whole out of homogeneous units. The idea of numerical infinity arises from neglecting the continuous nature of the unit, and therefore omitting the element which alone arrests computation at one number rather than at another. Thus an infinite number would be a number which is no particular number, for every particular number is finite.

(3) It follows from this that infinite number is unreal. This does not mean that there may not actually be more units of one kind or another than we can count in the time at our disposal or in any finite lapse of time. The statement deals with the nature of number, not with the extent of the sensuous universe. If, to put a common idea coarsely, we are asked, 'Supposing that you *could* travel through space for ever, and never come to an end of it, must not space contain an infinite number of units? or even if you can go on subdividing a given portion of matter for ever, must it not contain an infinite number of parts?' to such questions we could

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[1] A series which is *ex hypothesi* infinite comes under the head even if it seems to have a positive nature. For its nature *ex hypothesi* does not determine the number of its units.

only reply, 'Things or the parts of things may quite conceivably transcend our power to count. But except in view of a finite goal, number does not help us, does not tell us anything, grounds no ratio of parts to whole. We should in fact never give up counting any units that had interest for us, and should in doing so always be at some finite number. But if it could be miraculously revealed to us that there was no end, then I think we should stop counting, unless the units in question entered into subordinate or graduated totals which had an interest for us. Thus we go on counting the stars for definite reasons. Their relation to us is graduated, and several subordinate totals within their number have already been completed by enumeration; e.g. stars of the first eight magnitudes have been identified and counted. In counting them we have always in view some definable total to be constructed or to be corrected. Who counts the waves of the sea? The hope of complete enumeration is the justification of counting.'

Then why do we count the years and centuries? Do we pretend to know that they will have a numerable sum? and when they have reached it, do we imagine that the race will survive to take an interest in the completed enumeration? In the first place, for each of us time seems to have an end; and in the second place, all history is parcelled out in overlapping epochs which we have an interest in measuring. We do not in fact ordinarily know or compute the whole numerable series of years that has elapsed since the first events of ascertainable date in history; we adopt this or that era according to some overpowering historical interest which makes it seem to mark a fresh beginning. We do not count the years to ascertain their total quantity, but to give them names by which we can fix events; and as a means to fixing the relations of events we no doubt desire to note the quantitative relations within the total of historical time which has elapsed down to any given present. If we are pressed further, and told, 'But, after all, the years may go on for ever and the human race may go on counting them for ever', we can only reply that the faculties with which we are endowed refuse to express this 'ever', that at any point taken in the series we should be at some finite number, and that if a conviction of the endlessness of the series could be miraculously impressed upon our minds we could only conclude that, except as a record of the past, counting the years was an unmeaning form, seeing that the nature of the series could not be represented in number. A very simple case of enumeration *ad infinitum* would be that in which, by persistent errors of identification, we should count the same objects over again, round and round, without being aware that we were doing so. In such a case it is obvious that the conception of number would be destroyed so far as these objects themselves were concerned, though if they were at known intervals of space we might none the less use them as a measure for other things. When we measure with a foot-rule we do in fact count the inches marked on it over and over again in this way, not for their own sake or the process would be infinite, but only for the sake of some other quantity which we characterise by them. In this respect the inch-marks on the foot-rule correspond to the physical changes which indicate the day and year, and which serve as a measure for occurrences other than themselves.

An infinite series, then, is not anything which we can represent in the form of number, and therefore cannot be, *qua* infinite series, a fact in our world. Relations may indeed be given as actual which only an infinite series could represent on their quantitative side, such as the ratio of the diameter to the circumference of a circle. But for this very reason they never are adequately represented on that side, although we may know and argue from the positive character of the series, which *ex hypothesi* its prolongation to infinity is not to change. Our constructive judgment requires parts *and* a whole to give it meaning. Parts unrelated to any whole cannot be judged real by our thought. Their significance is gone, and they are parts of nothing.

Thus it is nonsense to speak of any definite number, say 100, as a portion of number, in the sense in which a foot is a portion of a yard, or a minute of an hour. The question 'what portion'? at once disposes of any such relation. Number as such cannot be identified with any particular total such that a given number is a definite fraction of it.

Closely allied to infinite number, and in a great measure depending upon it, are the conceptions of abstract or ideal time, and of abstract or ideal space, tending respectively to generate ideas of infinite time and of infinite space.

Abstract and Infinite Time

θ. In speaking of Comparison we saw that every 'now' tends to become a part within 'now' and 'then', and every 'then' again within 'then' and 'then'. This analysis is very gradually brought about, speaking historically, in the evolution of the tense-system. The sense of Time is in the first instance the mere consciousness of continuity in succession, that is, the mere perception of *a* succession or process of change. This sense however being only possible through setting off the succession against a comparatively permanent background of consciousness, is in embryo the comparison of successions, with the development of which comparison measurement of Time, and with this the abstract idea of Time, are brought into existence. The measurement of Time consists in the equation of one set of perceptible changes identified by a common nature, to another set of perceptible changes, in the sense that the beginning and end of a numbered series of the one coincide with the beginning and end of a numbered series of the other. The enumeration of phases of one series that coincide with one or more phases of the other series might conceivably be undertaken apart from any belief that either series has a constant duration in time, i.e. if repeated, would occupy the same amount of duration as before; but in enquiring whether such a belief actually exists we must distinguish between the reasonable doubt whether any portion of any series ever can or will be repeated absolutely without physical modifications which may affect its duration, and the unmeaning doubt whether a series assumed to be repeated without physical or causal variation may not nevertheless have varied in the absolute amount of duration which it occupied. The former kind of doubt will only lead to a demand for criticism and reciprocal adjustment of our time-measures, together with the temperate scepticism which our lack of exhaustive knowledge must produce about our acquaintance with even those natural conditions which we have most fully investigated. The latter kind of doubt, if pressed to its conclusion, would reduce the enumeration of successive changes to a statement of mere numerical fact wholly devoid of significance. Such enumeration would not be impossible, but would scarcely fulfil the requirements of judgment. It would be on a level with the mere repetition of the names of numbers. It is hardly necessary to give instances ; every one can see at once that if we entertain the idea of variation in the measures of time occurring without any reason, all equation of successions becomes futile, and ceases to afford any ground of expectation or of inference.

But though a doubt of this nature has been mentioned by great writers, yet it has never been extended to affect the only element of our time-perception which is essential to its utility, viz. the constant ratio obtaining between the successions employed as measures of duration. This limitation of the doubt arises from the confusion in which it originates, the confusion which consists in treating the whole as if it were a part. Time, for us, is a relation, a ratio, and its constancy is the constancy of this ratio. If all processes in time maintain a constant ratio to one another so far as they are unaffected by physical modifications, then there is no meaning in suggesting that tried by some unknown or impossible standard[1] they may be variable. But yet this suggestion is a natural suggestion arising out of a natural confusion. We naturally frame an idea of duration in itself, as that which has successive parts really and absolutely equal, because equality of successive parts *in terms of one of our time-measures* is what we are always endeavouring to ascertain about some other time-measure. And we forget that equality, if we exclude from it the idea of measurement, ceases to be an intelligible conception. This ideal of duration then, whose successive parts, though *ex hypothesi* not measurable, are assumed to be equal, we actually set up as an imaginary measure against the totality of consenting measures and processes the ratio of which to one another forms our world of time, and is merely represented in an abstract and ideal form by 'duration in itself'. 'Duration in itself', says

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[1] E.g. by measurement of successive phases against each other, which is *ex hypothesi* impossible.

Locke,[1] 'is to be considered as going on in one constant, equal, uniform course. But none of the measures which we make use of can be known to do so; nor can it be assumed that their assigned parts or periods are equal in duration one to another'.

But 'duration in itself' not being a relation of successions, could not be perceived as time, nor could any being that perceives as we perceive be aware of equality or the opposite in its successive parts; for equation is necessary to equality. If, on the other hand, we set it against our time-measures, then it becomes one measure among many, and in case of a discrepancy that should remain unaccountable it is only caprice that could choose which we should regard as right. The question in fact would be unmeaning, for the whole discussion obviously originates in the attempt to transfer an attribute which depends on a comparison to a set of terms considered in themselves and apart from comparison, and then to suggest a comparison between them (the supposed equal parts of ideal duration) and the totality of our time-measures of whose reciprocal relations those parts are an idealised form.

But this confusion does not naturally originate a doubt of the constant ratio, physical disturbances being allowed for, of our measures of time *compared with one another*. For it is this constant ratio from which the idea of duration as such, hypostasised by abstraction into duration in itself with equal successive parts, is derived; and to doubt the constancy of this ratio would be to deprive ourselves of that idea of duration on which the confusion itself depends. If oscillations of a pendulum of fixed length, such as are normally equal when measured by the rotation of the earth, may vary without a physical cause affecting either of the compared motions, the conception of uniform duration is destroyed, and no equation of successions is more than an isolated fact. On such a hypothesis there would be nothing to generate the idea of uniform duration, and no measure of time with which to compare that idea.

The antithesis between duration in itself and our measures of it is merely a case of the antithesis between the thing-in-itself and our knowledge of it. It is possible, though unmeaning, to doubt whether our knowledge as a whole is real knowledge—i.e. corresponds to some test or condition which we may imagine as imposed upon it from without. The possibility arises from our possession of an ideal of knowledge, which by an act of abstraction can be set in antithesis to the actual whole of knowledge from which it is abstracted. But it is impossible when operating in detail upon the matter of experience to doubt the existence of rational connection in any one particular group of appearances; for by so doing we paralyse the understanding, which can only act in the search for causes, and leave neither knowledge to condemn nor an ideal by which to condemn it. We shall have to return to this subject in a later chapter, when we speak of the postulates of knowledge.

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[1] Essay, Book II, 14, 21.

We have just seen that it is idle to treat the whole as if it were a part—our reciprocally adjusted measures of time as if they were one measure among many. It is also worth while to observe in the same instance the impossibility of making a part do duty for the whole, which is strikingly exemplified in the ultimate data of time. No process of perceptible change is a trustworthy measure of time except in as far as it is equated with other processes of known constancy, and observed to be in itself free from physical causes of variation. That is to say, the part can only be taken as a definite standard when it has been criticised in the light of the whole. This is true of all premises of knowledge.

Amount of Time, like all quantity, is measured by the enumeration of units which have a known value. But, unlike any other kind of quantity with the exception of space, it follows number not only into mediate counting (all quantity does this), but into enumeration *ad infinitum*.

Simple enumeration in the case of Time gives such judgments as 'He died three years ago', 'It is seven days since I saw him',—which expand by reference to a standard of succession the mere indications—probably of various nature and origin [1]—conveyed by the tenses.

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[1] Tense need not have originated in the idea of succession at all; and may often have arisen out of the expression of emotions or anticipations or out of the mere negation of presence as a perception of a certain kind.

Mediate or complex counting in Time gives such judgments as the equation '365 days = one year'. Such judgments deal with our real and ideal time as the result gained by comparison and equation of actual processes in experience. But the abstraction which stands as subject tends to assert itself as a something apart from the actual processes whose relation it is, and thus as we have seen generates the conception of 'duration in itself' or 'absolute time', which again having lost the element of totality precipitates us into the idea of infinite time.

Enumeration to infinity, when applied to the parts of time, has characteristics analogous to those of infinite number, but more striking, inasmuch as time is closely bound up with the attributes of actual existence. Infinite time is, like infinite number, an *unending whole*, which is a contradiction in terms. That is to say, we are prevented by the nature of our minds, if by no other cause, from attaching any meaning to infinite time as a quantitative expression indicating an aggregate expressible by enumerative judgments. Whether the problem (for so we must consider it) which is put before us in this form is capable of becoming, not a problem, but a fact, in some other form, is a question which does not concern us here. The instances which are alleged to show that an infinite series may be *given* point somewhat in this direction; I allude to such an instance as that of the relation between diameter and circumference. However this may be, it remains true that infinite time, *as a mere quantity of duration*, is a phantom generated by a meaningless abstraction.

But, as we asked whether there may not *be* endless numerable units, so we may be challenged to ask, May there not or must there not have been an endless chain of actual occurrences in succession? And if we are prepared to deny reality to every endless series, must we not first of all deny it to the actual world as in time, and we may add by anticipation of the next section, as in space? There can be no doubt that the relativity of events and appearances in time and space does involve for our minds an infinite progression in the way of referring one thing or appearance to another as its cause or explanation, or at least as in some way its determinant.

We can hardly conceive that we are really counting in a circle, but our position is just as hopeless as if we were. Our ideas of reference, determination, causation, do not allow us to fancy a first event, a beginning of time, or a limit of space. On the other hand, so far as we can understand, it is impossible for such a succession as we postulate to be actual in the sense in which a known section of history is actual. It is not merely something more than we do construct as a whole; it is something the essence of which is to be incapable of construction as a whole. We may say if we like that we are bound to think of such a succession as actual in the sense that it is a problem inseparable from the relativity of our world; but we cannot take the endless series *qua* endless as a positive element in our organised experience.

In dealing however with the succession of actual events having positive character and content we are in spite of their serial appearance on different ground from that of abstract time or equable succession as such. Actual events—the history of our world—have more in them than a mere series. We may say either that the world in time and space is not mere succession and externality, or that the real world which intelligent perception presents to us is not merely a world in time and space. The human mind and will are always busy in turning a series into a coherent and almost individual whole, projected more or less definitely on a scene of time and space, but not exhausted in its meaning by the nature of that background. Greece for instance or England are not mere 'geographical expressions'; and if they were they would still have a physical unity of a deeper kind than the juxtaposition of extended units or the sequence of a series. They are historical realities, but their coherence lies in their meaning. Therefore in denying that for us, in our way of understanding, an endless progress can be a real and controlling factor of organised experience, we do not deny the reality of the phenomenal world *as presented to intelligent perception.*

Abstract and Infinite Space.

1. The nature of Space is in many respects analogous to that of time, and bears on the whole a similar relation to the system of number, with its degrees of simple or categorical

counting, mediate or complex counting, and counting *ad infinitum*. The corresponding grades of abstraction in dealing with space may be identified as—

a. The measurement of actual distances.

b. The theoretical relations of spatial qualities, including the whole of the mathematical sciences, excepting what may be included in the mere system of number, viz. all forms of simple and generalised (algebraical) arithmetic; and

c. The conception of infinite space.

In defence of the subdivision here adopted, I venture with a good deal of diffidence to suggest that the idea of infinite space is not an idea belonging to geometrical science. The employment of the idea of infinity in geometrical reasoning belongs, if the account above given of it is correct, to the second of the above heads, being really an abstract mode of describing a geometrical whole. If enough space is given to make manifest the positive nature of the whole before us, it would seem that no addition can really affect the case. Quantitative infinity may be a roundabout description of a direction or a quality. That a certain straight line is infinite may only mean that its direction is such that it can never meet a certain other straight line.

Measurement of actual distances

a. The measurement of actual distances is *prima facie* a case of simple measurement, and must obviously arise at the point where positions distinguished by the comparative judgment are discovered to have relations of distance and direction reducible to degree. Degrees of distance from assigned points, and proportions of such degrees, considered as properties of objects, pass into the determination of concrete individuals and of their characteristic attributes. In this respect we have considered them above. But also, receiving their significance from a system of equations by which all spatial magnitudes are brought to a common denomination, they contribute to reflection upon space in the abstract, a reflection which is developed by the process of enumeration applied to the parts of space when thus idealised and considered for their own sake. The system of measures, as we have seen above, is a connecting link between simple and complex measurement. It is *prima facie* a system of ratios, and requires an arbitrary starting-point to give it meaning. But when taken as general and typical it passes into a system of proportions, as we see in such a judgment as 'This map is on the scale of twelve inches to the mile'. There need not be twelve inches in any direction in the actual map before us; the phrase expresses a proportion, not a simple fact of enumeration. It is not necessary to know, for the purpose of the scale, how much absolutely (i.e. in the mass of other relations) an inch or a mile is; we can interpret the scale if we know how many inches there are in a mile. The abstract numerical expression 1 : 63360, which is a *proportion* as governing the relation between every part of the map and the corresponding part of reality, represents what the formula 'one inch to the mile' really means to say. We may compare this case with the financial expression 'five per cent'; some newspapers *will* print this as £5 per cent, which is a confusion between singular or merely actual, and generalised or proportional ratio. They do not really mean that £5 need be concerned; their predication is as true if the interest in question only amounts to five shillings. The expression five per cent is simply a fractional or proportional expression.

Now, when, e. g. in the two instances just quoted, the reference to an arbitrary magnitude is dropped out, and when moreover the generalised equation is taken as expressing the relations between distances in space combined in a certain way, then we have ideal or mediate enumeration as it exists in geometrical science. 'Mediate' because the proportion contained in the equation is affirmed of reality as qualified by definite spatial attributes, which therefore enter into the content of the equation as interrelated conditions. The equation of an ellipse is a hypothetical judgment asserting that the axes of an ellipse (or other quantities involved in an ellipse) being treated in a certain way will be always equal to themselves treated in a certain other way. It is obvious that being mediate this geometrical enumeration or computation is also complex;[1] for the conditions by which it is mediated may involve units of any degree of numerical complexity. Thus judgments dealing with squares and cubes, still more with conic sections, presuppose, *on their numerical side*, more or less elaborate enumerative processes as involved in the formation or apprehension of the spatial unit.

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[1] See pp. 160–1 above.

Geometry.

b. The generalised relations of spatial attributes form a systematic science with a distinctive object-matter.

Individuality of Spatial Figures.

(1) 'Any two sides of a triangle are together greater than the third side.' 'Triangles upon equal bases and between the same parallels are equal to each other.' 'The square of the hypotenuse is equal to the squares of the containing sides.' 'The angles at the base of an isosceles triangle are equal to one another.'

Such judgments as these are among the simplest results of mediate enumeration[1] as applied to space, and they present an obvious peculiarity shared with them in a high degree by complex judgments of mere number, and in a less degree by judgments that compute time. Although geometrical science proper consists, as a science, exclusively of the equation with each other of variously described spatial units, which, if we further consider the sciences of motion, we must take as referred to units of time, yet under this 'variously described' there lurks a whole classificatory science of forms possessed of structural unity and quasi-organic relations between part and whole. Thus in the definitions and definitory judgments of geometry, as in the inductive judgment of enumeration, there is a revival of that structural subordination of part to whole which, though dormant, yet is never dead so long as judgment has a meaning. The triangle, the square, the circle, the ellipse, though each of them capable of being exhaustively defined by generalised enumerative processes dealing with homogeneous units, have also an aspect of structural unity and subordination of parts to the dominant quality of the whole. Both clauses of this statement are however subject to reservation.

In the first clause, the expression 'homogeneous units' is not entirely adequate to the facts. Distances in the three spatial dimensions are indeed as distances homogeneous, but it can hardly be said that an angle is homogeneous with a distance or direction, although a proportion of distances is an exhaustive measure of an angle. Therefore, though an angle may be represented in terms of distance, yet in considering the elements in the spatial structure of a square or triangle we must assume the angle as well as the straight line. The apprehension of converging direction is hardly given in the mere apprehension of direction.

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[1] For the further distinction between arithmetical and geometrical reasoning, see II, chap. ii.

In the same clause we must qualify the assertion that *all* geometrical shapes can be exhaustively defined by equation of quantities. I presume that the impossibility of squaring the circle implies a difference of kind between circle and straight line which is disguised but not destroyed by the efficient methods employed to express the one in terms of the other.[1] Here again, in the adequacy, as distinguished from exhaustiveness, with which quantitative equations represent characteristics of kind not wholly reducible to quantity, we find an analogy with the various systems of necessity which are superimposed upon one another in the organic and moral worlds.

And the truth of our second clause would be a good deal interfered with, if not annihilated, by enlarging our list of geometrical figures, and by regarding every figure in all its possible variations. As to the first point, every spatial figure is a geometrical figure; and those which we mentioned, together with others that occur to the mind at once as commonly considered in plane and in solid geometry, have no real prerogative of existence to the exclusion of trapeziums or any other irregular figures, even if partly curved and partly rectilinear in outline. The world would be easier to explain than it is (or so it seems to a superficial glance) if irregularity, ugliness, and disease did not, as they do, exist by law and necessity[2] just as much as symmetry, beauty, and health. It may be said in defence of treating the regular figures as if there were no others, that all others are reducible to them, —all rectilineal figures to triangles, and so on. But this is a mere ideal reduction by measurement, and in no sense a deduction of the existence of the one from that of the other.

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[1] The quadrature of the parabola shows that the impossibility of squaring the circle does not arise from the mere difference between curve and right line.

[2] I do not say by the same necessity; I only mean that they result mechanically from natural conditions. Any limitation of which this statement may be capable must refer to a distinction of kinds of necessity.

And again, most 'figures' of which we speak are really classes of figures, even if we take all 'similar' figures as the same, i. e. disregard size and only look to characteristic proportion. I do not know why we should not take the conic section as 'a figure' and treat the circle simply as a case of it. Anyhow the ellipse comprises in itself a whole class of figures which are qualitatively quite different from each other, and pass by imperceptible gradations into figures which are not elliptical. By insisting on facts like these we might melt away the individuality of typical figures, and exhibit every group of geometrical shapes as destitute of common characteristics beyond those which flow from the mere genetic relation itself. Thus, for instance, an ellipse when just passing into a circle or into a straight line has none of the characteristics which we associate with elliptical form, although the analytic relation by which its nature is theoretically determined may be within the limits which must as a matter of geometrical classification be assigned to that figure. Such a treatment would be the triumph of explanatory theory and necessary connection over individuality and characteristic quality. But it is a treatment of which, in geometry, the facts admit, and which even in the organic world is rendered indispensable by the idea of continuous evolution.

It is right that attention should be drawn to the above reservation. No habit is more pernicious than that of assuming what is obvious and familiar in a certain sphere to exhaust the contents of that sphere. Every figure is geometrical; and even if all irregular figures can be reduced for purposes of reckoning to the more regular types, this does not justify us in speaking theoretically as though there were no figures in space besides those which have been selected as typical by geometricians.

We must meet such conceptions now as we shall have to meet them again, perhaps more than once, in the evolution of the judgment, by the reflection that one positive existence cannot, by the mere fact of its existence, cancel another. There may be non-elliptical ellipses, figures which fulfil one set of conditions that mark the ellipse, but lack attributes without which we hardly recognise the figure ; and there may in the same sense be or have been (time makes no difference) non-human human beings. And explanatory theory may be able in both cases to trace step by step without *saltus* or miracle the transition from the one phase into the other by development of elements fundamental in both. But this will not obscure our perception of the elliptical shape of a characteristic ellipse, or of the humanity of a typical human being. The typical human being is made typical, as we shall see, by real teleology. The typical ellipse is only typical through a subjective quasi-teleology. This is the difference between the two cases.

Thus, in spite of the above reservations, it remains true that in complex enumeration as applied to Space the antithesis of individuality and necessity is strikingly illustrated. A generalised relation of distances, not obviously differing in kind from any other generalised enumerative relation, when interpreted into an actual figure, *may* at least produce a structural totality complete in itself and of a marked individual character. It is enough to mention such simple instances as the equilateral triangle, the square, and the circle. These totalities do not refer outside themselves for definition in the obvious sense in which simple magnitudes are forced to do so. They are primarily cases of internal proportion, of proportion, that is, between one and another element of a single totality. Relativity appears in them in another shape than that of simple equation with absolute (in the sense of arbitrary) standards of magnitude. A particular triangle or ellipse is relative and finite chiefly in the sense of being derived from arbitrary conditions, no one value of which has any prerogative of existence as against any other, and an extreme modification of which will always destroy the essence of the figure. There is also a further sense in which all natural existences are finite as compared with mind, because they cannot refer to themselves, but only *are referred* whether to themselves or to external conditions.

The subordinate totalities of Time, such as the hour, day, and year, are not really cases of the same principle, for they are mere aggregates of units without a limiting totality, and are simply formed by arresting enumeration at intervals prescribed by external interests. A better parallel may be found in the numbers of the numerical series which are distinguished by any peculiarity, e. g. prime numbers, squares, cubes, &c., if we consider these peculiar cases in their relation to the one unvarying process of adding together homogeneous units. This simple synthesis of number may be regarded as analogous to the mechanical or necessary aspect of an individual thing or figure, and its peculiarity as square or cube to structural totality.

Existence of Figures in Space.

(2) Figures in space then, and numerical relations, although abstract and ideal, and arbitrarily selected out of continuous sets of value of which no one has existence by preference to the other, yet seem capable in a sense of possessing characteristic quality and self-sufficing totality. What is to be thought of their *existence*? Do such judgments as are mentioned on p. 177 involve the assertion that the qualifications of Reality which form their subjects are actual, and if so, in what sense are they actual? The difficulty of the question is only displayed in its true extent if we add to the above instances some ideas for which actual existence, as diagrams in books or as thoughts in the minds of individual students, cannot with probability be claimed; e. g. the idea of a polygon with a thousand and one equal sides, of any trapezium chosen at will, or of any irregular figure or high number.

It is clear that the square or triangle *qua* spatial figure has no actuality which does not equally belong to all such less familiar shapes, and therefore mere presence in the individual mind is not the existence in question. And indeed to speak of it as such would be to enter upon a vicious circle which would stultify the judgment; for it is essential to the judgment to affirm a reality outside itself, and it would be too ridiculous that your judgment should refer merely to the content of mine as the reality asserted, and mine in turn to that of yours. It is obvious that the two manifestations of the thought-function are on a level, and if each refers to the other, each might just as well refer to itself, i. e. find its truth in the simple fact that it is made.[1] A triangle must be just as real[a] when no one is thinking about it as when many students are engaged upon the conception of one. This is of course not the same as to say that spatial figures do not depend on the spatial consciousness, or on consciousness at all. We are only saying that they cannot depend for actuality on one particular reflective consciousness of those particular figures. The world as it is for perception and intelligence is the object-matter of our whole enquiry, and we have no occasion to raise a question that assumes the destruction of the object which we are considering.

The absence of material existence and also of any mode whatever of particularisation in determinate forms (the selection e. g. of a typical ellipse not being justifiable on purely geometrical grounds) must however make a distinction between the actuality of the contents under discussion and that of material things or their sensible properties. Admitting, as we must admit for logical purposes, that Space is to be reckoned with as having a peculiar actuality of its own, still it is not clear in what relation geometrical figures, apart from the shapes of actual objects, stand to actual space. Geometrical figures as such, the subjects of judgment in geometrical science, are not the shapes of actual objects; they are not identical with any perceptible figure; they are not distributed through space nor present as special characteristics in any portion of it. I do not know how to describe them better than as a peculiar class of *laws* or *attributes* of the spatial relation as such, which are concrete in the nature of their content, though abstract in their medium of presentation. And in the same way the characteristic totalities of number must be taken as laws—embodied laws—of the enumerative relation of part and whole.

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[1] Reference to the world of meanings or objective reference (Introduction, sect. 7) is not merely reference to judgments in fact made by others. It is an inadequately conditioned reference to reality.

[a] This is overstated. Every thought, we must suppose, contributes something, however practically inappreciable, to the reality of what it thinks of.

We may illustrate the nature of such attributes by comparing them with any purely imaginary ideal content which bears (so far as an imaginary content can, for it is always, in my opinion, found wanting somewhere) the character of self-sufficing totality. Such an ideal content for instance is Shakespeare's Hamlet,[1] or the material spheres of ancient astronomy, or, to come nearer to our present subject, the conception of 'Flatland' as space in two dimensions only, with sentient beings confined to it. These conceptions, though doubtless based on elements of fact and illustrative of real conditions, yet exist only in the minds of those who read and think about them, or more strictly in the identical reference which these minds are stimulated to make to a world of meanings, but a world of meanings *explicitly* discontinuous with and detached from the actual world of fact, or what we call in short an imaginary world. Such a world is indeed maintained by judgment, but it is judgment of a peculiar kind and under peculiar conditions—subject, not like the common world of meanings to a wholly indeterminate, but to a *conscious* and explicit, abstraction from reality, which becomes semi-conscious in artistic fiction, and utterly lost and obliterated in mere error and superstition.

The figures which represent the properties of actual space are not imaginary in this extreme sense of the word. It might however be a question whether the difference between them and such ideas as have been mentioned is one of kind or one of degree. Both kinds of ideas it might be said involve abstraction from concrete perceived reality, both kinds are therefore hypothetical and not actual existences, and how far the abstraction is carried cannot be a question of principle. Nor, I must add, do I mean to insist on the manifest *contradiction* with experience, or self-contradiction when viewed in the light of experience, that some of my instances of imaginary ideas may be held to present. The distinction which I desire to draw is simply between abstract but real, and purely imaginary contents, when employed as subjects in judgment.

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[1] Omitting the considerations which arise out of the artistic truth of the conception, and taking it merely as an illustration of an imaginary idea.

Perhaps the distinction might be found as above suggested to consist formally in the *nature* of the abstraction to which the two kinds of contents are severally subject. A merely abstract content is subject only to the abstraction which its ostensible nature implies. The name of a figure in space is the name of a figure in space and not the name of a man or a mineral or of any material object. But the name of Hamlet for instance is and yet is not the name of a man. The name of a knot tied on an endless string is and yet is not the name of a reality in space. These imaginations are subject not merely to the abstraction which separates every content from all that is not included in it, but to a further abstraction which says, 'This is *a* meaning, but not *the* meaning you would take it for:' in other words, it is conditional within a world which itself can only be predicated conditionally and not directly of the reality with which we are in contact by means of perception.

Thus we may say, if we choose, that our ideas of actual space have for their meaning only possibilities, but these are at least *real* possibilities, that is to say, their fundamental generating relations actually exist in the world which centres in present perception. They are therefore as real as colour in the dark or as sweetness which we do not taste. And if we pronounce these attributes unreal outside the moment of perception, we have laid the axe to the root of the perceptible world. We might as well say that the wall in front of me is actual and that behind me is not. But what corresponds with these ideas to the actuality which colour and taste do partake of when perceived? Here we find a real difference. Such actuality in sensuous presentation they cannot have. But we do not of course admit that Reality is restricted to sensuous presentation. All we can say is that in all relations of actual things these spatial attributes make themselves evident as controlling conditions, and are introduced as conceptions without which the mind fails to construe the phenomena. They are abstract characteristics of the actual spatial relations of things, and are as much a fact for logic as any secondary quality, *qua* general quality; and they are not on a level with mere imaginations and fictions, even if these are consistent, or not notoriously inconsistent, with reality.

The quasi-generic judgment.

(3) The judgments mentioned on p. 177 are thus found to display in their own line of abstract evolution which we are now pursuing a character analogous *mutatis mutandis* to that which the Generic Judgment will be found to display in the concrete evolution of thought. The subject of each such judgment is Reality qualified as a structural whole which embodies properties rooted in an *actual* relation and controlling the consequences of that relation at every turn. The figures in question do not claim sensuous particularity and are not capable of it. The judgment therefore is a degree less generic and more hypothetical than classificatory judgments which retain much of the meaning of the collective judgment.

Yet the judgment in question *is* generic. In the first place, the particular figures which arise if particular conditions are assumed are individual totalities, not indeed having sensuous singleness, but self-identical as laws of space. And in the second place, although no doubt any series of such figures (triangles, ellipses, &c.) is in one point of view an infinite series [a] (the transition from value to value of the generating conditions being absolutely continuous), yet the whole falls within known limits, and is bound together by a characteristic quality which might probably be found to vary with the variations of the generating factors.[1]

Therefore a judgment like this presupposes, *not* as do the judgment of zoology a limited even if very large number of actual individuals forming a real historical unity though spoken of mediately and by help of an abstract qualification of Reality, but a series of laws regulative of form, or rather a law expressed in a series of forms, having positive common characteristics and bounded by definable limits within which the whole series must fall. 'All triangles' is hindered from meaning every particular triangle (i. e. variety of triangle—I abstract from particular sensuous presentations all through this discussion) not only as 'all men' is hindered from meaning every particular man, by the *practical* impossibility of dealing with such a meaning, but also by a theoretical absurdity, for all particular triangles would be an infinite series, which need not theoretically be the case with all particular men. But, as we saw above, a series which has a known positive character or falls within known limits may be treated as an actual unity in spite of its infinity. 'All triangles have their angles equal to two right angles' is a judgment about what is really a single continuous relation, but embodied to the mind's eye in certain salient types resulting from geometrical classification. It resembles the generic judgment more strikingly than appears at first sight, for the generic judgment too deals with a section of evolution in which a vast though not infinite array of transitional types has really bridged the gulf between the marked species which are familiar.

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[a] 'Then there are infinite series,' a critic may say. The case illustrates my point. If you could make actual all the cases of a continuous series, it would be a given infinite series. But I presume you cannot.

[1] It seems obvious that as one generating factor, e.g. one axis of an ellipse, approaches disappearance, we should expect the characteristic quality to diminish. But any such conception is not easy to carry out. E.g. if we take equality of the axes as the characteristic point in a figure of the conic section class, we get the circle as the characteristic type to the exclusion of the ellipse. The question has perhaps only an aesthetic interest.

Infinite space.

c. Infinite enumeration applied to the parts of space is the last result of abstraction in this region. Here, as in time and number, we have the idea of the absolutely homogeneous part, i. e. the part whose repetition has no tendency to generate a whole. The idea of infinite space is the idea of the endless synthesis of such parts, which must always present to us the appearance of an unsolved problem. If the problem has a solution, it must consist in changing the point of view from which we regard it, as if, to repeat an illustration which I employed above, we were suddenly to awake to the fact that we had been counting parts extended in a circle and not in a straight line.

Will not the doctrine known as the subjectivity of space and time help us to explain the nature of this contradictory reality? Up to a certain point it has undoubtedly done good service by showing that the difficulties which attach to sensuous reality are rooted in the nature of the percipient intelligence itself, and must be reckoned with as inherent in sensuous experience. But I am unable to see that the 'subjectivity' of these forms of apprehension can carry us further, unless or until we are enabled to put something better in their place. At present we seem only to have learnt that the difficulties of knowledge are not external to it, but are inbred and inevitable, at least so far as concerns the series of sensuous phenomena. But we gain nothing, so far as I can understand, by attempting to erect a world beyond as a non-sensuous counterpart of the sensuous series. If a counterpart, then it would seem to share the difficulties attaching to this series, while as non-sensuous it lacks the compulsory reality of sense-perception.[1] Our present knowledge rather points to the conclusion that *if* we are to attain something less contradictory, more capable of self-sufficing reality, or if we like to use the phrase, more above sense, we must look for it in facts and purposes which deepen the significance of life, not in a shadowy counterpart which repeats the world of sense without enhancing its value. Mere series, mere space and time, we must always remark are mere abstractions; and though no human knowledge is free from relativity, i. e. from the reference to what falls outside it; yet on the other hand no actual human knowledge is, like the abstract infinities, mere relativity and nothing more. There would be some justification for saying that, as *contrasted with the concrete structure of individual things*, a 'subjective', i. e. artificial and unreal, character might be attributed to number, space and time as infinite wholes; on the ground that they conflict with the nature of actual fact however comprehensive, and that the extension to infinity deprives them of relation to the phenomena in which they are known to us. But the distinction is hardly sound, for it is at best one of degree. The difficulties of relativity do not wholly cease as soon as we turn away from the abstract infinity of mere number, time, and space; in common ideas of causation they affect the actual content of phenomena. From the very beginning of knowledge, as I have tried to point out, Absoluteness co-exists with Relativity; but it is impossible to form categorical judgments of a comprehensive type until the idea of causation has been freed from its primary implication of an endless series.

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[1] Lotze has well brought out the difficulties attaching to the conception of an 'intelligible' counterpart to the world of sense. The strange thing is that they do not appear in any way to make him discontented with that conception.

The question might be asked, why are we exclusively tempted to demand the reality of infinite time and infinite space when there are other abstract conceptions of homogeneous parts not subordinate to any whole, which might in the same way be pushed to infinity? In the first place, it may be replied that abstractions of this class are not so common as might appear. The essence of them is that progressive enumeration shall not tend to modify their character. Thus intensive quantities, such as infinite force, infinite velocity, are conceptions of a heightening the later stages of which would modify the earlier and not remain indifferently beside them. Therefore although the phenomena of velocity or of force do suggest the idea of quantitative infinities of those kinds, yet they do not impel us to judge those infinities to be real, because the perceived forces do not in their nature refer to and presuppose infinite degrees of themselves, but rather each manifestation *per se* excludes the infinity which would involve a qualitative change in itself. Infinite force or velocity is as contradictory an idea as infinite space, but is not in the same sense a problem or a paradox, because it does not in the same sense claim reality.

And secondly, Space and Time may be called the Categories of sense. That is to say, they are the only principles according to which the world of sense-perception, both of our own immediate feelings and of external objects, appear to us to be possible. The question is not merely whether we can imagine the absence of either or both. I take it that experience would reply to this by saying that we cannot *seriously* imagine (i. e. conceive with full consideration) the absence of either *in a world of sense-perception.*[1]

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[1] Psychologically speaking, I should suppose that we may lose consciousness of either, perhaps more readily of space, e.g. when listening to music. Time is perhaps the more importunate of the two ideas because it extends to our inner feelings, &c., and I suspect this to be the reason of Lotze's notion that time is more 'objective' than space. Yet we may, of course, in a fit of absorption lose consciousness of time. I do

But the real point is not merely psychological, although of course in dealing with it we must appeal to facts of the mind. The point is that the very character and essence of sense is isolation, and therefore in apprehending variety, series. So extreme is this character of isolation that the presentation of sensuous contents even as a series is due to an intelligence that goes beyond sense. Space and Time are *for us* the first work of knowledge, as the conquest of them is the second. But all we are concerned with here is that, assuming the impulse to construct out of our sensuous perception a whole of the same nature as itself, we cannot but attempt to erect space and time also into wholes, an attempt which is frustrated as we have seen.

Mechanical view of Universe.

iii. But, lastly, reflective science in pursuing an analogous attempt does meet with analogous difficulties. It assumes as further characteristics of the sensible world the abstractions of matter and motion. Matter and motion are the abstractions in which the sensuous world is reduced to homogeneity in order to be susceptible of quantitative treatment, and in this treatment they are able to a large extent to represent genuine and actual relations of that world. In this respect they correspond to the structural classifications of geometry, and form the content of mechanical science. It would hardly be true I suppose to say that the infinity of matter and motion in space is an inevitable paradox to the scientific consciousness. It appears possible to conceive of the universe as a coexistent finite mechanical whole, demanding no determination from without. But this is only because the determination from without is thrown back in time by the doctrine of the eternity of motion, which, with a similar doctrine applied to matter, introduce the infinite series under the guarantee of the law of causation. Here again we have the insoluble problem which arises from the relativity of the sensuous world and presses upon us in its naked form as soon as, by reduction to homogeneity, the element of absoluteness or totality which helped to balance it is destroyed. It is to the latter element that we really look

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not suppose that these half-illusory states are at all perfect in their neglect of the non-obtruding element.

for a solution in the degree in which it is possible. The infinite series cannot itself be reality, but so far as we can transmute the series into an articulated whole, so far we can gain a reality out of it.

This distinction suffices to justify the well-grounded conclusions of science respecting the past and future of the material universe. Such conclusions are contributions to the projection that forms the actual world in which as percipient and intelligent beings we live. But with eternity in the shape of infinite regress and progress such a projection can have nothing to do.

We have thus traced to its climax in mechanical science that form of identity and difference in which an identity is regarded as the sum of the differences in which it is presented. This one-sided aspect of identity and difference is what takes the shape of whole and part in the strict or quantitative meaning of those words, the meaning in which the whole is taken as equated to a relation, whether particular or generalised, of homogeneous parts.

It naturally occurs to us to ask at this point, how, if quantity is homogeneous, and if *proportion* is, as we have reckoned it to be, generically a quantitative relation, the concrete individual (see. p. 127) whose characteristic quality takes the shape of proportion differs *in content* from a relation in number, figure in space, or system of motions which, though purely quantitative, is also, as we have seen, characteristic and self-contained. In the first place, we have spoken of the quasi-individuality which does attach to the structures that embody geometrical and, we may now add, mechanical laws. And we must remember that their quasi-individuality is only made possible by a certain revival of the qualitative element within the whole of quantity, even if the quality so present throughout the parts is, like the curvature of a circle, constant in all of them. And in the second place, we have to point out that in a *true* concrete individual its individuality exists in the form of a conscious purpose, a real teleology, and is the cause of its homogeneousness, the proof of this being that if the elements are isolated and removed from the individual they fall back into disparateness. The proportion in which its parts are held together is as we saw[1] secondary and not primary; it is a proportion between proportions. And though it is true that in the simplest forms of comparison, such as matching a colour, judgment and equation are hardly to be distinguished, being in fact as yet in their common germ, yet the peculiar secondary unity of a complex whole characterised by internal proportions is not fully expressed by reciprocal equation of its elements. Thus the concrete individual is from the first characterised by rather than equated to pure quantitative relations of parts. With abstract totalities just the reverse is the case. Their elements, homogeneous to begin with, are placed arbitrarily in any whole (in as far as the elements of various curves resist such construction they are not pure quantity), and only acquire the semblance of a relation to a whole by their non-resistance in such a construction *ab extra*. In the former case the *differences* involve the character of *parts* as the concrete involves the abstract; thus head, arms, legs in a man have quantitative relations to each other and to his whole figure, which vary only within narrow limits, and which sculpture or painting must not violate; or again, his whole life is only possible subject to definite quantitative relations of energy supplied to work done. In the latter case the parts, indifferent in themselves, are forced by construction into the function of differences. The reservations to which this last assertion is subject have been explained above (see pp. 177–8).

Thus equation and judgment are no doubt closely related in their origin; and this is further illustrated by the facility with which judgment drops back or crosses into the equational form, which demands altogether less effort and insight than the attempt to grasp the differentiated structure of things. I cannot refrain from quoting in illustration a paragraph from a powerful and sensible writer whose one fault is the love of moving in the lower categories and avoiding the effort to grasp entire realities as they are. 'As to the general result[2] [of human progress] what is it? Say, roughly, three hundred million Chinese, two hundred million natives

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[1] See p. 131, above.

[2] James Fitzjames Stephen, Liberty, Equality, and Fraternity, p. 177.

[illegible] hundred million Europeans and North Americans, and a miscellaneous hundred million or so,—Central Asians, Borneans, Javanese, South Sea Islanders, and all sorts and conditions of blacks;' and over and above all the rest, the library at the British Museum. This is the net result of an indefinitely long struggle between the forces of men and the weights of various kinds in the attempt to move which these forces display themselves. Enthusiasts for progress are to me strange enough. "Glory, glory: the time is coming when there will be six hundred million Chinese, five hundred million Hindoos, four hundred million Europeans, and Heaven only knows how many hundred million blacks of various shades, and when there will be two British Museums, each with a library."' The numbers here are not merely descriptive; they are essential; otherwise the element of progress could not consist in their augmentation. Of all instances that show in what thin abstractions a writer who prides himself on contact with realities may live and move, I know none more grotesquely striking than this; and it cannot be defended by suggesting that its absurdities are in some degree imputed rather than adopted. For they can only be so imputed because they are adopted. The faith in progress *need* involve no assumption of numerical increase of population. Had the writer ever heard of virtue or knowledge?[1] That his main thesis in the passage is somewhat of a truism does not justify so gross a misconception in supporting it. The more we examine the more we shall find that it is indolence which makes us drop into the equation when our subject-matter demands the judgment.

I have finished the account of the equation before proceeding with the judgment proper chiefly with a view to coherence in treatment, and not with an intention of representing the former as inferior to or less ultimate than the latter, though in a sense which may appear from p. 85 above, such a representation might be held justifiable. But the conception of divergence, of a co-ordinate evolution generated by abstraction, seems more appropriate to the matter before us than that of linear development.[1] The equation is, as we saw, hardly even a momentary phase in the growth of genuine judgment ; the two functions part company almost as soon as their significance reveals itself.

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[1] Contrast Dante's lines:—

'Considerate la vostra semenza;
Fatti non foste a viver come bruti
Ma per seguir virtude e conoscenza.'
Inferno, xxvi. 118.

And quantity is more than one among many categories ; as the simplest point of view which admits of difference and system it aspires to be, and in one sense is, *the sole* category, or ultimate ideal of knowledge. It may be treated as sole category falsely or truly. It may be sole in the sense that though abstract, yet, subject to the reservations involved in its abstractness, it has universal applicability. Every science, as we read in Aristotle, assumes its subject-matter, and does not give an account of it. The schematic world of space, time, and mass is in this sense, as an object of science, beyond question ; it has only to serve as an abstract postulate in working with perceptible facts, and from this point of view is *a* truth, if not *the* truth about the universe as a whole. As enabling a coherent reflective view to be obtained of perceptible phenomena as a quasi-totality (always encumbered indeed by the infinite series), it is of immense scientific value and co-extensive with definite existence. For these reasons, again, the equation—the judgment of quantity—is rather co-ordinate with judgment than a phase in its development. And still more is there reason for so considering it if we take account of the false employment of quantity as sole category. This false employment arises, or would arise, supposing the category of quantity to be considered not merely as co-extensive with determinate existence, but as, in its abstraction, the ultimate reality of all determinate existence, and consequently as furnishing the final ideal of science. It is obvious that the true use of this as of every category slides easily into the false one. Every science is occupied with its own abstractions. Every individual mind tends to magnify that with which it is occupied. The category of quantity, for reasons mentioned above, lends itself to universal application. It seems a short step from universal application to sole application, but it is the step from truth to falsehood. It is not made exclusively by votaries of physical science, nor perhaps by them chiefly. It meets us in theology and in philosophy under the form of the quantitative infinite as a sublime attribute of the Deity, or of soul life, or of the universe as contrasted with the 'finite' mind of man. We find it again in barbaric or vulgar art, in as far as this relies for effect on mere magnitude, mere evidence of expended labour, or mere costliness of material. And we do also find it no doubt in a formulated shape wherever matter and motion are invested with the dignity of real existence in a sense and to a degree that degrades the individual and concrete realities of life into something secondary and fictitious. But it is plain that no such tendency is necessarily involved in the treatment of these abstractions as real characteristics of the perceptible world. There is no special virtue in non-atomic continuous extension, nor any especial iniquity in the resolution of material objects into systems of vortex-rings, if such resolution either is a good working hypothesis or represents a real fact. The only error is in taking either a hypothesis for a fact, or *a* fact for *the* sole fact—in confusion, not in mere abstraction.

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[1] Above, p. 85.

The category of Quantity is, as we have seen, in its nature wholly relative. It is therefore incapable of furnishing an absolute and ultimate account of things. It not only cannot escape from the reference *ad infinitum* from term to term and condition to condition, but is forced to make this contradictory conception the very basis and postulate of its scheme. We have seen that the moment characteristic quantity or proportion makes its appearance in the judgment, as in any quantitative judgment it may, the whole between parts of which the characteristic proportions obtain is tending to exhibit itself as an individual synthesis of true differences, not as a mere aggregate of indifferent parts. The pure quantitative judgment or mere equation [1] is possible only by abstraction from one aspect of the essential judgment-function.

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[1] An equation that embodies a characteristic proportion is not purely quantitative. It involves in its interpretation the material differences between the parts which are in the assigned ratios, e.g. between angle and arc. See pp. 177–8, above.

It is not easy to find a parallel to so comprehensive and systematic an employment of a single class of abstractions, except in any attempt which may have been made to regard the world as simply a congeries of qualities, say of pleasures and pains. No such system indeed exists—the point of view excludes system; but one may conjecture of some such state of feeling as forming the consciousness of children and childish adults who have no judgment to pass on things, persons, or events beyond the expression of their likes and dislikes.

Thus I have thought it desirable to treat as in some degree co-ordinate developments the two series of judgments which diverge from the simplest measurement or equation such as a colour-match. On the one side we have the full evolution of concrete thought, as it builds up the actual and individual world within the series of relativity; on the other side we have the truncated evolution which embodies relativity almost pure and simple, but, as the abstraction is never quite complete (for then it would annihilate itself), may in particular matter revive its relation to totality, as we see in the exhaustive judgment of enumeration, and in the quasi-generic judgments of geometrical classification. And in the same way the more concrete judgment may in particular phases and under particular stimuli borrow determinations from or generate approximations to the abstract series. This happens when the life of a nation is subjected to statistical treatment; when the disjunctive judgment is taken in the weakened form of enumerated alternatives, and so gives rise to the calculus of probabilities; or when any one of the grooves or threads of relativity which compose the perceptible world is taken as a problem *per se* and tracked to its consequences by means of a pure hypothetical judgment.

CHAPTER V

SINGULAR AND UNIVERSAL JUDGMENT

Singular Judgment.

1. THE transition from the singular to the universal judgment is the transition from the affirmation of particular fact to that of general fact. This transition may be expressed by distinguishing two forms of the singular judgment, which may be called respectively the Individual and the Corporate Judgment.

Individual Judgment.

i. In approaching the Individual Judgment we are returning from the one-sided offshoots of measurement to the normal and concrete evolution of the judgment. We saw in chap. iii that it is the judgment of proportion which first reveals individual quality; that is, quality which, although particular and characteristic, yet does not refuse to admit diversity into itself and itself to enter into various contexts. This quality, however, we found, if merely indicated as the content of an abstract idea, stood in antagonism to the demonstrative indication of present perception which alone could attach it to actual reality. We found ourselves entangled in such judgments as 'This oak-tree has a leaf-spiral of $\frac{2}{5}$', 'This teasel has the bracts longer than the head', 'This tower diminishes in width from story to story'. Such judgments as these must rank as singular, for it is of their essence to qualify present perception by the meaning of ideas; yet their content is really ambiguous, for, as we saw, the characterising idea which ekes out the demonstrative reference to the concrete subject tends to grow into a condition and to make the judgment abstract, and in that sense universal.

Judgment with Proper Name.

a. The ambiguity which tends to split in two the imperfect singular judgment of the above type, which we have ranked among the judgments of measurement, is apparently removed in the class of singular judgments which we now proceed to consider, and which are avowedly based on the fact of recognisable individuality. Such are judgments whose subjects are designated by Proper Names. Enough has been said in the Introduction of the essential nature of Proper Names. We have now only to consider the logical value of the judgments which are made by their means.

The determinate idea, present in the judgment of proportion, is omitted in the judgment made by means of a proper name. On the other hand, the demonstrative particle, which by itself is helpless, being only an indefinite reference to presentation, is replaced in the proper name by the indication, not to be effected without some kind of meaning, of a particular individual. Thus it might be said that the two elements of the subject in such a judgment as 'That young soldier is the victor of Actium' are fused together in the subject of 'Caius Julius Caesar Octavianus is the victor of Actium'. But the union is effected at the cost of a mutilation of the significance; although as in the present example the diminution of determinate content may be more than compensated by the accession of suggested ideas. Identification no doubt involves ideas, but with the proper name, as we saw, identification is the end and ideas are only the means. In unfamiliar matter, say in a chronicle of remote date, we might conceivably identify the unknown possessor of some name as figuring in several scenes or incidents without being sure what he, she or it might be; whether a man or a woman, or a favourite horse. In this sense the judgment that deals with a proper name is merely particular. It has no meaning that can carry its application beyond the unique individual to whom it is taken to refer.

But, though subject to this imperfection, yet Singular Judgments of the class now before us form a real advance on the Singular Judgment of Proportion. They rest upon the fact of characteristic individuality capable of change and persistence without sacrifice of identity. Instead of an abstraction limited by pointing as if with the finger they refer us to a unique[a] concrete thing in its continuous permanence. In this sense, because attached to a continuous element of reality not shut up within a particular time or a given perception, the Individual Judgment is universal, and as we shall see leads up to a transition which takes us in one respect beyond the Singular Judgment. But universality—concrete universality—is not, we must understand, antagonistic to individuality. 'Caesar crossed the Rubicon' is an Individual Judgment; yet in it we are not[1] confining the reference of Caesar to the moment in which he was engaged in fording the river. If we thus refused to refer the predicated content to the whole extended identity of Caesar the significance of the judgment would be destroyed, and an eristic error committed by reducing an assertion to a tautology. Where is the significance of crossing the Rubicon if we do not affirm it of the conqueror of Gaul, the rival of Pompey, and the true founder of the Roman monarchy? Thus the judgment regarding a person, place or other object that bears a proper name introduces a reference that is determinate without being abstract, and particular without being confined to present perception. It should be observed that not all classes of objects are suited to be distinguished by proper names. This fact is akin to the inapplicability of significant names indicating *a thing* to many objects endowed with material existence. The range of proper names fall within that of significant names of things, and the spheres of application of these two kinds of symbols compared with each other and with the sphere in which neither applies have a curious bearing on the subject of individuality (see above, p. 128 ff).

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[a] I say 'unique' because that is the purpose of the proper name. Of course it has a general nature of its own, and cannot really guarantee uniqueness.

Judgment with Name and Ideas.

β. With these Individual Judgments must be classed all predications dealing with particular events, individuals, or objects, in which the demonstrative particle is dispensed with and replaced by a symbol referring to the individual. It will be found that these particular events, persons or objects have ultimately to be designated by reference to a proper name, or to some symbol which nearly approaches the nature of a proper name. As such e. g. may be ranked all chronological indications—'Christmas Day, 1885 A. D'.

It is obvious that such a symbol, or a proper name, as a fixed point in history, may be supplemented by any amount of definitely significant ideas; and as the proper name or the date is often understood or presupposed, e. g. in a continuous narrative, we are apt greatly to under-estimate the part played in judgment by the content of such symbols. Page after page of discussion about political or social tendencies may chance to be found in a reflective history, say in Lanfrey's History of Napoleon, without the mention of a proper name. But in so far as these discussions are to be taken as significant of the actual conditions of an actual epoch, they are understood as ideal content predicated of the nation, age and persons with whom the narrative is concerned. Chronological symbols exhibit the transition from the demonstrative to the proper name in a peculiarly clear light. 'To-day,' 'yesterday,' 'last year,' are just on the line between demonstratives and proper names. 'To-day' seems naturally to = 'This day', a demonstrative indicating mere relation to the percipient subject.[1] 'Last year,' 'twenty years ago,' show the relation to the percipient subject growing into an objective system. And when we come to the employment of an era, A.U.C., B.C., A.D., we have the system transferred from the accidental percipient subject, and attached to the content of a proper name. Wherever we have 'I' or 'my' &c., as points of reference in narration, we are dealing with something between a demonstrative and a proper name. And every narrative judgment which goes beyond a mere impersonal or demonstrative reference to present reality or to my own perception may *prima facie* be said to involve a reference to some proper name. No *mere* abstract idea can form the subject of historical predication. How far this *prima facie* conception must be corrected by allowing for judgments which may be capable of uniting without mutilation the powers of unique reference and of determinate notions is a question which will occupy us in the sequel.

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[1] Contrast Lotze, Logik, sect. 58.

Corporate Judgment

ii. Closely allied to the Individual Judgment, and perhaps in rigid technicality not distinguishable from it, is what for want of a better name I call the corporate judgment. This title is meant to include all such affirmations as deal with comprehensive totalities or aggregates which we *bona fide* take in their corporate or singular aspect, and do not consider either as in the collective judgment, in the light of sums of enumerated particulars, or as in the hypothetical judgment, in the light of mere abstractions whose very existence is not absolutely postulated. Such a judgment may be expressed indifferently by a singular or by a plural enunciation, so long as the name used in the plural is a *bona fide* designation of a known or knowable unity in respect of its characteristic features: e. g. 'The ancient Greeks were at once a most scientifically and a most imaginatively minded race.' This judgment obviously = 'The Greek race was', &c. This is not a collective judgment in the sense which has above been given to that title. It is not capable of being obtained by successive synthesis of the component units—by enumeration—and retains therefore no special extensional reference to the individuals who as an aggregate constituted the ancient Greek race; it starts from the idea of a common stock as a historical entity with peculiar endowments and with its own rise, decline and fall. Other instances are such as 'Europe has acted harshly to the modern Greek nation'. Europe is here not a geographical expression, nor even a mere body of nameable states; it is an organisation acting upon definite resolutions and through a known combination and proportion of forces. Several states in geographical Europe probably do not count in political or concerted Europe. Again, we may say 'The House of Commons detests a bore'. This might be transformed into 'All members of the House of Commons' &c., but the two affirmations are only equivalent if the latter is understood of the members *quâ* members, i. e. as engaged in debate in the House; otherwise this latter becomes a mere collective judgment, dealing with the members as a collection of individuals who share two attributes, viz. belonging to the House of Commons and hating bores, but wholly neglecting any reference to the House in itself as a single body with its own functions and peculiarities, among which is the one predicated in the judgment. The glacial period, The French Revolution, The Italian renaissance, The solar system, are 'corporate' realities and subjects of singular judgments of the species of which I am now speaking.

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[1] I take what I believe to be the actual meaning in use. Philology may or may not support it as the original meaning. We are bound to take philology into account as evidence of evolution and as a guide to observation; but it cannot override present usage.

The distinction between such individualities and those of really single objects or persons such as form the subjects of individual judgments is not a matter of principle, for in both the existence of the subject is affirmed, or, as I prefer to say, absolutely postulated. Nevertheless, the fact that such individualities as we have now before us do on one side consist of immense aggregates of particulars, and are therefore capable of being regarded at any moment from points of view antagonistic to that of their individuality, either as abstract ideas or again as series of numerable units, makes it desirable to mark by a specific distinction the fact that *in their unity* they can be regarded as individuals.

On the other hand, the subjects in question are again not easy to distinguish absolutely from the content of scientific class conceptions whose individual unity and actuality are disputable. The line which I have drawn is intended to correspond with the distinction between history on the one hand and truly physical science on the other. History deals on the whole with contents into the essence of which time enters, and which therefore, however comprehensive, are ultimately particular within the phenomenal series. Abstract science as a rule deals with timeless connections and systems of attributes, though it will be necessary in exhibiting this tendency to make allowances for a considerable admixture of and recurrence to the historical attitude, for in the light of evolution, time, or at least amount of change, enters into the essence of most things. And besides Abstract Science we must not leave out of account Classificatory Science on the one hand and Philosophy on the other, both of which, though in different senses, may claim to deal with actual realities.

The Corporate Judgment marks the fullest development of the affirmation of particular fact, and at the same time a near approach to the affirmation of general fact. In other words, the affirmation with which we have so far been dealing is beyond possibility of dispute Categorical affirmation, treating of contents which the judgments affirm, or at least absolutely postulate to be features that have or have had actuality in the world continuous with present perception. But from this point onwards the Categorical character of judgments becomes a matter of theory and of analysis. There is indeed, according to the scheme here adopted, no absolute separation between the Singular and the Universal Judgment. Still the distinction, unknown to Formal Logic, which is the ground of that separation as made by recent writers, is the basis of our scheme also. The Historical or Singular Judgment, and *a fortiori* the perceptive or imperfect singular judgments that precede it in our scheme, are not on all fours with the judgments of science, whether classificatory or analytical, or of philosophy. Before attempting to trace in these latter types of judgment the elements which we have observed in the former, though in other proportions and combinations, it will be well to examine more particularly the one leading difference between the two chief stages of affirmation.

Time and existence in Singular Judgment.

iii. The Judgment that asserts particular fact coincides with the Categorical Judgment in the primary sense of the latter title. According to the standpoint which for the present we have adopted, a judgment is Categorical when it asserts some thing or event to belong to the actual world in which we live. This assertion is made both by the Individual and by the Corporate Judgment. For the subject in these judgments is something that can exist as a particular reality, and is therefore taken or presupposed to be a real particular, while at the same time its reality is so far determinate that it can intelligibly be denied, which we saw not to be the case with the 'this', 'here', or 'now' of the Imperfect Singular Judgment. In these judgments, therefore, though not necessarily in them alone, we have existential assertion. And the simplest test of the presence of such an assertion is to ask whether the non-existence in reality of the content which enters into the judgment renders that judgment false. To such a question the primary answer is that *at least* in all judgments where time enters into the content — i. e. which assert facts in time—the non-existence of this content renders the judgment false. And these judgments will be found to be coextensive with the Historical

Judgment, affirming as it does not necessary connection, but particular reality within the phenomenal series. And thus, as said above, the Historical or Singular Judgment is equivalent to the Categorical Judgment in the sense here under consideration.

This answer, however, demands explanation and limitation. If non-existence of the content or subject of a judgment renders the judgment false, we have further to ask, Non-existence *when* ?

In every judgment we must distinguish between the time *of* predication and the time *in* predication. The time *of* predication, i.e. the time at which some thinking being makes the judgment, is relatively to the content of the judgment a mere accident, and alters actually while we are occupied in judging, and *a fortiori* when the judgment is rethought after a lapse of days or years. The time *in* predication is the relation of the predicated content to the total content of the temporal series of events which we construct and contemplate as objective. The time *in* predication, if any, is affirmed as an attribute by the judgment ; the time *of* predication is not affirmed as an attribute by the judgment, which therefore is not made false by any relations whether negative or positive between its content and that time. Hence it follows that the non-existence of the individual subject or content which falsifies a Singular Judgment is non-existence at the time *in* the predication, not non-existence at the time *of* the predication. 'Thucydides is among the greatest historians of the world' is not false, although Thucydides is in fact, when we speak, not a living man.[1] 'The House of Commons is an integral part of the British Constitution' does not depend for its truth on Parliament not being in dissolution at the moment of predication, but only on the co-existence of a House of Commons with the British Constitution in the sense and to the extent demanded by the import of the judgment. It appears to be a corollary from this principle that if no time in particular is involved in the import of the judgment, which is the case at any rate with geometrical truth, it becomes all but impossible to convict a judgment of falsity on the score of non-existence of its content; although the demonstration of utter non-existence, i.e. I suppose of impossibility, would so convict it. This bears on the categorical character of the generic and hypothetical judgment.

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[1] It may be objected that it is false to say 'Thucydides *is* a general in the Peloponnesian war', simply because Thucydides does not exist at the time of predication. Where we have true tense this is so ; we have then not got rid of the personal era. See p. 199. It must be remembered that in 'Thucydides *was*' &c., Thucydides *is* non-existent, and yet the judgment is true.

A subtlety is introduced into the problem by the phenomena of *tense* which include the time *of* predication, or a relation to that time or personal era, within the content of the judgment, and so within the allegation of time *in* the predication. The content so superadded is of the most fugitive and relative nature, and is constantly neglected even by historical narrative. Picturesque history neglects it by the use of the historical present, and philosophical history by the use of the logical present. Nevertheless where a past, future, or true present tense (neither historical nor logical) is intentionally employed, its relation to the personal era or date at which the narrative is drawn up beyond a doubt enters into the judgment and makes an assertion which demands a particular limited existence on the part of certain objects and events, and is false if this existence is not as affirmed. In the case of the present tense there is a peculiar subtlety arising from the confusion between the atomic or nearly atomic and the continuous or ultimately non-temporal present. The former is the import of the present as a true tense; the latter of the present as it approaches to a mere vehicle of affirmation. Compare the following examples: 'The Derby is being run at this very moment'; 'The horses are leaving the paddock for the Derby of 1861' (from a narrative written in 1883); 'The Derby is run once a year'; 'The Derby race in England is an instance of those customs which owe nothing to government but yet amount to national institutions'. The first of these judgments if true now must be false after the lapse of five minutes. The second is only false if there was no paddock or no Derby in 1861. The third is only false if at the time *of* its predication the race has altogether ceased to be an annual event; but its present is well able to comprehend within itself the intervals of time which the import of the judgments admits and requires, and is not falsified by reason of the non-existence of the race during these annual intervals. And the fourth is false only if there never was nor will be such a thing as a Derby race having the importance alleged in English life. I am discussing of course only how far the respective judgments become false if we assume non-existence of the content; I am not raising the question of the material truth of the attributes enunciated. It is only in the first of these four examples that we see the present operative as a true tense. In the case of the past and future the difficulty of tense does not arise in its full extent; the tenses which indicate them must of course introduce the personal era, but for that very reason cannot be confused with a mere form of predication, for which they have not the appropriateness that the present possesses. Therefore they do not risk the reduction of every possible judgment to a statement about a momentary date or epoch, which would be the result of neglecting the above distinctions in the use of the present tense. And moreover, these very tenses 'was' and 'will be' prove that it is at least not necessary to the truth of a judgment that its content should exist in the moment *of* predication. It is however necessary for judgments dealing with past and future that their content should have the alleged *relation*, however fugitive and accidental, *to* the time of predication. But at any rate in none of these cases, neither in present, past nor future, need the content be shut up *within* the time of predication or the time related to that personal era. Past and future do not pretend to be momentary, and the present cannot possibly succeed in being so. It must be credited either with duration in itself or with a continuity that shades by degrees into the past and future.

Thus, though the time of predication has the slightest possible relation to the content of judgments, yet no doubt, when predicated as an attribute by help of true tense, a relation to the time of predication or subjective era enters into predication. And in so far as history as a whole falls in the past, the use of the past tense in narrative bears true witness to the essential particularity and limitation of the existences with which history deals. But after all, the past is not a point but a line; and so for precise temporal import even of narrative propositions in the past tense we must go, not to the tense, but to the import of the judgment. And *a fortiori* this is the case with the present, where the true tense is difficult to distinguish from the same tense used as the mere form of predication.

We have thus seen in what sense and to what extent Singular and Perceptive Judgments are identifiable with Categorical affirmation. The Judgment whose content involves a limitation of time is plainly false if in the time-relation prescribed by such a limitation its content is non-existent. This test shows that all such Judgments are assertions of particular fact. Even the example (on p. 204) which goes beyond common historical usage nevertheless asserts fact which is essentially in time, although the truth of the assertion is not relative to the date of predication. If the Derby race never existed the judgment would be false, and *its* existence involves a multitude of temporal relations which are necessary to its being what it is. It is essentially a fact in time.

When we get away from the proper name and the relations of events in the temporal series, we find it much less easy to say what non-existence of particulars, if any, would imply the falsity of the judgment, and in what sense therefore, if in any, the judgment alleges actual fact belonging to the real world. We must bear in mind that we have up to this point been dealing with unanalysed perceptions and with proper names and their expansions. We have therefore not got rid of the element of irrational concreteness which attends all judgment whose subject is given as if by simple pointing with the finger; for *qua* a mere symbol of identification the proper name itself partakes of the character of simple pointing; and until we reached the proper name we were encumbered with the demonstrative particle.

Therefore the judgments thus far considered, omitting the one-sided forms that arise out of measurement, possess certain peculiarities. They assert, to begin with, the existence of things or events in time. In doing so, they restrict and burden with irrelevant matter the application of determinate ideas, which, as the only symbols of meanings, they cannot avoid employing. Taken therefore as rational connections of attributes, a point of view which determinate ideas challenge, and towards which, as we saw, the import even of proper names tends to develope, the judgments which we have been considering are *false*, being burdened with irrelevant and deficient in relevant matter. This is the same thing as to say that they present an aspect of necessity, in which aspect they are defective and so false.

The Universal Judgment.

2. By the title of 'Universal Judgments' I mean to designate, in all their phases of import, the assertions usually typified by such well-worn examples as 'All men are mortal', 'All fire burns', and 'All triangles have their three angles equal to two right angles'. It has been shown above that perceptive and singular judgments, and more especially those which employ proper names, possess an element of universality as predicating identities into which differences enter or which persist through differences. But in spite of this fundamental unity of the judging function, the distinction between the 'Singular' and the 'Universal' Judgment has an importance for us which it has not for traditional Logic; which indeed treats the Singular and the Universal Judgment as on the same level in a strictly logical point of view, and both, consequently, together with the particular judgment, as species of Categorical enunciation. The attitude which we have to adopt towards this arrangement was defined in an early part of the present book,[1] where we saw that it was impossible for us to retain these species of enunciation in their traditional relations. It is beyond a doubt that the Universal assertion must at some point of its development assume in some aspect or in some degree a hypothetical character; while if the genus Categorical, which certainly includes the Singular Judgment, extends into or over the domain of the Universal, it can only do so subject to reservations which are unnecessary so long as it is confined to the judgments of present perception or of simple narrative.

The Universal Judgment, then, is not distinguished from the Singular Judgment by the mere feature of Universality, but by a special phase of Universality, that is to say, by the predication of a *universal law of connection*. It has, indeed, been obvious to us throughout the analysis of judgment that universal connection was everywhere at work in the background, exploiting any qualification expressed or implied in the Subject, for the benefit of systematic connection or necessity, and at the expense of the simple perceptive or narrative conjunction of contents; until, surrendering for the moment the task of intelligible qualification of reality, we fell back, in the proper name, on an attempt at unique designation.[1] But here again we fail to escape from universal connection or necessity, and in the names of illustrious individuals, as in those of nations, epochs, or movements, we have found that every significance tends to break down the mere conjunction of *data*, and to exhibit itself as a connection of reason and consequent. We attempted to draw the line at events and individuals into whose content time entered, and to show that judgments dealing with such matters as these were inevitably allegations of fact, and not of abstract connection. Now we adhere to this distinction, and it is perfectly true that any fact which is especially involved in one portion, however extended, of the temporal series of phenomena, must have existence within that portion if an affirmative judgment about it is to be true. The assertion of such a fact in its accidental concreteness is therefore radically different from the assertion of a mere law or relation, and if taken in this latter sense would necessarily be false. But we found in the corporate judgment that it was not perfectly easy to distinguish the facts of history from the truths of science; for sets of events greatly extended in time appear to pass by a sort of *sorites* into sets of events which, though in respect of existence precisely on all fours with actualities limited in time, are nevertheless either not treated as relative to time or really are not held to be so relative. In order, while marking the distinction demanded by the absence of limited particularity, to give full weight to the continuity of import between these types of judgment, I shall not follow Lotze and other modern writers in identifying the universal *ab initio* with the hypothetical judgment. I shall prefer to distinguish within it two species, of which the first, the *generic* judgment, alone belongs to the main evolution of thought, and the second, the *pure hypothetical*, is regarded as an abstraction of a quasi-mechanical character, and consequently as a divergence in the direction of the arithmetical and geometrical judgment.

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[1] See *supra*, chap. i, p. 87 ff.

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[1] I have said above (Introduction) that I do not give this as a historical account of the genesis of proper names, which must no doubt have been applied and recognised by a gradual differentiation. But none the less, wherever language has fixed them as a class of words, they perform the function and are subject to the modifications indicated in the text.

The Generic Judgment.

i. The generic judgment is the qualification of reality under the aspect of a Natural Kind by attributes or relations incident to that Kind. A Natural Kind is for our purpose a Kind accepted and treated as such by any science. We have already seen that the geometrical sciences are in this respect in a peculiar position, and we have discussed the limits under which their quasi-generic judgments may be taken to embody truth about actual reality. The question of alternative classification, which arises on account of the different points of view introduced by different sciences, was treated in the Introduction, and should cause us no difficulty *if* we are once able to understand the nature of the truth embodied in *any* science. For every science employs some abstraction and idealisation, though there is an all-important difference of degree between contents which are merely abstract as not sensuous, and contents which are abstract as not concrete.

The restriction 'incident to that kind' is not intended to exclude relations which one kind shares with other cognate kinds. A purist logic, following a suggestion to be found in Aristotle, might indeed require that every generic judgment should be 'commensurate' or characteristic. It would then have to deny that the possession of breathing apparatus was incident to man, because he shares the respiratory function with the whole organic world. Such a view would be illustrated by the fact which was commented on above, that e. g. the animal properties of man are, in man, modified by their relation to his humanity; so that in order to represent them as features of *man* they ought ideally to have certain modifications assigned to them, while in their abstraction they can only be set down as characters correlative to the no less abstract idea of animality as such. But this principle seems needlessly purist. The attributes which man shares with the animal world are elements of identity, however partial, between him and it; and there can be no reason against characterising him by these identities which would not tell equally against any knowledge which falls short of perfection. The rule to be borne in mind about such cases is that imperfect knowledge only becomes *false* when mistaken for perfect knowledge. In as far therefore as the form of the judgment implies a truth completely adequate to its subject, something may be said for the view which has just been stated. But we shall find if we press the matter home that this ideal is to be regarded as the vital principle active in knowledge, but not as hostile to any genuine fact that is free from confusion.

Under 'incident to the kind' then are included the attributes and relations which lie at the root of the individuals' being, although shared by them with individuals of other kinds, or even with mechanical or geometrical wholes. It is only to be borne in mind that such relations, e.g. the characters of vertebrate organisms, are not thought as mere abstract properties, when applied to specific kinds, but are regarded as concrete schemes presenting both a general and a specific aspect. It is in this characteristic of graduated identity that the intelligible order of the world reveals itself to us.

The generic judgment has always been the battle-field of conflicting logical tendencies, corresponding to actual needs and features exhibited in various points of view by the judging activity. I shall endeavour, on the same plan which I have hitherto pursued, in some measure to satisfy these tendencies by distinguishing as different, in phase and line of evolution, intellectual acts which are often reckoned as one and the same. But it follows from the nature of thought that all such aspects have a real connection, and are in fact rather distinguishable than separable. Thus in distinguishing the generic judgment from other acts of thought we shall also be analysing this many-sided judgment itself and justifying to some extent the views of those who have recognised in it only one or other of its many sides.

The Quasi-collective Judgment.

a. The Generic Judgment—the Universal Judgment of common life and of classificatory science—was regarded by Aristotle, as is well known, under two connected aspects, as a judgment of Allness and as a judgment of Necessity. The former of these aspects has been seized on by formal logic, and the doctrine of logical universality has been adapted mainly to the consideration of subject and predicate as names or ideas applicable to groups of individuals. We examined the judgment from this point of view, as dealing with an aggregate arising out of enumeration, in a former chapter. We there saw its actual goal in the Collective Judgment and its reversion towards a more concrete and natural mode of thought in the Exhaustive Judgment, which amounts to nothing but a Generic Judgment very strictly taken in extension. It must however be remembered that we did not think it possible for any judgment, however closely confined to an aggregate resulting from enumeration, to avoid characterising the individuals by a common attribute, and predicating an attribute of them. Attributes are enunciated by extensional no less than by intensional judgments; but in the latter they are connected with attributes, in the former they are centred in the identity of individuals. Therefore it is possible to consider the Generic Judgment as differing from the Collective Judgment simply in degree, viz. by predicating attributes of an unknown or unlimited and not of a known or determinate aggregate, the actual means of predication being in both cases alike a general attribute, though in the latter case attached to reality by a proper name or its equivalent. But an unknown or unlimited aggregate of individuals is a contradiction,[1] a numerical problem which proclaims itself insoluble by enumeration, and therefore the judgment which is couched in this shape, and which in fact conveys a perfectly intelligible meaning, must derive this meaning from some other source than from such an enumeration as that on which the collective judgment rests. The Exhaustive judgment must be interpreted by the Generic and not by the Collective.

Examples of the Quasi-collective judgment of which we are speaking are—'All men are mortal,' 'All organisms both breathe and assimilate,' 'All unstriped muscle in the human body is inaccessible to the control of the will.' This last example, however, is suggestive. The Exhaustive judgment, i. e. the Generic judgment in its aspect of 'allness', is helpless in the face of the most trivial exception. Thus '*Nearly* all striped muscle is under the control of the will', but the muscles of the heart form an exceptional case, and, though striped, are normally inaccessible to volition. Nevertheless there can hardly be a doubt that the coincidence expressed by the judgment must indicate some sort of connection, however circuitous, between the appearance of the muscle and the degree in which it is under control, and that the exceptional case must be accounted for by special conditions. But on its purely enumerative side the judgment has nothing to say to this; it only knows that the sum-total of Enumerative judgments cannot be made, and the judgment of allness is therefore unwarranted. It is obvious that the affirmation of universal connection which in such an instance we feel to be all but warranted is not approached from the side of the individual units, but from the side of the common or continuous nature which binds them into a whole.

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[1] See p. 166, supra.

True Generic Judgment.

β. As dealing with a common or continuous nature the Generic judgment may be more properly expressed in the form, 'Man is mortal,' 'Water boils, under one atmosphere, at 212° Fahrenheit,' 'A society organised on a purely commercial basis treats the working classes as little better than slaves.' These propositions are accepted as practically equivalent to 'All men are' &c., 'All water' &c., 'All societies' &c. Here however it is plainly the connection of attributes that warrants the affirmation concerning individuals and not *vice versa*. When thus regarded, the Generic judgment challenges comparison with the Singular judgment in both its forms, both as Individual and as Corporate.

Ordinary Generic, or Analogical Judgment.

a. 'Man is an animal capable of social life,' 'The bacillus is a septic organism,' 'Throughout the vast Orchidean order, including 433 genera and probably about 6,000 species, the act of fertilisation is almost invariably left to insects.' These are affirmations that unquestionably refer to something real, but yet employ neither perception nor a demonstrative nor a proper name. Therefore, *prima facie*, it seems as if the determinate idea had come to its rights, and were no longer obscured by any irrelevant elements of the phenomenal concrete. The act of thought ought, it would appear, to fall at once into the groove of abstract necessity: '*If* man, then social;' '*If* Orchid, *then* insect-fertilised.' But ideas such as those now before us offer a resistance to such treatment. The determinate idea is abstract, indeed, as all thought is abstract, but nevertheless it may have a content which is concrete, and in the example before us we have such concrete contents. These, therefore, bear the morphological character of individuality, by which alone even the unique object named by a proper name is made recognisable, persistent, and so universal. Compared with such an individual subject the Generic subject has lost unique reference; but it has gained abstract significance, with which the proper name was incompatible. And it is in virtue of this significance, the significance of individual self-completeness, that the Generic subject persists as an identity through the differences which form its attributes. Now the individuality when reduced to a content is not single, but exists in instances. Thus, in attaching differences to the individually characteristic content as such the judgment goes altogether beyond the synthesis ot differences in an actual individual subject, and affirms such a synthesis mediately of a number of subjects, which may be taken as endless seeing that its limit is at this stage not held essential and not enquired after. Such a judgment, which treats a concrete individuality as an abstract universal, and extends its incidents to all individual instances, may be described as an *analogical judgment*. And this is the fundamental nature of the ordinary Generic assertion.

The introduction of the term analogy into the theory of judgment may indeed be objected to on the ground that analogy is a kind of inference. But the fact is, that apart from any *general* question of a connection between judgment and inference, we are now at any rate on the threshold of an activity of judgment in which inference is unmistakably present. All that we can do in order to avoid a premature discussion of inference is to approach the analogical judgment rather as a conclusion the content of which is open to analysis than as a complete inference whose process lies before us. It may be added, that logic is quite familiar with the idea of 'necessary' judgments. Yet necessity involves inference far more explicitly than does analogy.

In the analogical or *ordinary generic* judgment, then, we have neither implied reference to perception as in the impersonal judgment-form, nor the demonstrative 'this' or 'that', 'here' or 'there', nor a conventional implication of unique reality by means of a proper name. The subject—I speak of the immediate subject or subject within the judgment—is an idea, and *qua* idea, is abstract. But we must distinguish between abstractness as incident to thought in contrast with sense-perception, and abstractness as a character attaching to contents present in thought. In the former way of speaking all thought is abstract. In the latter, some is abstract and some is not. The ideas which form the subject of the generic judgment in the phase now before us are not abstract in the latter sense. They are ideas of totalities, existences complete in themselves, to which we cannot indeed venture to apply the conceptions of teleology proper except in so far as the wholes in question are products due to the human mind, but which must be regarded from the standpoint of that secondary finality which may be described as morphological unity or quasi-teleology. We have thus a character or complex of attributes which is at once general and individual, abstract in thought and concrete in content. As abstract, it defies enumeration of instances, and implies necessary sequence or connection of attributes. As individual and concrete, on the other hand, it refuses to be taken as a mere ideal antecedent in a relation of necessity, i. e. of reason and consequent. The conciliation demanded by these antagonistic elements of import is found in the judgment of analogy. The essence of this judgment is that it is neither purely subsumptive, as expressing a *de facto conjunction* of attributes in a single subject, nor purely constructive as expressing a *de jure connection* of attributes independent of the immediate subject in which they may exist, but is something intermediate, as expressing a perception or presumption that the content enunciated in the judgment is bound up with the characteristic individuality which forms the immediate subject.

The ultimate foundation of any such insight must be the final cause or teleological idea of the individual, which however, when considered as an immanent or embodied final cause, is most prudently treated on the level of morphological character. We may indeed safely say that the purpose or final cause for which we make a microscopic lens is to combine magnifying power with light and definition, and from this purpose, by help of a number of further judgments dealing with optical and mechanical truths, the physical attributes of a good lens may be constructed. But in dealing with things not made for a known purpose we cannot apply any such abstract rule, and must fall back on the idea that the thing discharges an actual function, or at least looks as if it had a function, which must be taken as immanent and identified with the thing in its concreteness. The judgment that pronounces what is involved in this content and what is not rests on the presumption of the individual unity of the content, and on the capacity of discerning from the structure of this unity aided by empirical knowledge of instances what is essential to it and in what degree. The insight in question has undoubtedly some kinship to aesthetic judgment, for both depend on the power of seizing the concretely presented import or principle of unity of a concrete whole. To judge the structure of a fossil creature from a vertebra, or to detect the affinity between two zoological species which are externally much modified, is a synthetic apprehension of the same nature as that which realises the construction of a picture or of a drama. Such judgment, however, is merely the condition precedent, and not the essence, of the true relation between the mind and fine art.

The analogical judgment, like the aesthetic judgment, is essentially outside relativity and necessity, and incapable of being resolved into them. It is true that judgments of abstract relation, drawn from the mechanical or geometrical sciences, are perpetually coming in aid of analogical truth, by indicating that this or that *de facto* service within the concrete individual *can* only be performed by its parts under this or that condition. To support a certain weight the plant stem or spinal column must have adequate strength. To impel a certain bulk and mass through the air at a certain velocity the bird's wings must have a certain area and striking rate, the arrangements necessary to which of course react on the whole muscular circulatory and respiratory apparatus. But even this merely rhetorical selection of an abstract final cause is really unjustifiable. Which comes first? why such a weight on the stem? why should the bird's body have such a bulk or mass? There is nothing to fix any one of these elements as a given final cause to which the others must be adapted. Adaptation to the bird's prey or the like is again simply *de facto*. An animal might have to change its prey by reason of a change in their relative powers, just as probably as it might develope new powers to keep pace with those of its prey. And further, in the background we may see such a law as that of the Conservation of Energy dominating the entire system and operations of everything that moves. In all these relations we observe the ultimate character of necessity, viz. the reference of a subject to a whole other than itself; e. g. the treatment of an animal as a part in the whole of moving matter, or as a figure having properties in space, so that in each of these relations it appears as determined by the character of a totality other than its concrete self. The nature of space for instance is *per se* a *datum* or fact; but when it determines the results entailed, e. g. by the shape of a leaf, it is exhibited as a whole prescribing the relation of its parts, which relation as regarding something that is not merely a part in space is external but constraining and so necessary.

Yet even if the entire construction of an individual content were laid before us in terms of mechanical analysis, still the analogical judgment would force itself upon us, as the aesthetic judgment would in a parallel case. Analogy would then indeed no longer be the chief instrument in discovery, or at least in presumption of universal connections, because these would be capable of constructive apprehension of a more direct and relevant kind. Such a state of knowledge may already in some degree be illustrated by the Darwinian analysis, say, of an orchid-blossom, in as far as it succeeds in tracing the mechanical modifications, which, each of them representing a definite physical adaptation to some external circumstance, have generated the present structure of the flower. The same observation might even be applied to the identification of types and their affinities. The mechanical history of any organic structure would, if ideally complete, include the nature, degree and physical causes of its deviation from kindred structures.

But all this would not interfere with the import of the generic or analogical judgment. For this import consists in the identification of individuals with a concrete content, and such an identification involves connections which differ in kind from the identification of abstract relations which are not Things. They take the content not in its external relativity, but in its relation to self or to an immanent final cause—a final cause identical with itself. We have examined this self-relation in the more difficult case of geometrical figures which are absolutely and adequately reducible to examples of general conditions, and seem merely to mimic the self-contained relation of the concrete thing. This relation is an element of import which does not wholly disappear even in those kinds of existence which are hardly ranked by common language in the category of individual things—as we saw to be the case e. g. with the elements, and in short with all unorganised substances. I do not restrict the meaning of the term unorganised to = 'inorganic' in the technical sense; but I employ it to designate any portion of matter, organic or inorganic, which is not shaped into a whole by human activity, or regarded in respect of its natural subordination of parts with interest due to its unity for our intelligence. Every element has no doubt its peculiar minute structure, and every fragment or portion of matter has no doubt its spatial or other relations which unite it into a whole. A pebble or a bit of rhomboidal spar or a nugget of gold has a self-relation, a characteristic peculiarity which makes it single, and distinguishes it as a persistent universal from things external to it. Much more has any organism a typical individuality which introduces the distinction of inner and outer, essential and relative, into what as a mere example of general laws has no self-relation [1] and no inner or outer.

And in these last two sentences I have omitted the strongest case, because it is so strong as to dispense with the reservation which we were trying to illustrate. But when we come to reflect on the conception of a *thing*, we must be struck with the fact that by far the greater amount of what we most readily recognise under that title are objects made by man for purposes which he consciously embodies in their structure. I cannot think that, apart from our familiarity with such objects, the conception of a thing would seem so simple as it does. A mountain, a waterfall, a wave of the sea, are *things* chiefly to the aesthetic perception ; and if we left this perception out of account, it would not be easy to assign the boundaries of their individuality, or to single out its essence. Complaint has been made [2] that those who lay stress on the progressive interpretation of the idea in nature do not find room in their theories for the achievements of the screw and the lever, *and* for the laws of the equilibrium of fluids, of pressure, and of tension. These examples may be regarded in two aspects. The screw and the lever are best known to us as *tools*, in which capacity they belong to the sphere of *mind*, as objects endowed by human foresight with an immanent significance depending on their adaptation to a determinate purpose. But as mere characteristics of matter mechanically considered they rank with any of its general attributes,—rigidity, gravity, inertia,—attributes which are the basis of all material organisation, but do not by themselves suffice to give individual interest to any fragment in which they are embodied ; and among such attributes must be ranked equilibrium of fluids, and the effects of tension or of pressure.

But what are we to say of the shell that screws itself into the sand, of the screw propulsion exerted by the porpoise's tail, or of the levers which form the limbs of animals ? We dare only speak, in relation to such phenomena, of a *de facto* purpose or actual function. In virtue of this function, this contribution to an obvious and real end, the total life and motion of the animal in which they are found, we claim for these arrangements a morphological unity which forces us to grant them the character of elements in things that have concrete individuality. We dare not ascribe to them the unity of an ideal purpose, as we safely may to the screw of a micrometer or to the lever of a balance ; but we treat them as elements in a unity *analogous to* that with which we are familiar in objects that represent the purposes of our mind. The rudest mechanical contrivance is in this respect on a level with the products of fine art and superior to those of nature, that it unites the abstractness of thought with the concreteness of sensuous existence ; i. e. while in one aspect a mere material object, yet in another it embodies an idea, and does so determinately and without irrelevancy.

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[1] Of course such a point of view is unreal in its abstraction. If the thing were absolutely regarded without self-relation, its external relation would be gone too, for what would there be to determine it ? The point is that the centre of interest, in relativity, falls outside the self.

[2] Lotze, Mikrokosmus, English Translation, vol. i, p. 17.

Thus the conception of machinery has a double and not a single import for logic. If on the one hand it accents the fact that matter is indifferent to our purposes and simply acts and reacts according to its own nature—and it is this of which we are constantly being reminded as the mechanical or uniform aspect of the world—yet on the other hand it is the most obvious example of a concrete embodiment of mind in matter, and corroborates if it does not awaken the reflections of the understanding on the rationality of things. Of this rationality the existence of individual types as concrete universals recognisable by the analogical judgment is a higher phase, a phase more akin to individual intelligence, than matter in its abstract and general modes. And therefore the generic judgment resting on analogy, i. e. on the perception of concrete identity of content, is not capable of being superseded by the abstract judgment of pure relativity. The latter, if ideally complete, gives a true account of what occurs in terms of mass and motion, but necessarily omits the teleological or quasi-teleological import which gives the content of the judgment its interest and significance for knowledge.

I will analyse a single example. Exogenous trees display 'annual rings' in the wood, which are due to the augmented pressure of the bark as each year's new wood expands the stem, resulting in the flatter formation of the outer and later cells in every year.[1] As this stands I call it a generic or analogical judgment. It is indeed based on one of the above-mentioned simple mechanical relations, the effect of pressure ; but it predicates this relation within a concrete individuality which gives it an import that as a mere mechanical problem it would not possess. Let us reduce its essential points, however roughly, into the latter shape. We shall obtain some such residuum as this : 'A fabric gradually constructed under increasing pressure out of a material which hardens after a time will show increasing effects of pressure in its later-formed portion.' Here we have, in part at least, 'freed the direction,' to borrow an expressive phrase from Bacon, i. e. stripped off circumstances which are irrelevant to the production of the effect in question. But with these irrelevancies we have lost in the case before us not merely confused concomitants of perception, but the interest which gave the example its place in knowledge. The fabric is no longer wood, the gradation no longer that displayed in the annual rings of timber, the subject of the judgment is no longer reality embodied in the characteristic individuality of the exogenous trees. The import of the judgment is gone. 'But its content is subsumed under the simple mechanical relation, if this is rightly understood.' Perhaps ; but what does this mean ? If the import of the concrete thought is to be saved, it must mean that the analogical judgment is re-thought in its full depth, but with the explicit knowledge that it includes the abstract mechanical relation. The typical character of exogenous trees, though we must not call it a final cause, yet prescribes the extension and gives a definite reference to the content of the judgment. And I must here put the reader in mind, that, wishing to gain nothing from what may be called accidental ignorance, I have laid no stress on the present impossibility of constructing any living thing on purely mechanical principles. Individuality rests on a difference, not on a confusion, of categories. I am convinced that no organic nor spiritual movement accessible to human intelligence is without a mechanical aspect. I have therefore treated the present subject from a point of view which admits such an aspect to be knowable in all vital and spiritual processes. This point of view may seem absurdly fictitious when we consider the present state of exact explanation in the sphere of biology, or again of social science. The generic or analogical judgment now has, and seems likely long to retain, what we may describe as a secondary function; a function not merely of interpreting but of predicting—not merely of resuming sensible facts under higher categories, but of anticipating their actual occurrence. In all science that deals with subjects beyond our power to construct, we draw our conclusions by means of analogical judgment in this secondary sense. When we judge a particular plant of deadly nightshade to be poisonous, or a particular red stag to be dangerous at a certain time of year, we are judging on analogy; on an anticipation based upon a concrete character whose particulars we cannot construct. The precise nature of these inferences will occupy us in the theory of inference; it is plain that the larger part of inexact science consists of them.

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[1] Probably other causes concur in this process. I have purposely simplified it. The rings are annual only if the period of growth in the year is single. A second hot season may cause a second ring.

We must not confuse analogy in that secondary sense, as a *mere* anticipation of nature,[1] with the true generic or analogical judgment which is compatible with complete analytic perception of mechanical cause and effect within the subject considered. We have perfect examples of these latter judgments in the case of things made by man for a purpose; in which a complete and accurate perception of their structure, interior causal nexus, and inevitable course of movement in no way supersedes the summary of their import which a knowledge of their purpose enables us to embody in a generic judgment. When we wind up a watch of which we know the construction, we do not merely anticipate that it will go because we have seen other watches go; we can point to the specific causal connections by which it must (excluding accidents) result that the main-spring, unwinding itself, will draw round the wheels; that the motion passed through the wheels will at one point be regulated by the escapement, &c., &c. If we knew nothing of the use of a uniform measure of succession, but had some experience in mechanics, we should be quite certain that the watch must go, but we should have no notion of its generic content—we should not know 'what a watch is,' i.e. what its purpose is. We could not therefore make the pure generic judgment, 'A watch is a motion regulated by an escapement so as to maintain a uniform rate.' In order to this judgment we must know the purpose of the instrument, viz. to maintain a uniform rate. With this knowledge however we are in a position which, in strict theory, we can never attain with regard to any natural product. We can dictate the generic import of the watch; we can say that if any watch possesses this import imperfectly it is a bad watch; if it possesses it not at all, it is not a watch at all.

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[1] Anticipatio naturae.—Bacon.

Judgments such as this form the ideal to which the universal judgment in the form now under consideration always aspires. The properties expressed in such judgments are not merely anticipated or presumed; they are, or at least may be without altering the nature of the judgment, deducible with the utmost rigour. Yet, again, they are not mere causal sequences; it is possible to have before us all the causal sequences concerned in the object, and yet not to make the true generic judgment which unites them into a coherent system. In this class of objects we may fearlessly say that it is the purpose[a] which is the essence, and that generic judgment rests on the knowledge of essence. In all other classes of objects such a view has degrees of precariousness, and can only be applied to the purpose as immanent, and therefore as not determinate, and as uncertain in its boundaries. Nevertheless, when we predicate in the organic world 'growth', 'development,' 'self-preservation,' 'irritability,' we are really referring mechanical processes to an idea of life—an idea of self-relation, of 'inner' and 'outer', which is a higher result, though it *is* a result, of their purely mechanical nature.

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[a] Such a purpose or essence, however, is limited, and e.g. excludes the means, and therefore part of the whole which embodies it. Teleology in this current sense, therefore, cannot be used as a principle in explaining any whole, e.g. the universe.

The above is the best account that I can give of the normal generic judgment, which represents the really central phenomena that were designated under the title of Universal Judgment by Formal Logic. The point of view which emerged in comparing this Universal with the Collective Judgment, and which was suggested by the quasi-collective form of the plural subject with 'All', can never have been really felt to include what the judgment intended to affirm. But, as was said above, 'allness' is undoubtedly *an aspect* of universality.

Existential meaning.

b. In order to bring to a focus the nature of this judgment we have now to consider how its affirmation is to be classed—whether as asserting the existence of fact, or the connection of attributes.

To determine this question, we must recur to the distinction laid down above [1] between abstractness as a character of thought in contrast with sense-perception and abstractness as belonging to a kind of thought as distinguished from concrete thought. Abstractness in the former sense is compatible with individuality, while in the latter sense it is not; and it is in the former sense that we apply the term abstract to the ideas which are subjects in the generic judgment. Thus though we have no longer a proper name as in the singular judgment, yet we have a concrete idea, which being as a whole capable of reality, presupposes such a reality. We saw that in the case of the demonstratives 'This', 'Here,' &c. the reality which is the immediate subject cannot be intelligibly taken as affirmed to exist, but only as presupposed. The ideal qualification which sometimes accompanies such demonstratives showed us the point at which presupposition of existence tends to pass into affirmation, simply because a *significant* presupposition can be intelligibly denied. But this tendency is never absolutely fulfilled. The union of actual and ideal qualifications in the demonstrative judgment—e.g. '*This* bad man,' for though he were a good man he would still be *this*—always leaves the ideal qualification the option of being read as a condition. The proper name, again, in its primary function, being void of determinate meaning, presupposes rather than affirms the existence of its content. It only tells you that *some* individual is in question ; and you cannot deny that an individual may be in question. But then as the proper name becomes more charged with import, which may even be made explicit in ideal contents as it is in the Corporate Judgment, it also becomes as we saw capable of conditional meaning. Now as the primary function of the proper name can never be cancelled while it remains 'proper', there arises within the singular judgment a parallel ambiguity to that which arises within the demonstrative judgment. The presupposition that reference is in any case made to actual existence is at war with the determinate qualification which can and will only refer to some determinate existence that may or may not be forthcoming. A determinate, i. e. significant, ideal qualification standing as or in the subject of a judgment is never unambiguously affirmed to be an actual existence. It may always take refuge in a conditional meaning. The reference to actual existence is presupposed in the sense possible for the subject ; in perceptive judgments because we are never without perception, and in singular judgments because the form of the subject-idea suggests an individual, and reality consists of individuals.

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[1] p. 214, supra.

The independence of these two kinds of qualification, ideal and existential, and their consequent liability to contradict one another, is the very root both of existential and of conditional affirmation. In existential affirmation the two qualifications are taken as meant to coincide, though it is a purely material assumption or assertion that they do so ; in conditional affirmation the two are allowed to fall apart, i. e. the ideal qualification is not read as implying an individual reality that possesses it.

Returning now to the concrete idea which stands as subject in the generic judgment, 'society,' 'man,' 'art,' 'the bird,' 'the rose,' 'a time-piece,' 'a telescope,' we find the same elements of meaning, but in reversed proportions. The subject here consists technically and primarily of ideal qualification and nothing more. It has reference neither to an unnamed perception nor to a unique individual undetermined by abstract significance. Hence the presupposition that the subject is an actual reality is less prominent, while the abstract conditional import of the ideal content is more so. In the earlier types of judgment we feel that we are referring to reality, and we assent with reluctance to the analysis which shows us that our reference is conditional. In the more abstract forms at any rate of the generic judgment we feel that we are affirming conditionally, and we at once acknowledge our reference to actual reality to be merely implied or presupposed. It will be noticed that I do not admit this side of the antithesis to be completely developed in the simpler generic judgments. I do not think that in 'The rose has pinnate leaves and perigynous flowers' the existence of the subject is merely 'implied' and not 'asserted'. It is not uncommon to find in a manual of botany such and such a variety 'is no doubt a mistake of the observer', which shows that the observer's description asserts existence so far as existence is asserted by any judgment.[1] All this however is a mere question of degree. What I am here concerned to show is that the mere *implication* or presupposition of real existence to which in one way or another we do undoubtedly come in the Universal Judgment, is not extraneous to the affirmation and dependent on a mere fancy or habit of ours, but is the lineal descendant, *mutatis mutandis*, of that so-called existential affirmation which we have traced in perception and in narrative. And the strength of this implication depends on the concreteness of the idea which here forms the immediate subject in judgment. We have then here as before two elements in the content, or rather a content regarded in two lights. We have self-relation, existence, or a categorical aspect, and external relation, necessity, or a hypothetical aspect. But the nature of the generic affirmation, as analysed above, shows for the first time a trace of reconciliation between these two points of view. The concrete self-relation is no longer void of meaning and purely designative; it is a system of assignable import, and the analogy of which we have spoken is the anticipation or the insight based on this import. But again, this analogy introduces relativity and necessity, and as we saw interprets relations that unite the individuality in question with other totalities which prescribe to it either conditions or purposes. Thus the generic judgment is categorical in respect of its concrete self-relation, and hypothetical or necessary in respect of the analogical or constructive nexus to which the import of that self-relation gives rise.

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[1] It may be said that in such a case the observer has alleged the plant to have been found in a given spot at a given time. But this is not essential; he may simply send in the description.

But, it will be objected, this might be all very well if we were speaking of individuals, whose nature is to be unique, like the present King of England, or the centre of the material universe; but here we are speaking of an indefinite set or series of individuals whose common nature is nothing and nowhere but in them. This is, it may be urged, Scholastic Realism over again. What existence does a generic judgment presuppose, when and where? In reply to such an objection, I insist in the first place that we cannot treat any imperfection of knowledge as incident to knowledge unless we can prove that it necessarily is so, and that to treat a natural kind as an indefinite set or series of individuals is an imperfection of knowledge which can be shown not to be necessary. This consideration however belongs to the subsequent section.

But in the second place I reply that even without treating a kind as an actual unity, and though in fact we do not treat it so in judgments which are true of each individual singly (as common generic judgments are), yet still the individuality of the content dictates its own time, place and measure of existence. And it is this time, place and measure, wholly without reference to subjective era,[1] place or fancy, that is affirmed[2] in generic judgment. It is characteristic of the rose to exist in a certain epoch of evolution and within certain limits on the earth's surface. Existence within this time and place, subject to such variation as the nature of the content allows, is what the generic judgment affirms (or implies) of the rose; that is to say, in affirming that 'the rose has perigynous flowers' we mean that individual actual roses, found within these limits, have the attribute in question. If there are none such, then the rose is like any genus or species that has been imagined to exist by a mistake of identification; the kind in question would in that case not exist, and the judgment would beyond question be false. Of course in every-day subjective judging the place and time &c. of existence is but roughly indicated by what we happen to know or believe about the actual subject of judgment, but it is never referred to the time or place in which we judge, unless *per accidens* our knowledge is limited to, or the content especially concerns, that time and place. Rose in the abstract does not exist. But it is a concrete universal which has power, in the context of the real world to which we refer it, to dictate the epoch, place and quantity of its individual embodiment.

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[1] See above, p. 204, discussion on tense in judgment.

[2] I should not object to replacing 'affirmed' by 'implied', in order to mark the unquestioned line between the singular and the universal judgment, if it were admitted that the implication is an integral part of the judgment, and not a fancy of our own. The point is that existence is implied or affirmed in these judgments, just as Necessary connection is implied or affirmed in Perceptive and Singular judgments.

I need hardly guard myself against the misapprehension that I am alleging that anything and everything exists which we choose to fancy. I am maintaining just the opposite, viz. that if we attempt to embody fancied realities in judgment, such judgment is false; for all judgment is a definition of real reality. We can only escape this result if the fancied content is such as is in its logical nature debarred from being real, i.e. a mere abstraction, and is therefore incapable of claiming to stand for a reality.

Individual Generic Judgment.

c. The reality involved in a concrete universal will be made plain by insisting on a third aspect of the generic judgment, viz. that in which it challenges comparison with the Corporate Judgment. For this purpose we must think of generic judgments which are not merely analogical, but which for want of a better term I may designate as Individual. These judgments are characterised by not being true of any and every individual singly, but only of the kind taken as an individual. Such judgments are, 'The animal world represents an evolution coordinate with that of the plant world,' 'The Orchidean order includes 433 genera and probably about 6000 species,' 'Space has three dimensions,' 'Humanity is the object of worship to Positivists,' 'Monarchy disappears with the advance of civilisation.'

In comparing such judgments as the above with that form of the Singular Judgment which we called the Corporate Judgment, we find at first sight little distinction between them beyond the fact that these Generic Judgments do not employ a proper name in the subject, whereas the Corporate Judgments do. And even this distinction is in some degree bridged over when we call to mind that such determinations as 'now,' 'last year,' 'this,' and 'mine' appeared to us essentially to rank either with proper names or with demonstratives; and also that there is a tendency on the part of proper names themselves to assume abstract significance, so that a proper name is not always easily distinguished from a generic name. The Greek race, Europe, are proper names; but it is more doubtful how we should class 'the Aryan languages,' 'the North Pole,' and 'the Mahometan religion.' Again, 'the earth,' 'the solar system,' seem free from all arbitrary reference; but in speaking of them we really imply 'our earth,' 'our solar system,' and so fall back into some form of the singular judgment.

The difficulty is worth noticing; but it is simply one of those which must arise from the Sorites-like character of any continuous evolution. It is hard to say whether 'the Mahometan religion', as we mean to employ the term, involves a reference strictly of the order of a proper name,[1] or on the other hand is simply an ideal content, and, so far as ideal, abstract. The reason of this is that the conception is in fact on the border-land between the proper name and the mere determinate content, and in all probability it is sometimes employed in the one sense and sometimes in the other. The question is whether the general meaning or the individual identification comes first in the mind. As we saw above, it is not improbable that in primitive careless and unscientific thought the significant word is not made distinct from the proper name—in other words, the intension which is the mere means to identification is the only intension signified, but for this very reason the purpose of identification is not distinguished, so as to be considered primary, from the purpose of definition. In fact, when we now speak colloquially of 'the bay mare,' 'the low pasture-field,' we are using 'bay' and 'low' merely as signs of identification, though of course by help of their meaning. Such is the type of usage which we may imagine to have been the common root of the significant and the proper name. The terms of the Linnaean arrangement of plants, in as far as they are subservient to mere recognition, are a somewhat similar case in point.

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[1] In every case we must keep etymology out of the question. The reference to Mahomet as a historical individual is certainly not the chief element, and perhaps hardly an essential element at all, in the direct significance of 'Mahometan' at the present day. Compare the rhetorical antithesis that has been drawn between 'Christianity' and 'the religion of Jesus'.

But when all deductions are made, there remains a clear distinction of principle between judgments which use proper names, and judgments which do not. From this point of view the generic judgments now before us agree with those last discussed and contrast with the singular judgment. They are able to convey their reference to reality by means of a determinate ideal content.

On the other hand, in the *nature* of their reference to reality they agree with the singular judgment and differ from the common generic judgment. They do not rest on analogy. The individual to which they refer is a real and a single individual, and not a mere individuality. So far from being mediated predications about a number of particulars they are not even true of the particulars that enter into their content. When we said, in the former section, 'The rose has perigynous flowers,' we were treating the individuality of all roses as one by analogy.[1] But when we say, 'The rose family is a descendant of x, a divergence from y, and a transition towards z,' such a judgment is not made about each particular plant within the tribe, nor even about each particular species. If true, it is true of the whole section of plant-life in which every particular rose-plant is a distinct and separate progressive or divergent phase. There can be no doubt, I think, that from an ideal point of view every natural kind in the concrete sensible world must be thus regarded; and of course when we consider existences in which intelligence is more definitely active—society, mankind or at least civilised man, fine art, or morality—in these phenomena the totality is more real and concrete, and the reciprocal relations of its parts exist not merely for the microscope of analysis, but as patent every-day facts.

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[1] Subtlety of transition must have an end somewhere *in writing*, but *in fact* it has none. Thus the reader may object that if it is nonsense to say that the rose family as an actual individual has perigynous flowers, yet we may always safely say in such a case that it prescribes that the particular rose shall have perigynous flowers. I can only admit the objection; the fact is so *if* we are *bona fide* regarding the genus as a whole of evolution, whose actual individuality expresses itself in this and in other common predicates. But if, as is most probable, we realise nothing of the kind, but are merely going by analogy to a common property of roses, then we are treating the kind as a mere individuality, and must not pretend to be treating it as an individual.

Although universal, the Generic judgment in the aspect now before us is fully Categorical. It is in this respect wholly on a level with the Singular judgment, being in fact related to the judgment of Analogy with its dual nature much as the Singular judgment is related to the proportional or comparative judgments that are introduced by a demonstrative. The Singular judgment may be regarded as a *premature* attempt to concentrate individuality—the 'characteristic quality' which the proportional judgment had revealed—into an individual; resulting as we saw in the omission of determinate quality from the individual content by the use of Proper Names. The generic judgment raises (in its Analogical form) and meets (in its Individual form) the same problem in a more adequate way, concentrating individuality into an individual by completion and not by omission. It is as a system of such individuals, united perhaps in a yet more concrete individual reality, that we must conceive of the world known to us through space and time, if we are to assign it any existence beyond the present of presentation. *For us*, it is plain, such individuals are intellectual constructions, and only attached to, not shut up within, the actual present perception. The distinction between concrete realities and abstract truths is not, for us at any rate, that the latter are intellectually initiated and the former are not; it is not a question of origin, but a question of nature, i.e. of the degree in which a content is capable of being regarded as something that exists as a whole and can be considered in relation to itself, or on the other hand is incapable of being given as a whole and affords no matter for consideration in relation to itself. All contents must theoretically be regarded as combining these two characters; and as an important application of this idea I may instance the answer to a question which arises when we make the categorical nature of assertion depend upon the degree of concrete self-relation.

Is it possible, we shall be asked, to lay down a hard and fast line, by which abstract shall be divided from concrete contents? And if not, does not our view surrender the self-dependence of reality and make it purely relative to fancies and notions in the individual mind? Does it not enable us to treat as actual any content however abstract or trivial, and any however concrete or significant as a mere element in hypothesis, simply by varying the point of view from which we regard them? And the answer is, that as reality unites these two characteristics, we can always emphasise either at will; and further, we commit no error in so doing, unless we assume and assert the relation which we happen to be considering to be the only relation that there is. Our knowledge always falls short of reality, and apart from false identification of relations—with which false antithesis is at bottom the same—we have a right to see all that we can either of absoluteness or of relativity in any content whatever. Reality is such that any element or feature of it, however slight or superficial, can be raised by our intellectual gaze to the position of a self-related significant whole. The nature of mind is present in everything; the only difficulty is to see it there. And such an elevation is not false, except in as far as it is exceptional; in as far, that is, as we fail to view the remaining contents of reality with the same constructive insight. Not merely a fragment of stone or metal, but a colour, a curve, a relation of size or weight, is ideally capable of being passed through the stages of generic judgment, of being regarded first as an individuality, and then as an individual. What is false or forced in such a mode of contemplation depends on the want of proportion between it and our ordinary careless vision of organisms and fine art, of men and of society. All contents are relative except the absolute;

but the import and degree of their relativity is not the same.

A further corollary may be worth drawing in a few words from the above considerations. Our present treatment of logic starts from the individual mind, as that within which we have the actual facts of intelligence which we are attempting to interpret into a system. But our consequent preoccupation with the phenomena of the individual mind, with its imperfect grasp of reality and the varying aims and tendencies of its thought, brings with it a double danger which haunts every phrase and every idea in a logical treatise. Either one may speak as if reality were simply relative to the individual mind, a ridiculous idea, but one which the very caution required of a modern writer is apt to encourage; for he hardly dares to allude to Mind as such or in itself; or one may become interested in tracing the germination and growth of ideas in the individual mind as typical facts indeed, but only as one animal's habits are typical of those of others, and so we may slur over the primary basis of logic, which is its relation to reality. For mental facts unrelated to Reality are not knowledge, and therefore have no place in Logic. The difficulty is, in other words, simply that modern Logic has a hard task to hold its own between Metaphysics and Psychology. I entertain no doubt that in content Logic is one with Metaphysics, and differs if at all simply in mode of treatment—in tracing the evolution of knowledge in the light of its value and import, instead of attempting to summarise its value and import apart from the details of its evolution. My object however in mentioning the difficulty at this point is merely to protest that though I assume reality as the norm of the mind, in constructing which it is reconstructing and not creating *de novo*[a] out of itself, yet I can entertain no doubt that intelligence is essential to the being of Reality, and that an abstraction which tries to regard the one apart from the other is a hopeless and helpless self-contradiction. As a working conception in Logic we are forced to adopt some such idea as that of a normal intelligence operative in all human minds, but subject to the accidental limitations of each. The evolution of knowledge is, as Plato long ago portrayed it, the emancipation of individual minds from their accidental limitations, and their education into the knowledge of the one real and intelligible world.[a] But the duty of modern science is to preserve the continuity of this evolution, and to admit no *saltus* at any point between the world in which we live and the world which is real and intelligible. And in this continuity we have a standpoint which Plato, although he reached it, did not consistently maintain. Objective Intelligence presents itself in Logic as the mere postulate required by such a continuity, and, starting as we have done from the individual consciousness in time, it is merely as a postulate that we propose to treat it. To say that the real world is the intelligible world is only to repeat what we found ourselves obliged to suggest as an elucidation of the earlier stages of judgment, that reality is something at which we arrive by a constructive process.[1]

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[a] See ii, p. 319. Reconstructing, for me, never meant representing or copying. And thought is as creative as anything in the universe can be. But creation *de novo* is a contradiction in terms, because it forbids the conception of a creative nature, and so of any interest or continuity throughout reality.

We are now to consider the consequence of emphasising the abstract or relative aspect of the Analogical judgment. We are thus led to a form of thought which is antithetical to the Individual Generic judgment of which we have just been speaking, and consequently must be regarded as a divergence from the concrete evolution of thought towards the mechanical or analytic judgments which begin with enumeration.

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[a] It does not seem necessary in Logic to develop this point further. A 'first philosophy' would have more to say about the sense in which the one Real world is intelligible, and would not admit that the limitations of individual minds ought to be called, in principle, accidental, little as we may be able to explain them.

[1] See above, Introduction, p. 38 ff.

CHAPTER VI

UNIVERSAL JUDGMENT (*continued*)

Pure Hypothetical Judgment. Its relation to previous forms.

ii. THE Universal Judgment, when pushed to the extreme point of abstraction, becomes the Hypothetical Judgment.[a]

a. The Hypothetical Judgment is distinguished from all which have thus far been spoken of, by its essentially abstract character; abstract not merely as thought is said to be abstract when compared with sense-perception, but as the thought of an ideally isolated attribute is abstract compared with the thought of a self-dependent and self-related individual. It represents the fourth of the elements or aspects which have been confounded, or at any rate have not been duly distinguished, by traditional logic within the so-called Universal Judgment. Its differentia is that it does not refer to a concrete subject, not even to what we called an individuality or the concrete self-related content in its aspect of self-relatedness; and that consequently we do not consider whether its subject is given in actuality or not. For it is essentially the judgment of necessity or relativity, in which the subject is taken, not given, and taken not for its own sake nor with reference to its individuality, but for the sake of that which is to follow from it, that is, for the sake of its relativity. It is a judgment which follows out the single thread of a nexus of attributes, and does not heed the import of the pattern into which it enters. If a gravitating body is set free to fall, it falls with an acceleration proportional to the squares of the times, whether it is a drop of rain, or a tortoise with the head of Aeschylus below it. Here we have, in an explicit shape, the relativity of knowledge which has haunted us throughout the evolution of judgment, forbidding us to feel satisfied in connecting together any data which we might merely chance to light upon in conjunction, and requiring that every idea should always be limited and controlled by its reference to something else, and not simply taken as we find it in perception or in ordinary life. And just because this principle has so haunted us, the judgment that embodies it cannot be sharply severed in meaning from the earlier forms of the universal judgment, and even the quasi-collective 'All gravitating bodies &c.' may, and most frequently in this case does, contain what is really meant as a hypothetical affirmation. 'In this case,' for the distinction really goes, as we have maintained all through, not by the shape of the proposition, but by the content of the judgment. The connection however between all the types of universal judgment is intimate and essential,

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[a] A word must be said on the distinction between Conditional and Hypothetical propositions, to which Mr. Keynes, following Mr. W. E. Johnson (Keynes, p. 249), attaches much importance.

Conditionals, as I understand, affirm a connection between phenomena, and their elements are events or combinations of properties, the connection of which in a subject or in time or place is affirmed. Hypotheticals have for their elements independent judgments, propositions of independent import, a relation between the truth or falsehood of which is affirmed. The characteristic phrases of Conditionals are 'Whenever,' 'In all cases in which,' 'If any—then that.' The unambiguous form of a Hypothetical is 'If — is true, then — is true.'

I think this distinction tends to obscure the meaning of the pure Hypothetical. I note some points:

(1) I do not see why the elements of a pure Hypothetical should be taken as propositions or judgments. Its characteristic, I hold, is to trace a thread of logical necessity, and I see no gain in throwing the sequence into the form of two or more propositions. And this opinion encourages

(2) the idea that true Hypotheticals coincide with what I have called 'broken-backed sequences' (p. 236 below), the propositions retaining their full meaning when separated from one another (Keynes, p. 251). For the criticism of this view I may refer to my discussion on p. 236.

(3) It would have made a neat series if we could have said that Categoricals (in shape) are only assertorical and existential; Conditionals (adding 'Because,' 'Since,' to the above-mentioned phrases) unite modal, assertorical, and existential meaning; and true Hypotheticals are modal only. But Mr. Keynes rejects any such distinctions, which are indeed incompatible with usage. Therefore I see little or no *raison d'être* for the class of Conditionals. It is hard enough to distinguish Categoricals (in shape) from pure Hypotheticals.

(4) I am glad to agree strongly that it is not the differentia of propositions of the type If A is B, C is D, to imply *doubt*.

(5) The student will distinguish from the doctrine I am discussing

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Mr. Bradley's view that all judgments are conditional, i. e. that there is always a supplement, which is never completely known, necessary to make the predicate fully true of the subject. Appearance, ed. 2, p. 361.

so that in popular usage one easily slides into the other, or even combines the other with itself as its ground or consequence. When I say 'All animals need food' I am probably expressing a *quasi-collective* conclusion about a property shared by all species of animals, taking its significance from an *analogical* perception of the generic function and immanent purpose of animal life, but ultimately resting on the *hypothetical judgment*, expressing a *necessary or relative* principle, 'If force is to be expended it must be supplied.' It will be observed that the second type of generic judgment, which for want of a better name I have called Individual Generic, is omitted from this combination of aspects. It represents a tendency divergent from that of the Hypothetical assertion, while the Analogical judgment is undecided between the two. If the Individual Generic judgment is capable of combination with the Hypothetical, we must look for the result in the Disjunctive and not in the Universal affirmation.

External Form.

β. The type of the Hypothetical Judgment in traditional logic, so far as it is recognised at all, is stated in one of three forms: 'If A is B is;' 'If A is B, then C is D;' and 'If A is B, then it is C.' The third of these forms is that which guides us to the true import of the judgment, though conformably to the habitual irrelevancy of popular thought the second is that most commonly in use. But this second is obviously a broken-backed sequence, in which no point of unity is formally recognised between the antecedent and the consequent. When, indeed, significant words are substituted for letters, the unity would generally be obvious, supposing the sequence to have scientific value at all; but in such a case the expression is not essentially distinguishable from that of the third form. The first form, 'If A is then B is,' has been said to be an abbreviation in which letters stand for clauses; in that sense of course it must be reducible to either the second or the third form. We get the same result if we try to take it as a combination of single-word or impersonal predications. Contents may undoubtedly be ascribed in judgment to an unanalysed present, but an unanalysed present can form no bond of union for a necessary sequence. 'If guilty, then death' is a mere linguistic abbreviation for 'If he is guilty then he will be put to death'. And even 'Where there is smoke, there is fire' superadds to the impersonal 'There is' a true local particle in the 'where' of the antecedent, and this reacts by a curious equivocation on the impersonal 'there' of the consequent. No doubt, but for the unpleasantness of the sound, we should say, '*Where* there is smoke, *there* there is fire.' Here again, then, we have in essential meaning the third type of the Hypothetical judgment, 'If A is B, it (A) is C.' I will next illustrate the transformation of type ii. into type iii. 'If the barometer (A) falls (B), the weather (C) becomes stormy (D).' 'If the atmosphere (A) decreases its pressure locally (B), it (A) must leave a gradient for wind (C).' But now if we take the lines,

> '. . . when in Salamanca's cave
> Him listed his magic wand to wave,
> The bells would ring in Notre Dame,'

we find that the *saltus* from antecedent to consequent is all but essential to the judgment ; the point of the mystery is that we cannot get at the underlying unity. Thus we see the extreme case of type ii. in a judgment which has for its object to assert magical, i.e. irrational, connection. Of course the general scheme of reduction would have to be, 'When his magical power (A) was exerted (B), it (A) could act at any distance (C).' It is in this sense that Schopenhauer calls some of Euclid's demonstrations conjuring tricks, because, although in a demonstration some unity of course must be shown between antecedent and consequent, yet the unity shown is often not central or fundamental,[1] and is therefore a *causa cognoscendi*, and not a *causa essendi*. In the pure type, 'If A is B, then C is D,' we have no indication even of a *causa cognoscendi*.

Much more might be said about the forms of conditional sentence ; but the subject is really grammatical rather than logical, for the hypothetical judgment can be expressed without a conditional sentence at all. Hypothetical and Categorical Judgment, as we understand the terms, are a question of content, not of grammatical form, and the hypothetical judgment is found wherever we frame assertions about an abstract content, in the above sense of abstractness; although there is a difference of adequacy in different grammatical expressions for any kind of judgment, and the conditional sentence resists any attempt to embody in it a purely categorical meaning. 'If this man dies, our cause is lost,' takes 'this' as a sign of unanalysed content and not as a point of attachment in reality. We know that the reason is somewhere in the unanalysed content, and so take it as an antecedent in the lump.[1]

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[1] Schopenhauer, Werke, i. 136 ff.

Assertion made by Hypothetical Judgment.

γ. What is the precise nature of the assertion conveyed by a Hypothetical Judgment? In answer to this question I shall speak first of the idea of Ground or nexus in general, secondly of Ground as compared with Cause, and thirdly illustrate our view by the attitude of the individual mind in hypothesis or supposition.

The idea of Ground.

α. The contrast which we have anticipated throughout the above account of judgment determines the central attribute which we are now to consider. The content of a true hypothetical judgment is abstract; abstract in itself and not merely by the absence of sensuous perception. In other words, the subject of a hypothetical judgment is not an individual, not a whole, nor anything considered as a whole, i. e. as a self-related system. On the contrary, the content of a hypothetical judgment is composed of ground and consequent, each referring to something other than itself, and hence essentially a part. For a system as a whole, such as space, or the totality of gravitating matter, or the British Constitution, is a mere fact, complete in itself, and neither a ground for nor a conclusion from anything else.[2] It is only as parts within a system that elements can be so relative to one another that '*if* this is *so*, then that must be *so*'.

It is only a question of detail how far the system in and by which the nexus subsists, is itself made explicit as a content within the hypothetical judgment. We may say, if we like, that the ideal of logical sequence demands that the system as a totality should be so made explicit, because the system is the real ground of the nexus ; and if the system does not appear in the content, the real ground does not appear in the content. But this argument, from the comprehensiveness of the real ground, does not overcome the essential principle which is involved in there being a ground, as ground, at all. Ground implies a consequent other than, though fundamentally one with, itself. This transition or otherness ceases to exist if the content does not formally present itself as part to part. For say that the totality of the system is explicit in the ground, still this totality is depressed into the relation of a part by the fact that a part is selected to appear over again as consequent, and so as formally at least external to the ground. Thus it remains true that the elements of content in a hypothetical judgment are related as other to other within an identity which determines the one on the basis of the other. Such an identity, as far as exhibited in the one term that, in virtue of it, determines the other, is what we mean by Ground. It is obviously capable of all degrees of completeness up to the ultimate fact or whole which embraces in itself as parts both ground and consequent commonly so called. The various degrees of imperfection or broken-backedness in Hypothetical judgments, such for instance as were illustrated in the last section, are simply the degrees in which the system that determines the nexus fails to manifest itself within the content connected. It must not be forgotten however that we have refused to treat the grammatical form of propositions as decisive of the character of judgments. Where no rational nexus is traceable, but only a coincidence in fact, however general, we cannot admit that the essentials of hypothetical judgment are present. But then if we are impelled to make a judgment in hypothetical form, there always is some presumption of a rational nexus. We shall consider in the following section what attributes of true hypothetical judgments are shared by analogical assertions such as ‘ If he is a negro, he has woolly hair ’. It is not worth while to insist more fully on these degrees of imperfection, except in so far as they will come under our notice in dealing with the doctrine of causation and with the kindred subject of the negative in hypothetical judgment.

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[1] I take this analysis, which appears to me exceedingly felicitous, from Bradley's Principles of Logic, pp. 89–90.

[2] This applies to the examples given only when considered with reference to their internal nature. It may be said that space implies an intelligence; but this is as within a further whole. There is, of course, no ultimately Absolute whole except *the* Absolute whole.

Let us attempt to make perfectly clear, before we go further, the nature of the relativity within a system which we ascribe to the contents before us. The simplest cases of such relativity are drawn from the field of numerical or geometrical construction. A Chinese puzzle or dissected map may give us a first idea. Any selected piece out of such an arrangement determines nothing by itself, but when a second piece is given some relation between them emerges, though perhaps only a negative one. It is further possible for a piece entirely enclosed by others to have its place relatively to them determined long before the whole arrangement is completed ; but this determination will really be partial, for the place of the whole group of pieces cannot be determined till the whole puzzle is put together. Now the arrangement *as a whole* is a mere matter of fact ; it is only within it and by reason of it that each piece has a prescribed place in virtue of its own shape combined with the shapes of all the other pieces. All the pieces being given, of course the arrangement is given too ; but if nothing is given, of course all is in the air, and one arrangement and set of shapes is as likely as another.

Or again in the region of number, we may take as equivalent to Hypothetical judgments those which in treating of Enumeration we called Mediate. $50 \times 3 = 25 \times 6$ would run in conditional language, 'If 50 is multiplied by 3, the product is equal to,' &c. Here we have one form of the numerical whole 150 presented as a term from which another term, viz. another form of the same whole, may be inferred. The system within which the relation exists is of course not the whole 150, but the system of number as such ; or we may say if we prefer, the whole 150 as involving and exemplifying the nature of the whole of number as such. This system may be brought to bear either by simple counting, the process which is so to speak the medium in which number exists ; or by developing any of the relations which are embodied in the several places of the series with their individual names.

Thus the above statement may at once be reduced to a tautology by taking 50 as 25 × 2 and 6 as 2 × 3. The combinations which might be made with the same result might be pursued as long as we cared to continue the enumeration of places in the series, and from whatever point we started in the system we should obtain the same result so long as both sides of the equation were subjected to treatment that was equivalent according to the rules of the system. On the other hand, if we were to assume the invalidity of the equation, we should at once make the whole numerical system inconceivable—a unit, say, would have to be taken as varying in value without being subjected to any arithmetical process, and such variation is incompatible with the fundamental principle of number.

The same may be shown in space, treating it not *qua* extended whole of parts outside one another, but in respect of the connection of its attributes. 'If two straight lines have the same direction, they can never meet.' This is a consequence drawn from the conception of direction in homogeneous space. If we destroy the idea of homogeneous space, the relation, which only holds within a totality having that attribute, is annihilated. If there can be a change of direction which yet is not a change of direction—I do not know how else to express the notion of direction in space which is itself curved—then, I presume, the judgment from which we started is no longer true.

The same characteristic might be pointed out in relation to gravity, inertia, or any property which is the basis of exact inference. The consequences of gravity hold only within the totality of gravitating matter, of inertia in the combination of motions, and so forth. Every sphere of this kind, every set of relations within which certain nexus of attributes hold good, is itself ultimately a *fact* or datum, relative no doubt within some further totality, but absolute relatively to the inferences drawn within it. Hence we are brought to a conclusion of the last importance. All hypothetical judgment rests on a categorical basis. That is to say, all relativity rests on an absolute datum and all necessity on fact. Why then is mechanism, necessity or relativity, opposed to individuality, fact or absoluteness, if all mechanical relations are themselves characteristics given in some individual whole?[1] The answer seems to be dictated by what has been said. Individuality is in self-relation, Necessity is in external relation. But as all relation is within some whole, it follows that wherever we have necessity or relativity we are concerned with more than one whole or individual content; that is to say, we have a whole or content with its own import and significance taken as a part within a wider or completer totality.[2] The ground or necessity which forms the affirmed nexus of attributes lies then in the systematic nature of the wider whole, that within which the terms of the nexus are capable of being opposed as part to part.

We will examine two or three instances of ground and consequent in the light of the above doctrine. 'A picture 6 ft. × 7 ft. cannot be hung in a space 5 ft. × 6 ft.' Here we are taking the picture as a whole in itself, but as a part within space, and as therefore having external relations determined by the spatial system as including other objects. The arrangement of other spatial objects so as to leave only the area 5 ft. × 6 ft. is incompatible with the occupation of the area 6 ft. × 7 ft. by the picture. But again, we may judge that 'If the boat in the right foreground of the picture were erased, the arrangement of the distances would become confused.' In this judgment we still call attention to spatial determinations, but only as involved in the concrete individuality of the work of art, which assigns them their meaning and value. We are thus taking not space as such, but the individual picture before us, as the totality or determining system, and contemplating the necessary relations which this fact or whole, from its nature and structure, imposes on its several parts Of course, apart from the effect of the whole picture, there would be no such necessity or relation between the parts. This is an illustration of the ultimate nature of logical necessity or relativity and its relation to fact, which is, if not specially felicitous, at least true in every detail. In a true work of art we have the bearing of every part on every other, the innumerable details, none of which could be altered without necessitating the alteration of others, all concentrated in a unity which is itself constituted by all these parts, and yet, as a whole, prescribes the relations existing between them. And yet the whole is itself a comprehensive *fact*, and apart from it or outside it all these prescribed relations lose their necessity and disappear.

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[1] Space and Time are, as we saw, imperfect individualities. But it is their individuality and not their imperfection that makes them sources of general relations in things.

[2] See on Measurement, chap. iii, supra.

Again, take such a judgment as 'The sound of the violin is of peculiarly piercing quality.' If we describe the sound merely in terms of the mechanical system of vibratory movements, it is governed by the necessary relation, 'If a string be dragged by a bow, slipping from it at intervals, its vibration is of highly angular form,' i. e. in terms of its effect on the air, produces sharp and not gradual transitions from increase to decrease of condensation. Omitting the effect on our hearing, this is a nexus of attributes grounded in the properties of vibrating bodies, and in the laws of friction and of undulatory transmission. But here again the whole system of physical properties, though comprehensive, is a datum, and except in it no necessity could be shown why motion communicated by a bow must have this particular form, or why this particular form should find a correlative in a peculiar type of impulse communicated to the air. It is this system which as an identity in differences appears first in the effect of the bow on the string, and then in the peculiar impulses communicated to the air by the sounding-board. It is only as having such a unity behind both of them that the one of these phenomena can condition the other. And here again we may obtain a relative absolute by considering the compound tone of the violin string as perceived by the mind through the ear, simply on its own merits. It then becomes an ultimate fact, embodying certain relations between musical sounds. And within this fact we may distinguish the necessary relation, 'When a tone is piercing in quality, the higher overtones are strongly marked in it.' And finally, we may bring the physical and the musical system together under the complex fact of the correspondence between the shape of oscillations and the character of tones, and say on the faith of this complex fact that where the oscillations are angular, the higher overtones are audible, and proceed to the whole system of deductions made possible by Fourier's analysis of all vibrations into combined single oscillations.

The idea on which we have been insisting—that of a system or unity which prescribes the relation between its parts or differences—is the idea of Ground, which includes the sphere of the Hypothetical Judgment, and indeed wherever it appears may be said to involve a Hypothetical element. It is difficult to express the essence of this conception otherwise than by saying that the system is the same in the one difference or aspect as it is in the other. We thus appeal to the notion of identity in difference, which we have taken throughout to be the content of judgment. Only, as Ground, it is not *mere* identity, but *systematic* Identity,[a] a notion easier to illustrate than to define, but apparently equivalent to 'identity such that the differences in which it is manifested have definite relations to one another.' Of course any such definition only repeats the characteristic which the account of the Hypothetical Judgment presupposes.

Assuming however these characteristics as summarised in the above definition, we can draw from them two consequences that affect the idea of Ground. First, it is plain that when once a Ground is rightly stated, in conformity with the true nature of the system which it presupposes, and with which it is in fact identical, such a Ground is unalterable except by alteration of this system itself. With what justification, theoretically, we refuse to contemplate such alteration of the universe as a whole, or how far practically we permit ourselves to contemplate it in respect of subordinate systems, e. g. man's moral nature or the type of disease, are questions that must be reserved for a general discussion of the postulates of knowledge. Formally, we may say, the whole cannot alter, because any alteration must be included in the whole. But we shall see that so purely formal a postulate would not satisfy the purposes for which a postulate is required.

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[a] The question has been raised (Keynes, p. 264) whether the consequent in a true Hypothetical should be called an inference from the antecedent. My answer is given by the doctrine of Ground. I certainly agree that the consequent in a typical Hypothetical does not follow from the antecedent alone, if that means that it would follow if the two stood, without further implications, alone in a universe. On the other hand, I hold that the warrant of the sequence lies in the relevance and adaptation of the clauses to the systematic world which they presuppose (see p. 245 below), and assuming this world as a basis, I should say that the consequent is always a necessary inference from the antecedent. It appears to me off the track of the distinctive Hypothetical type to treat the consequent as inferrible only when it is a sort of converse of the antecedent, or when the antecedent has the full premisses of a syllogism put into it (Keynes, p. 264). Even here, after all, you have to presuppose some law of identity or Contradiction, so that you do not get rid of the assumption of a systematic world.

And secondly, it is plain that a ground is not rightly stated unless it either embodies the whole essence of the system which constitutes the ground, or at least is exactly relevant to or compatible with that system, and to the particular bearing which a given interest in any context imposes upon it. In the former case it is clear from what has been said that the hypothetical judgment must tend to expand itself into a categorical one. When we go to the root e. g. of geometrical truths we find ourselves affirming facts regarding the nature of space. We shall thus at a later stage have to face the conception of judgments at once categorical and necessary; we have indeed anticipated something of the kind in speaking of the individual generic judgments. The latter case is that which gives rise to hypothetical judgments having strict reference to a systematic ground, which they therefore imply, but do not need to express. Such are the ordinary statements of 'pure cases' in exact science, or geometrical truths as commonly treated without raising fundamental questions of the nature of space. A 'pure case' is a nexus of differences reduced to their expression as the differences of the system in which they have their nexus. Without knowledge of such a system we may analyse *ad infinitum* and yet never be sure that we have obtained a 'pure case'. This has, as we shall see, an important bearing on the theory of Induction. I gave a rough instance of a pure case on p. 220 in reference to the annual rings of exogenous trees. Such, again, are the mechanical and chemical elements in the vital processes of man, e. g. the pumping action of the heart, the oxidation of the carbon in the blood and so forth, in stating any one of which as a necessary sequence of ground and consequent it must be treated as belonging to its own mechanical or chemical world, and not as an element in human life. All such judgments are abstract in the fullest sense, and analytic ; their very point is that they disregard the import which constitutes the individual. On the other hand, a system which is the combination of individuals in their full import, i. e. the state in relation to moral beings within it, is most naturally dealt with not in hypothetical but in categorical judgments. For the subject is either the concrete system itself, or an individuality subordinate to it, taken in its full concreteness. It would be sheer pedantry to speak in hypothetical language of man's moral being, its ground, and its necessary relations.

It is a corollary from the idea of Ground as a relation purely relevant to a positive determinate system that the hypothetical judgment, when ideally complete, must be a reciprocal judgment. 'If A is B, it is C' must justify the inference 'If A is C, it is B'. We are of course in the habit of dealing with hypothetical judgments which will not admit of any such conversion, and the rules of logic accept this limitation as they accept the custom of ordinary speech as to the comparative range of subject and predication. Some cases of non-reciprocal sequence and their justification will be considered in the next section. But here we are only concerned to explain the principle upon which necessary sequence must ultimately rest ; and according to that principle, the unity of a system in its determinations, it follows that if A B necessitates A C, then A C must also necessitate A B. We are not now speaking of causation, but simply of coherence in principle, and it is obvious that the idea of coherence in a system is reciprocal. A cannot cohere with B unless B coheres with A. If in actual fact this is found not to hold good, and A B is found to involve A C while A C does not involve A B, it is plain that what was relevant to A C was not really A B but some element α β within it. 'But may not the irrelevant element be just the element which made A B into A B as distinct from A C, so that by abstracting from it A B is reduced to A C, and the judgment is made a tautology, i. e. destroyed ?' 'If he is drowned, he is dead,' e.g. is not reciprocal. .But it may be objected that the judgment I desiderate is really 'If he is dead, he is dead'. The suggestion is tempting, because it aims at cutting up by the root a troublesome scientific problem, viz. the statement of connected attributes as purely relevant to one another and yet as distinct. We constantly tend either to insert irrelevancies by way of distinction, or to let both attributes fall back into the undistinguished abstract relation which connects them. To grasp a distinction in unity is an effort, and we dislike effort. Nevertheless, if it were impossible, the idea of system, of the one in the many, would be gone. In the above instance, the reciprocal judgment required would be something like 'If his heart has stopped for good, he is dead', or 'If he is drowned, he is dead through suffocation by water'. The former shape obtains reciprocity through the expansion of the antecedent, the latter through the limitation of the consequent.[a] A systematic relation is always within an individual whole, and the priority or antecedence of its elements belongs to the imperfection of knowledge, and not to the relation in itself. I am not saying that every individual reality exists endlessly in time, but I am saying that every whole in as far as its parts form a system has a nature which is independent of time, or (what really comes to the same thing) continues positively and actively through the fugitive moments of time.

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[a] Mr. Keynes's remark (p. 271) called my attention to my omission of the case in which reciprocity is secured by dealing with the consequent (see however p. 255 below). My use of the term expansion is not his. Applying it to the consequent, which *ex hypothesi* in a non-reciprocal Hypothetical is wider than the antecedent, he ought to mean expansion of *the expression*, but limitation of *the case*. But from his use of the phrase 'a more complete statement of the consequence' as equivalent to 'expansion', I think he has in mind his cases in which something like a middle term is introduced in the antecedent and dropped in the natural statement of the consequent; and that he hardly notes the point that as a rule the consequent, as a case, has to be cut down and not expanded in order to secure reciprocity. My reason for insisting on the ideal of reciprocity has to do with my general attitude to Logic and to the Hypothetical Judgment (Preface, and p. 236 above). I do, as Mr. Keynes says I ought, maintain that the proposition A is imperfect. See Knowledge and Reality, pp. 190-2.

But apart from time on the one hand and irrelevant elements on the other, I cannot see how the relation of conditioning differs from that of being conditioned. Every B that is conditioned by A is the condition of A being such as to condition B, i. e. of A being what A is; and if the being of A were wholly relevant to B, this would be equivalent to saying that the existence of B involves the existence of A. In other words, if there is nothing in A beyond what is necessary to B, then B involves A just as much as A involves B.[a] But if A contains irrelevant elements, then of course the relation becomes one-sided, as if we were to say that a plane section of a sphere has its radii equal. The mention of the sphere makes the relation of coherence one-sided; the circle need not be regarded as a section of a sphere. But, always assuming the homogeneous nature of Space, the relation between equidistance from a central point and uniformity of curve is inseparable, and it is impossible to see that either of these essential differences of the circle is prior to the other. It may however be questioned whether in an ultimate sense any incomplete case can be pure, i. e. whether irrelevancy can be wholly avoided except by including the whole fact to which the judgment belongs. What, e. g. has distance to do with curvature? The only answer is in the nature of space. This amounts to a doubt whether in the end any Hypothetical Judgment can be true, and points us again to a further type of judgment in which such deficiencies may be made good.

The relation of Ground is thus essentially reciprocal, and it is only because the 'grounds' alleged in every-day life are burdened with irrelevant matter or confused with causation in time, that we consider the Hypothetical Judgment to be in its nature not reversible. The habit of thought is to proceed by determining an undetermined datum ; and this habit is never wholly laid aside even in the Hypothetical Judgment which is theoretically its negation. But a given condition, though interpreted in a single aspect by the judgment which draws its consequence, may have other consequences just for the same reason for which its consequent may have other (alternative, not merely co-operative) conditions. The 'other' conditions arise through a variation of the irrelevancy present in the given condition ; as, if a circle has been said to arise through cutting a sphere in a plane, this condition may be varied by altering the superfluous relation in which a plane figure bounded by a line equidistant from the centre can be regarded ; e.g. it may be taken as a section of a cone, or as an ellipse with equal axes. And just as each of these irrelevancies would present the antecedent of circular curvature in different garb, so the presence of an irrelevancy which is thus capable of variation involves all the independent consequences that follow from the irrelevant idea—in this case that of a sphere-section—which has been included in the condition. If we restrict ourselves to the relation of equidistance in a plane, we can get no result beyond that of a circular figure with the properties which belong to it in the geometrical system.

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[a] Mr. Bradley's remark should be noted. 'I certainly agree here that if the judgments are pure, the relation holds both ways [ref. to this place]. But if in the end they remain impure, and must be qualified always by an unspecified background, that circumstance must be taken into consideration.' Appearance and Reality, ed. 2, 362 note. I presume that the effect would be to demand great caution in asserting a reciprocal, e. g. not to assert it unless we knew by experiment or other special consideration that *ad hoc* the background might be neglected.

A ground that admits of such variation is not only partial or abstract, i.e. one which leaves the true ground in a measure to be understood, but is actually in part 'impure', i.e. burdened with matter which gives rise to diverging consequences, and makes the ground itself one among many converging grounds.

We have thus seen the idea of Ground in three aspects ; as an actual system, interpreted in its bearing upon its parts ; as a 'pure case', i.e. a factor within a system stated in terms precisely relevant to the system and entering into a nexus in virtue of that system ; and as an 'impure case', i.e. a condition weighted with irrelevant matter and so failing to express the real nexus which is aimed at. The first of these three is necessarily categorical in import, and may perhaps be identified with Schopenhauer's 'Seyns-Grund', or Ground of Being ; the two others are primarily hypothetical and only *imply* reality behind them, and correspond together to his *Erkenntnissgrund*, or Ground of Knowledge. It need hardly be remarked that the ground of Being is also the only genuine and complete ground of knowledge. In respect of reciprocal character they must be divided differently; the first two, the complete and the abstract ground, being necessarily reciprocal with their consequents, and the third being as obviously not so.

The idea of cause.

b. Cause may correspond either to the complete form or to the incomplete forms of Ground. In the former sense it can scarcely be taken to differ from ground at all. In the latter sense it is a distinct species which is included in a common genus with the incomplete forms of Ground.

Cause as complete Ground.

(1) Cause[1] as corresponding in meaning to complete Ground would consist in the exhibition of some selected attribute or event—the effect—in the totality of systematic relations which constitute its necessity. And in such meaning it can scarcely be taken to differ from Ground, because the temporal succession, which seems the natural differentia of Causation, disappears in the reference of the effect to a positive and continuous system. Mere temporal relation is negative, is nothing. It is only the unity behind the temporal relation that can bind cause to its effect; and in the real or complete Ground this unity is made explicit. The cause of the earth's being where it is at this moment may indeed be popularly indicated by saying that it was, wherever it was, at the previous moment; but strictly of course the relation of the present position to the last position when fixed before the mind as discrete and successive in time is simply that the one is not the other, which is so far the same relation that subsists between the earth's present position and the sun's or moon's last position, and amounts to nothing at all. The cause of the earth's present position is the persistent velocity, together with the persistent influences regulating the direction, of its passage through space. This meaning of Cause is the ideal logical import of the term, and is what Mill meant to indicate when he defined Cause as the 'sum of the conditions'. The word 'sum' is unfortunate, because it indicates a special

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[1] On the conception of Cause, see some very acute remarks of Professor Clifford, Lectures and Essays, i. 150.

way which may be inappropriate of combining the factors. The totality of the relations would be a better phrase than the sum of the conditions.

The only difference between Cause in this sense and Ground would be that Cause, though not a sensible event, still retains an import relative to the explanation of sensible events or of attributes entering into events (character, health, &c.), and is therefore not coextensive with Ground, which includes e. g. geometrical relations where the phenomena of process in time are wholly wanting. There would be no sense in saying that the attributes of a triangle are the cause of those of parallel straight lines, or *vice versa*.[1] The distinction, however, is more one of usage than of theory. On the one hand, the effect, subsequent in time, which is exhibited as one relation or difference within a necessary nexus, is necessary to the persistence of the whole system and to the evolution of its significance, so that the parts of the unity or system are reciprocally necessary in *complete* Cause as in complete Ground; and on the other hand, if in investigating a ground, say in geometrical matter, we go back to the whole system of fact which is at the root of the necessary connection, we shall be justified in treating this fact as a Cause. We could hardly be censured for saying that the nature of space is not only the ground, but the cause, of the attributes of triangles and of parallels alike.

Cause then, in its largest sense, is a real ground, and ultimately there is no complete ground which is not a real ground. Ground and Cause are thus not identical but convergent conceptions, i. e. as they are completed they tend to coincide, and the striking differences between them depend on a comparison of their imperfect and ultimately self-contradictory forms.

Complete Cause, like complete Ground, corresponds to a Hypothetical Judgment whose condition and consequent are reciprocal. If, as is perhaps the case in Mill, the phrase 'sum of the conditions' is not limited to relevant conditions, and the hypothetical judgment which expresses the nexus of such a sum with its effect is consequently not reciprocal,[1] the notion of sum of conditions loses the only merit which it appeared to possess. But if it means, as it seems to mean, a persistent and systematic fact, then it agrees with other indications in suggesting that for the complete Ground *or* the complete Cause we must go beyond the Hypothetical Judgment.

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[1] Schopenhauer's 'Seynsgrund' describes the relation of such cases, supposing the rational connection to be central and fundamental to the contents connected.

Cause as an event, distinct from Ground.

(2) Cause as an event in time is thus an imperfect conception. Indeed it is hardly possible to formulate the idea of one event in time as the cause of another that falls, in time, wholly outside the first. Cause is always taken to be more or less of a complication of relations and circumstances; and these, as acknowledged to bear on one another, are not mere events in time. It may indeed be retorted that *mere* time is an unreality and that no one ever said that causation was in mere time, i. e. in succession taken as discrete; but that real time involves continuity as well as discreteness, and in such real time causation really is. Such a retort might be ill-founded as a statement of common logical opinion, but would in substance express the principle which I am endeavouring to explain. *Mere* time is mere succession; but real time involves something that is not in succession, though it remains through succession. The consciousness for which there is time has begun a process which tends to abolish time. To say that in this sense Causation is in real time is to say that Cause corresponds to an incomplete ground, i. e. a partially known unity including the factors which are in question as Cause and Effect. But when we come to speak of an incomplete Ground, the difference between thought and reality emerges, for it is only the complete Ground that is the real ground. When the ground in thought is distinguished from the ground in fact, then the cause is one with the ground in fact and is separable from the ground in thought, which latter is sometimes called by analogy the *causa cognoscendi*.

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[1] Cp. Essays in Philosophical Criticism, p. 96 note. Of course if Cause as = sum of conditions is compatible with Mill's plurality of causes, the Cause as sum of conditions cannot = Ground. But it ought to be incompatible; for in any concrete circumstance that may be named as condition, what is not relevant is not condition. So the sum of conditions ought to be restricted to the relevant or minimum conditions.

Of course the *causa essendi*[1] must be a *causa cognoscendi*, but a *causa cognoscendi* need not be a *causa essendi*. As a matter of fact the ground in thought or *causa cognoscendi* often belongs to the effect in time,[2] but may be any element whatever related to the real ground, whether cause, effect, or abstract principle.

The root of these distinctions is that the nexus of ground and consequence is at this stage still charged with irrelevancy. The cause and effect, ground and consequent, are all of them at this stage concrete events, or groups of incomplete relations, among which the special aspects belonging to any *nexus* that may be in question have not yet been *freed* by analysis. As a result of this state of things the hypothetical judgment which embodies such a connection follows the analogy of singular judgments or of imperfect universals, and has an antecedent which is not affirmed by affirming the consequent, just as the subject of a singular or generic judgment is not affirmed by affirming its predication. And in so far as the hypothetical judgment is taken to be the natural vehicle in which to assert causation, this characteristic of it agrees with the popular view that the same cause always has the same effect, but the same effect need not always be due to the same cause. This doctrine, formulated by Mill under the name of the plurality of causes, and wholly incompatible with his view which treats Cause as 'the sum of the conditions', is a mere translation into analytic science of the notion of subject and attribute, here quite out of place. The degree of truth which the view possesses depends solely on an imperfection of knowledge and not in any way on the nature of causation.

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[1] Schopenhauer's distinction between the *Causa essendi* (Seynsgrund) and the *Causa fiendi* (Ursache, cause proper) is reduced by the view taken in the text to a distinction of degree. Effect cannot be in succession to cause in the sense of falling outside it; there must be a real whole which includes both.

[2] It is worth remarking as a matter of usage that 'antecedent' in the conditional sentence has absolutely no allusion to the temporal relation of the events connected. Its name may have come from the usual grammatical place of the condition, or from some profounder idea of priority. But this would be a mere historical connection. Mill's application of the term to succession in time creates quite an unfounded idea of correspondence between Causation and the Hypothetical Judgment.

It is an old story that if, having said that 'All men are mortal', you then further say that 'A. B. is a man', you are committed to the assertion that he is also mortal; but if you prefer to make the more cautious assertion that he is mortal, you do not thereby pledge yourself that he is a man. The same maxim in relation to the Hypothetical Judgment is summarised in the formula 'Assert the antecedent or deny the consequent'. It makes no sort of difference in the application of this formula whether cause is antecedent and effect consequent or *vice versa.*

Thus the reason why the law of Causation has been stated in the form 'The same cause always has the same effect' rather than in the complementary form 'The same effect always has the same cause' is that popular philosophy tends to start from the event which comes first in time, as logical antecedent, because the primary source of knowledge is simply to observe processes in time: and so the further determination of any datum or circumstance *when effected by this elementary method* corresponds to the succession of events in time, and that event which comes first is taken as the datum to be further determined, and that which comes after is regarded as its determination. Of course then the same datum always has the same determination, for every content—and a datum is a content—is an identity, and having attended to an identity in respect of one of its differences we are quite safe in saying that this identity—this datum—will always have the same difference. For if it seems not to have, we may say either that the difference is disguised, or that the datum is not the same. And so we come first to the principle that the same cause always has the same effect; and sometimes, to make quite clear that we are simply regarding a real content in respect of a difference which we have selected out of its concrete nature, we add 'the same cause under the same conditions', or 'in the same relation'; thereby showing that we know very well that the concrete cause has all sorts of different effects under different conditions and in different relations.

It is usually presupposing the truth of the first principle that we go on to consider whether the same effect always has the same cause; and neglecting in the effect, which we take at first as a goal in which thought can rest, the idea of a limiting reference to a particular antecedent, we are impressed by the variety of relations and conditions compatible with the undetermined result, as contrasted with the single aspect in which we watch the operation of a cause; and we forget that each set of these generates the effect in a slightly different form. It seems so common-sense to say 'If a man is drowned he is dead, but if he is dead he need not therefore have been drowned', that we forget that, if he is dead *in the particular way produced by drowning*, then he has been drowned. We might, from the very first consideration of an effect, draw a parallel to the popular form of the Law of Causation, 'Same cause in same relation, same effect,' by a Law of Effect which should affirm, 'Same effect, in same form, same cause.' But, as our first impression in starting from Cause is of the Identity of Effects, so our first impression in starting from Effect, because there is no simple guide to further determination, is of the Plurality of Causes. Really however we have to supplement these ideas by those of the Plurality of Effects and of the Identity of Causes. It is, technically speaking, an accident which of these four points of view attracts our attention first. The knowledge that the same effect has the same Cause is not necessarily later than or dependent on that of the converse maxim. 'If a man is dead his heart has stopped' does not involve a knowledge whether stoppage of the heart must always cause death. Still, as we saw, the common law of Causation is most readily suggested by our experience of simple observation, and has a certain real preeminence because of this experience. Experimentally we only follow up Cause into Effect, not Effect into Cause. And thus the natural tendency is to identify Cause with Antecedent, and the common law of Causation 'Same cause has same effect' is the resulting one-sided application to Cause and Effect of the commonplace rules of the Hypothetical Judgment. Of course, when we have both principles together we have what is truer than either alone; but in itself neither *prima facie* involves the other.

We have seen, then, that even the incomplete or partially known Cause can always enter into a Hypothetical Judgment either as Ground or as Consequent. In the same way it is possible for Effect to be either Ground or Consequent in Hypothetical Judgment. But Effect can never be Cause, unless we go back to the doctrine of *complete* Ground in which the boundary between Cause and Effect really melts away. Effect can never be Cause, and yet Effect may be as inevitable, as essential to the sequence, as necessary a Ground of hypothetical nexus as ever Cause could be. It is a well-known saying that we cannot conceive a storm to have been less violent than it actually was without the difference involving differences in a series of physical processes going back *ad infinitum* in the causal nexus. Yet we cannot bring ourselves to treat the storm as the cause of the previous physical processes which, as we say, resulted in it. The distinction which is at the root of our inability to do so is of course the distinction of Time. The operation of this distinction has never been more trenchantly stated than by Aristotle,[1] who lays down the general doctrine of Ground with perfect clearness, but in going on to deal with causation in succession doubts the security of all arguments from cause to a subsequent effect. For 'in the moment between the two, it would be false to say that the second has taken place, although the first has already taken place'. It may of course be rejoined that the cause cannot have completely taken place if the effect has not begun. This rejoinder however depends on the postulated unity of the causal process and on the consequent continuity of time. If we press this point of view, it takes us back to the doctrine of complete ground, which consists in exhibiting the unity or continuity of causation regardless of succession in time. But we are anxious at this moment to do all we can in the way of elucidating the problem involved in the natural conception of causation as sequence, and therefore we will not simply fall back on this notion of complete ground. Granted that time and sequence are continuous, yet they are also discrete. There is indeed no empty μεταξὺ or interval in which we can stand and say, 'The cause is past and the effect is not begun.' But unquestionably we can make a stand at any point in the continuous sequence and say, 'So much is (or "has been") real, the rest is not yet real.' And what is not yet real cannot be the cause of what is or has been real.

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[1] Anal. Post. ii. 95 a, 30 ff.

This appears to be the root of our whole conviction about cause and effect in time. Even after the entire sequence has been realised, and when all of it is alike real or unreal, as we may choose to count the past, still the objective temporal order into which we project our experiences embodies the succession of relative reality and non-reality which attached to the order in its original constitution. We remember that this became real while that was still unrealised, and we therefore feel that however certainly they may reveal themselves as parts of a single whole, we can never hold the event which came after to be an element in the actual determination of that which went before. It would involve to our minds the absurdity of treating the existent as caused by the non-existent.

I do not mean to deny the reality of this distinction. It amounts to just what it is. Time is a condition, is *the* condition, is we may almost say the inmost nature, of our sensitive experience. The *first* operation of our intellectual synthesis is to build up an ideal objective order which, though itself not in time, yet contrasts as a more or less completed reality with the sensitive experience which is always passing into it. It is obvious that we can only construct our anticipation of reality out of its positive content so far as known to us; and its positive content so far as known to us belongs to the past. We may fill up gaps in the past out of other parts of itself, but we can get nothing out of nothing, and therefore can draw no anticipations from the future. Therefore *at any given* moment we have no choice but to say that the future is conditioned by the past, and the past not by the future; effect by cause, and cause not by effect. Cause, at any such given moment, is what we have, and effect what we have not. And further, taking the past as a representation of all that is, for it is the only positive content that we have to represent anything, we are right in saying that the past as a whole is the cause of the future as a whole. What is, is, and will act as it will act; and what we already know of it is the only source from which we can anticipate its action. But of course the past *as a mere series* of events *is* past; it has ceased to be real just as truly as the future is not yet real; the relation between two nothings is nothing, and cannot cause anything. The same applies to particular events; it is hard to find words which describe the negative relation of effect to cause, and which do not apply equally to that of cause to effect. Effect cannot be the cause of its cause, for the reason that it is absurd to find the cause of something existent in what does not (yet) exist. But is it less absurd to find the cause of what is now entering upon existence in what does not (any longer) exist? Yet this is what we do if we take cause as an event and effect as a subsequent event. Hence we are driven to the *second* operation of our intellectual synthesis, which is, after erecting an objective temporal order not itself in time, to strip this temporal order of the importance attaching to its successiveness, and to treat it more and more as the expression of a plan or unity. *Except* as the expression of such a unity, causation, as we have seen, disappears; but *as* the expression of such a unity, the causal relation ceases to be in time, because the positive connection between cause and effect being made manifest, the two are united in the complete ground. This must be carefully distinguished from saying that time may be introduced anyhow without making a difference; it does not mean that eggs boil water, or that death produces a revolver-shot. It simply means that the order of succession, which has a largely negative aspect, disappears in the significance of a positive systematic connection, and that we do not in fact, in considering a past sequence, regard what came later in time as less fundamental or elucidatory than what came before.

Then is not Time real? I answer that everything is real, so long as we do not take it for what it is not. Time is real as a condition of the experience of sensitive subjects, but it is not a form which profoundly exhibits the unity of things. And when we transfer the true judgment 'What has not yet happened must be a manifestation of the same unity which is involved in what I already positively know' to a totality which is already in all its parts equally real, we confuse, and give ' time ' a reality which it has not. Such a confusion is involved in the idea that there can be no more in the effect than there was in the temporal cause, and in the tremendous power consequently exercised by historical analysis over common minds. The confusion is reinforced by another aspect of causation. Practice, like sensitive experience, is in time. In translating a plan into practice, the relations of succession hold good. A sequence is what it is, and nothing else ; and a reversed sequence would simply be different. Therefore for practice the earlier event is more important, in the sense in which the means is more important than the end. For knowledge of the end does not give power to produce, but knowledge of the means does. But this importance begins and ends with practice, and even there it only exists in virtue of the unity whose nature is expressed *for us* in the fabric of ideal reality—a fabric which is not in time.

Thus it is easy to see the relation between Cause and Ground in the imperfect stage in which they are distinguishable conceptions. Ground is a content which is perceived, by reason of any systematic relation whatever, to involve the determination of another content. Cause is also a Content perceived to involve the determination of another content, and is therefore a kind of Ground, but is primarily confined to the special case in which the determining content is real and the determined content unreal. But as this is merely a negative relation, even the first presumption of causation in some degree supplements it by the postulate of a positive nexus, and we know very well that in practice if no positive nexus, no continued identity of process, can be alleged, we do not allege causation. A familiar illustration is the sequence of day and night. It is generally urged that if causation were mere succession, day must be the cause of night, but that really day is not the cause of night, because both are effects of a common cause, and either might very well go on without the other. This is one of those trivial examples that seem hardly worth arguing about, and yet, if argued about, must be treated at length. I have shown that I at least think

'Causation = mere constant succession' plainly false and in contradiction with facts; but if I must discuss this case I should like to have the terms defined. The question, as I take it, is, Does 'day' in the sense in which it is here used include a unity, a system, a principle, which is continuous in and responsible for night? If so, day is the cause of night; if not, not. The reason why we think it wrong to call day the cause of night is not because night has the same claim to be called the cause of day. There is no contradiction here. In our ordinary way of treating imperfect causation there is no reason why the daylight hours of Monday should not be the cause of Monday night, Monday night of Tuesday, Tuesday of Tuesday night, and so on. The difficulty does not lie in the sequence being of this alternate nature—a single oscillation of a pendulum is certainly the cause, though not the complete ground, of the next—but *that the persistent unity which lies at the root of both phenomena does not fall within the natural definition of either.* If day meant not merely the presence of light on the earth's surface, but, in relation to any given point on the earth's surface, that portion of the earth's rotation which carried that point from its sunrise to its sunset, then I do not see how it could be denied that this portion of rotation, in as far as it determined the position of the selected point throughout the immediately succeeding section of the rotation, was the cause or principal condition of the ensuing night. But of course the name 'day' is applied e. g. to the six months' day of the poles only by metaphor, being a chronological idea which has become largely independent of the relation of a particular place to the sun's illumination, and having legal and social meanings which do not admit of an antithesis with night. So the true reason why we do not like to predicate causation of this sequence is simply that, owing to their varied accessory significations, the terms day and night do not apply to successive stages of a continuous natural process, but are mere chance distinctions that are drawn, according to our shifting purposes, on the surface of that process.

But though we do not allege causation where we cannot allege a positive nexus, yet, as I pointed out above, there is a considerable distinction of degree between the objective temporal order and the intelligible unity of things. The less we advance beyond the stage of perception and narrative to that of science and intelligence, the more does the negative distinction of time retain its significance. Strictly speaking, the distinction between cause and effect in time is only real at an arbitrary moment in which we draw an ideal line across the temporal process of sensitive experience, between the real and the unreal. When cause and effect are both absorbed in the past, the distinction is only transferred by memory into the content of reality, which thus takes the form of the objective temporal order. This order would be an intolerable chaos but for a certain presumption of causation, i.e. of unity, which binds it together according to some sort of system; there is no real history apart from the idea of causation. Nevertheless this unity remains for the most part inchoate, i.e. only in some degree explicit; and so, though not itself in time, presents the scheme of a *de facto* evolution in time as a sort of extended memory, with the transferred character of determination of unreal by real, i.e. of effect by cause. Now it is plain from what has been said that the distinction of Cause and Effect is self-destructive. It is utterly impossible to be successful in the investigation of a causal relation without reducing it to the intelligible unity of a complete ground. History therefore, in the sense of the mere record of remembered fact, would seem to have for its ideal to disappear into systems of hypothetical judgment, in which complete ground should do duty for cause and effect, and the relation of time should disappear.

This conclusion is true, in my judgment, in relation to the mere phenomena of the past, and the resulting connection between causation and ground. But as regards what we really mean by history, such a conclusion is repugnant to our feelings and inconceivable to our understanding. The reason is plain. History is not merely a name for the recorded past. A series of astronomical observations is not history; it is science, and has no value but for science, unless by chance it throws light on the observer's character or on the state of science in his time as an element in the condition of man. What we mean by History is the revelation of man's nature in action and intelligence. And when we deprecate the reduction of history to a system of hypothetical judgments founded in some single abstract individuality—to a science like abstract mechanics or abstract economics—what we really mean is that man's nature reveals itself in individuals, in actions, in forms of intelligence, and we do not want to lose these realities in abstractions of relativity and necessity. But if we consider that hypothetical necessary or relative judgment is entirely based upon categorical judgments, that all nexus is within an individuality, we shall see that history may be received into the intelligible unity of knowledge without sacrificing its concrete import and characteristic significance. This could only be destroyed if we insisted on predetermining within *what* whole or system we should find the facts of history to be necessarily related.

And no doubt a suspicion of some such prejudice is operative in the reluctance to absorb history in 'science' to which I have adverted. If science meant exclusively the sciences which grow out of the one-sided forms of measurement, then we should rightly deny that there is a science of history, and, for the same reasons, that there is a science of art, of political form, or of religion. We escape however from such suspicions if we remember that all connection is based on fact, and all analysis on individuality; and that the nature of the facts or of the individual whole or system with which a science deals can *ex hypothesi* be only for that science itself to determine.

Thus the conception of Cause as an event in time anterior to effect gives way on analysis, and forces us back to the conception of the complete Ground; and the conception of incomplete Ground (*causa cognoscendi*) as distinct from Cause, expands into the same unity, which, as we saw, is at once the complete Cause and the real Ground; i. e. the relation of part to part within an actual and systematic totality. This relation of part to part, either burdened with irrelevancy as in the ordinary hypothetical judgment, or pure and relevant as in the hypothetical judgment whose terms are reciprocal, forms the content of the abstract universal judgment. And this abstract judgment is a divergence from the concrete evolution of thought, and joins with the mediate and quasi-generic judgments of the sciences which arise out of one-sided measurement.[1] But it may also be regarded as an element in or aspect of the popular or transitional quasi-collective and the generic judgments which are enumerative or individual in form but analogical in meaning. Analogy, as we pointed out, is compatible with systematic necessary relation.

On the other hand, though a complementary aspect of the universal judgment, the pure hypothetical is destitute of finality, and incapable of standing alone. It demands a reversion to concrete thought by the fact that it presupposes a self-existent whole. Apart from such reversion it may become a wholly arbitrary and meaningless play of fancy, presupposing conditions which are not made explicit.

As I have not yet dealt with negation, I shall leave the negative forms of the hypothetical judgment to be dealt with in the same chapter as the disjunctive judgment, with which they are closely connected.

Supposition in universal judgment.

c. The above appears to me to be a fair account of the hypothetical or abstract universal judgment considered from a strictly logical point of view. In one important respect however it is *prima facie* at variance with conclusions which might be drawn from the grammatical shape of these predications as to the attitude of mind in which they are normally made. The point in doubt is the existential significance of the universal judgment. The account which I have given treats the existential implication which attaches—undoubtedly in very various degrees—to the different forms of the universal judgment, as cognate with the existential affirmation involved in the singular and in the perceptive judgments. But it has been trenchantly laid down by Mr. Bradley that a different view is suggested by the attitude of the mind in all purely abstract judging—abstract be it remembered not necessarily in content, but only as all thought is abstract when contrasted with sense-perception. I am unable to reconcile this view with the existential value of judgments about individuals designated by proper names, in which there is no direct reference to sense-perception nor to anything but a content, whose real existence is as I imagine taken to be asserted owing to its concrete nature. But I proceed at once to discuss the analysis of the abstract universal affirmation, from the point of view to which I allude.

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[1] See Scheme, p. 86.

It will be remembered that in the general discussion of the nature of judgment, we agreed that the ultimate subject in judgment was never an idea,—never that is to say even a logical idea or content, for the particular psychical image we found not to concern us in logic. The ultimate subject in judging was always, we held, the Real, which in the act of judgment is qualified by certain logical ideas. So long as the immediate subject was present perception, whether additionally qualified by significant ideas or not, all went smoothly, for the immediate subject was then simply the point of contact with the ultimate subject of judgment. Some difficulty, indeed, arose in explaining the real reference of the Singular judgment in which the subject may fall outside present perception and may have to be united therewith by a constructive process. Still, however, the immediate subject was a determinate element in the whole of Reality, having individual existence manifested within the sensible series, although e. g. in the case of an organism, still more in the case of a man, not of a kind that could really be presented by sense-perception as such. So far therefore we were, in judging, referring an ideal content to reality in some particular concrete aspect, and therefore our Judgment was still plainly existential, i. e. such as to become false if the concrete element of reality described had no place in the series of sensible events.

Now when we come to deal with the Universal Judgment it must be admitted that *at least* as a question of form the reference to reality becomes less easy to define. The pseudo-collective and the analogical judgment at any rate (dismissing the conception of the individual generic judgment, in which the singular judgment revives) are unquestionably capable of an interpretation which reduces them to the pure Hypothetical Judgment. In our old acquaintance 'All men are mortal' the 'all' is too obviously *not* collective to stand in the way for a moment. We certainly might be driven to confess that in so judging we had only asserted 'If man, then mortal' or 'Where man, there mortal.' Such an interpretation is involved no less in Mill's analysis of the import of propositions than in Lotze's or Bradley's treatment of the universal judgment. To affirm co-existence of attributes is not to affirm existence of subjects.[1] The analogical judgment has this aspect even more plainly. 'An organism as such is mortal' means, it may be urged, if taken strictly and without counting implications, 'If organism, then mortal,' 'Where organism, there mortal,' 'Whatever is organic is mortal.' If in consequence of such an assertion we take it that organisms are actual elements of the real world, this is implication—though very strong implication—and not assertion.

According to this analysis, the essence of which is to regard the implication of existence in these judgments as outside the matter affirmed, all abstract affirmation—abstract merely in the sense of not referring to present perception or to particular sensible events—is on the level of hypothesis, has for its immediate subject an idea not a reality, and consequently has no existential import, or 'deals purely with adjectivals.'[2]

The identification of the universal judgment as such with affirmation based on hypothesis being thus made, the further development of the view turns on the nature of hypothesis or supposition. The essence of supposition is that it is *qua* supposition, wholly arbitrary in its starting-point. Its content is taken, not given, is an idea, not a fact (if *per accidens* a fact, is not used as a fact, i. e. its existence is not argued from), and is considered not in itself, but for the sake of its relativity, i. e. for what flows from it, for its consequences. The essence of supposition is in short argument from content, and not from existence of content. The consideration of any proposed legislation, e. g. a Reform or Land Act, with reference to its consequences, is an example of supposition. 'Suppose every adult male to have a vote, it will be impossible to maintain indirect taxation;' 'Suppose Ireland to have a Statutory Parliament, the Imperial Parliament will by this fact itself become statutory.' Or again, 'Suppose beings endowed with perception but confined to a plane in the exercise of it, they must see all figures as lines or points.'

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[1] This comes out very emphatically in Mill's account of Definition, which when Real at all, he analyses into a meaning *and* a postulate of existence. In this he is pretty much at one with Mr. Bradley's account of universal judgment. See also Mill's account of mathematical truth.

[2] Bradley's Principles of Logic, p. 81.

The process is to select or to fabricate, apparently at pleasure, an ideal content, to think of it as in connection with some known reality, and to judge the result as a truth conditional on such connection.

It is plain that in this operation, subject to a certain reservation to be mentioned directly, the supposition selection or fabrication[1] of content is arbitrary, but the judgment proper is necessary.

What then, precisely, it has further been asked, is in such a case affirmed about ultimate Reality? Plainly, *not* the existence of the content as a fact in the context of our world. We may take Sigwart's instance, reproduced by Bradley. 'Si tacuisses, philosophus mansisses.' Something is here affirmed, but not the actuality of the content, which is by the strongest implication denied. Or again, 'All trespassers will be prosecuted.' The truth of this declaration does not depend on there being trespassers, though it cannot be tested unless there are trespassers.

When we have once accepted this point of view, and excluded as unessential the various implications of existence which attach to various Universal Judgments, the conclusion is inevitable. Judgments so regarded do not affirm as true of Reality any explicit content or even any connection of explicit content. We have seen that it is no impeachment of the judgment that its content never has been nor can be actual. 'The necessary may be impossible or non-existent.[2] And as for the *connection* of content, though it is necessary, yet it cannot be actual unless the content to be connected is actual, and the judgment may be true though the content be incapable of actuality. Therefore the truth of the judgment, according to this extreme analysis, depends neither on the actuality of its content, nor on the actuality of the connection it alleges within its content. What then does such a judgment assert of that Real which is the ultimate subject in every judgment ? Simply this, that the Real *is such* that under the ideal condition which forms the immediate subject of the judgment it will furnish the ideal consequent which is expressed in its predication. The hypothetical judgment would then be *illustrative* but not *enunciative* of Reality. The property of Reality which it illustrates may however be accessible to knowledge ; or again, according to the view before us, it may not. Simple examples of the former case form the best explanation of the conception which we are discussing. 'If you ask him for money he will refuse you.' The real quality of the real man, on which this prediction rests, may be that he is a miser ; then his miserliness *x* is the real fact, *not* apparent in the judgment, which the supposition and consequent *a* and *b* only illustrate by its effect in an ideal case ; and which is categorically affirmed, but only as an unknown *x*, in the Judgment. And it is conceivable that the property of Reality which lies at the root of the judgment may not be known, and wherever the connection envisaged as hypothetical is considered to be ultimate and not susceptible of further explanation, the property of Reality at its root *is* pro tanto unknown. Such a connection would be, I suppose, 'Whatever is material, has gravity.' The property *x* of reality which is here categorically asserted as the basis of this connection, 'If *a* then *b*,' is, I presume, and is likely to remain, unknown.

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[1] Selection and fabrication differ only in degree, not merely because all fabrication is selection, but also because all selection is fabrication—involves the constitution of an idea from a given or chosen point of view.

[2] Bradley's Principles of Logic, p. 186.

The real criticism which I have to offer upon this view is contained in the whole account of judgment which has been submitted to the reader. Its point and purpose have been to exhibit the aspects of fact, and of relativity or necessity, within the judging function as complementary and inseparable, but as differently predominant in dealing with different contents. Consequently, the abstractness on which their respective predominance has hinged, has been treated as the abstractness of contents, and not as that formal abstractness which is merely the mark of thought as opposed to sensuous perception. And the result has been to exhibit the graduated existential implication of universal judgment as falling within and not without the import of those judgments, and as homologous with the aspect of existential affirmation in perception and in historical narrative. But I propose to comment very briefly on two special problems raised by the view before us.

Simple and conditional Judgments.

(1) It is easy for any one to form for himself a *catena* of universal judgments, beginning where the proper name becomes significant, as with Europeans, Christians, Peelites, and passing on through the generic conceptions of classificatory sciences, to physical and mechanical principles, geometrical axioms or theorems, and finally to imaginary and impossible but self-coherent hypotheses, like that embodied in the ingenious *jeu d'esprit* entitled 'Flatland'. Such a *catena* is at the root of the view of Universal Judgment which I have endeavoured to formulate. Any one who will take the trouble to follow up and to fill out with instances familiar to himself the terms of such a series, will hardly be able, I think, to avoid forming the conviction that no single type of proposition is adequate in the same degree to the content of all universal affirmations. If, however, we are compelled to choose, the conditional proposition is the more adequate. But it is not adequate. Such a judgment as 'All Christians hold that God is a Spirit' combines collective and generic meaning with necessity. It indicates not merely that the doctrine is of the essence of Christianity, i. e. that 'If a man is a Christian he holds' &c., but that there is *a* Christian world, realised in *many* individuals, which is united in this conviction.

And on the other hand, after our too laborious consideration of the subject, it is not necessary to remind the reader that relativity is operative within the judgment from the moment of the first analysis introduced by perception into the data of sense; that is to say, even when the judgment assumes the external form of the impersonal proposition, which indicates that identifiable subjects are not yet constituted in virtue of determinate qualifications.

The fact then is this. Speech can express no logical relation except by making it the correlative of a word or clause. But the common types of speech, which have been made the basis of logical investigation, are direct and simple. They therefore embody only one aspect of the concrete logical thought, and leave all others to be guessed at from variety of context and the requirements of content. The process of recognising explicit linguistic correlatives of relations which in these direct forms are only implicit is a slow process—Aristotle did not recognise the conditional proposition on its merits—and begins, as is usual in such cases, by substituting one-sided abstraction for unanalysed confusion. Therefore by adopting either the direct or the conditional mode of expression we ignore in the first case, and formally exclude in the second, elements without which it is impossible for judgment to exist. The direct or 'categorical' form is used till it breaks down under the burden of an import for which it was not invented; and the conditional form then takes its place, to express the relative import, the determinate and restricted reference from part to part within a whole, which now insists on making itself predominant. But both forms, not one only, are inadequate to their content. The content of categorical assertion has relativity, and that of hypothetical assertion has absoluteness. But categorical assertion (I am speaking of grammatical form) leaves the former, and hypothetical assertion leaves the latter to be implied. The two forms however have not an equal right to the ground they commonly claim. The first comer has, naturally, occupied all it could get, and more than it could adequately deal with. It is against this encroachment of the categorical judgment-form that modern logicians have rightly espoused the cause of the hypothetical. The true frontier is, beyond a doubt, where the singular judgment ends. After that point, if we dismiss the Individual Universal and omit to consider the Disjunctive, the purpose of assertion is relativity or sequence, and absoluteness or existence is only its presupposition. Formal Logic granted this territory, that of the pseudo-collective and the analogical judgment, to the Categorical *Judgment*, simply because it found the Categorical or direct *Proposition* in possession.

And the further analysis of the irrelevant connections which encumber every perceptive and singular judgment, and which, if understood to be *generally* affirmed, immediately become false, may be regarded as a reprisal on behalf of the hypothetical judgment, which, now that attention has been drawn to the meaning made explicit in the conditional sentence, threatens to dislodge the categorical judgment even from the fields of perception and of history. The degree in which such a claim should be conceded has been indicated above.[a]

Thus the simple and the conditional[1] propositions are, the one of them an indeterminate and the other a one-sided type—both therefore imperfect expressions of thought. The latter is borrowed no doubt from, or is at any rate most appropriate to, the extreme and arbitrary attitude of mind known as supposition. It is natural, as we have seen, that the empire of the first comer should not be broken into unless a forcible demand arose for something utterly incompatible with its type. The representation which is correlative to supposition is thus the sole representation in ordinary speech of the aspect of necessary sequence within the judgment. This is why, when logic awakens to this aspect, it is tempted to find its essence in supposition.

But supposition is not the essence. Supposition is intentional abstraction or selection culminating in fabrication; and the essence of the judgment is not in the fact of intention, but in the logical ground and justification of the intention. The interference of the will is no *differentia* in a logical process. All thinking presupposes will, but the *guide* of thought can never be moral purpose, the purpose of will, but must always be logical necessity. What remains then out of the fact of supposition is that a content is taken, chosen, fabricated, if we will, with a view to its relative aspect, to its consequences. But what is presupposed in so taking it, and what are the conditions under which alone it can be so taken—whether, that is, it can be taken in the air and without either self-relation or relation to an actual system, are questions in no way touched by the mind's attitude in supposition. We shall see that speech finds the embodiment of a necessity resting on fact in the proposition which expresses the disjunctive judgment.

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[a] See p. 261 above, and note Mr. Bradley's later statement, 'Thus we really always have asserted subject to and at the mercy of the unknown. And hence our judgment, always but to a varying extent, must in the end be called conditional.' To the word 'unknown' he adds a footnote, beginning 'Hence in the end we must be held to have asserted the unknown. It is better, however, *not* to call this the predication of an unknown quality (Principles of Logic, p. 87), since quality either adds nothing, or else adds what is false'—Appearance and Reality, ed. 2, 362. The following discussion of supposition and the hypothetical judgment is not relevant to and does not deny Mr. Bradley's general position as formulated above. I retain it mainly for its treatment of illegitimate hypothesis, which seems to me an interesting point.

[1] The grammatical difference between the two is connected with their logical import in that the conditional sentence provides for an express analysis *ad hoc* of the subject-qualification of Reality, whereas the direct sentence simply indicates a subject-content by a name. If we insist on the name being the *right* subject name in the context, as in Aristotle's *καθόλου*, we have an intermediate stage.

The basis of Hypothetical Judgment.

(2) The character of Reality on which the sequence 'if *a* then *b*' depends must be in the ultimate analysis of judgment admitted to be unknown.[a] This follows from the ultimate reduction of all judgment whatever to judgment which is conditional. But if this were so in the case of the hypothetical judgment as commonly understood, the back of the sequence would be broken. The *ground* would be absent. Every hypothetical judgment is affirmed only within an actual system. What then are we to do with our ultimate sequence, e.g. with the nexus between resistance and gravity? I cannot but maintain that, *if* we have no explicit ground to go upon, we must either surrender the sequence altogether, or affirm it categorically, i.e. not *as* a sequence, but as a datum; not as a coherence, but as a conjunction. 'All matter (i.e. all that resists) gravitates' is no doubt a judgment in which we look for necessity. But it does not follow that we find it. It presupposes the judgment 'There is gravitating matter'. It is not adequately rendered by 'If or wherever there is matter it gravitates'. We are unable (or at least I am unable, which is all that my illustration requires) to assign any system which acts as ground and compels the sequence to be true within it. The world of

[a] See note, previous page.

matter given as resisting and gravitating, as a systematic fact, *is* the ground. And therefore it *is* on the other hand a true hypothetical judgment to say, 'If a material object is set free within range of a gravitating centre, it falls with a velocity accelerating as the squares of the times.' For the content of this judgment is *within* the system of gravitating bodies, and the sequence is compelled by that system, whose existence as a fact is required by the judgment, but not in this form explicitly asserted. In this case the unity of gravitating matter is the x on which the sequence a–b is founded.

May not the whole system be supposed, ground and all? Certainly not; and this is the fundamental point at stake. Every judgment is a qualification of reality by some ideal content, and when the basis of a sequence is the content by which reality is qualified, that basis is by the judgment affirmed to be actual. So far Mr. Bradley and I are together; my further contention is that this ground or basis must be known, and must be indicated in the judgment, of which it forms the essence. The degrees in which it is so indicated form the various complete and incomplete grounds which were discussed above. But an ultimate ground must be actual; it is the fact which is judged in a hypothetical judgment. We may of course freely suppose or imagine *a* system, as complex as we please; but if we proceed to *judge* about the consequences or results of such a system, it must thus be related to these consequences within some further system; and this further system must be actual. In other words, you can only suppose an antecedent, you cannot suppose a consequent; the consequent must be judged, not supposed; and in judging the consequent you assert the underlying ground to be actual.

This may be illustrated by the extreme cases in which we refuse to entertain a supposition. This means that we are not aware of any reality which furnishes a system such that the supposed case is capable of entering into it. When supposition begins to infect the nature of the reality, we are beginning to suppose and not to judge our sequence. It is quite doubtful whether the conclusions of 'Flatland' can be taken as true even *qua* hypothetical judgments. When your supposition has knocked the keystone out of actual reality, how is it to support a conclusion ?

The application of this conception would I think solve the curious cases in which a sequence is true, though it is possible for the content of supposition not to exist, or even impossible for it to exist. What must exist is a system that, subject to the supposition, necessitates the consequence drawn from the supposition. Whether the content itself exists or not depends upon whether it is an element essential to the system ; and how it exists, on the nature and selfcompleteness of the system. The former condition meets the case of impossible contents, which are in every case illegitimate suppositions—i. e. suppositions in which the consequent has to be supposed and not genuinely judged, because the supposed antecedent conflicts with the nature of the real system on the basis of which alone any conclusion about it can be drawn.

I will take some examples. 'Given a first cause, we can dispense with the idea of a regress to infinity.' But Cause, in any sense which it could be First, i. e. in temporal relation, means an element in a system of relativity. Therefore the idea of a first cause contradicts the whole actual system to which the idea of Cause refers, and it is utterly impossible to affirm anything about an idea which begins by destroying its own basis of affirmation—the causal system. Or again : 'If one man were throughout the whole period of his conscious life alone in the universe, his moral purpose could be nothing but to please himself.' Here we are judging on the basis of an existing moral world—for it is only this that gives a meaning to a judgment relating to moral purpose—but we are putting a case which contradicts the nature of man as a being existent in a moral world. I do not think that in this case any judgment can reasonably be made. But the purposes of supposition in argument are so various, for its object may be in different degrees to emphasise the impossibility of the content supposed, that the limits of legitimate supposal are exceedingly hard to define. Undoubtedly its use is one of the most fallacious if one of the most effective means of controversy. 'If A. B. were to turn coward'—'But he could not'—'But I am only putting a case'—'But if you put such a case I may put any consequence I choose as equally likely,' i.e. it is felt that the real basis on which judgment rests is annihilated.

Some real basis can never be dispensed with in judgment. The nearest approach to dispensing with it is made when elements of reality which would conflict with the suggested case are wilfully kept out of account by an act of abstraction; which act of abstraction may be either borne in mind, or forgotten. If the act of abstraction is borne in mind, we obtain such judgments as those in which mathematical science deals with imaginary quantities.[1] Thus judgments are subject to the reservation implied in the abstraction from reality which enables them to be made. Yet, in as far as they are judged at all, they must rest upon, and involve the affirmation of, properties of reality. If the abstraction is forgotten, we then obtain such judgments as apply imaginary conceptions without reservation to the real world; 'A conjuror can tie a knot in a string whose ends are held, because he understands the properties of four-dimensional space.'

I may further illustrate this last case by the example of artistic fiction, which I have discussed at length elsewhere. It consists of judgments both singular and universal,[2] made on the basis of human nature, but subject to a reservation which separates them from the world of past sensible events. While actually under the spell of romance or of the drama, we forget or half forget this reservation; but we do not and cannot forget the true and ultimate basis of the judgments, fundamental human nature, which is the ground and substance of the whole matter.

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[1] See p. 165 ff. on mathematical infinity.

[2] It may be doubted how far the universal judgments in a fiction ought to partake of the character of fiction. It depends on their grade of universality. Judgments of a reflective order, about human nature for instance, if not dramatic, are expected to be true without reserve. Judgments about parties, nations, &c., may be fictitious. Dramatic sayings are yet more complicated; they are not judgments of the author at all; the author's (fictitious) judgment is that they were uttered. Then their value or merit is compounded of their truth as estimated by the limited reality of the drama in which they occur (i. e. their appropriateness) and of their truth as estimated by real reality, i. e. their weight or depth, which of course involves the whole relation of the piece to real reality. See Knowledge and Reality, p. 140 ff.

But we raised a question not only whether the content exists, but *how* the content exists. This depends, we said, on the nature and self-completeness of the system within which it exists. Many universal judgments deal with sensible events, which are not, within our knowledge, contained in any really concrete whole in time and space. For these, according to the principle which we have followed throughout, the abstract universal or hypothetical judgment postulates such existence as they can have, according to their nature and that of the incomplete system to which they belong. There is nothing which cannot ultimately be taken up into some individuality by constructive thought. But as in ordinary judgment no such actual construction is operative, we have to substitute for it the mere assertion of the basis to which the sensible occurrence in question is known to have some relation. The imperfect explicitness of this relation is the note of distinction between the hypothetical and the disjunctive judgment. 'Pure red is ethereal undulation impinging on a normal eye with *x* vibrations to the second.' Here the ground or basis of affirmation is the existence of light, which is ultimately dependent on the existence of sentient organisms in a certain relation to the material world. Now this relation, though not known nor explicit in the judgment, must be taken as knowable and real. Space and Time indeed produce the illusion of endlessness; but no special positive content like sentience or light is involved in this illusion; we rather assume every positive content to have its own time, place and conditions within our actual individual system. No doubt we may be asked, Does the above hypothetical judgment assert the existence of red, or not? And can we claim to assert the reality of what is *for us* an indeterminate endless series which as a whole cannot be real? To these questions I should propose to reply: 'The hypothetical judgment in question asserts the existence of light as the categorical basis of the nexus which it selects. The existence of red light is involved in the existence of normal light-stimuli and normal eyes. Therefore the judgment in question asserts the existence of red light as a feature of the reality constructed by and for us, and subject to the reservations which its position in that reality imposes on it. We do not claim to assert the reality of an endless series of sensations as such; but in the first place a positive series as referred to an actual system is already placed beyond theoretical endlessness, and in the second place it is not as an occurrence in the way of sensation that we assert its reality, but as an attribute of things in the whole of consciousness, which, as constructed out of perception, is *for us* the only reality.' This is in effect the answer which was given above to the question whether the conceptions of geometry were judged to be real; and it follows inevitably from the considerations to which we were led in treating of the doctrine of Ground. We shall see that the real Ground, when made explicit, takes us into the province of the disjunctive judgment. The element of categorical assertion in the hypothetical judgment, consisting as it does in some underlying real attribute, also presents a close analogy to the positive basis of negation, as will appear in the following chapter.

CHAPTER VII

NEGATION, OPPOSITION, AND CONVERSION

1. THE Negative Judgment presents at first sight a paradoxical aspect. We are bound to take it, *qua* judgment, as playing some part in knowledge, and as at any rate capable of contributing some factor to the ideal fabric of reality. But it assumes the external shape of ignorance, or at least of failure, and the paradox consists in this—that in negation the work of positive knowledge appears to be performed by ignorance. The contradiction arises, as we have seen other contradictions arise, from the adoption by thought of a shape which at best expresses it but partially, and the retention of that shape when the aspect which it did express has come to be dwarfed by other aspects of knowledge. But of course the shape could neither be adopted nor retained did it not in some prominent aspect coincide with the requirements even of developed thought. Here then, as elsewhere, the key to our problem must be looked for in the conception of the individual mind working out its participation in reality by help of forms never wholly alien to this aim, but profoundly transmuted in proportion as it is attained.

Negation and Affirmation.

Negation is at first sight merely negative. It appears to say nothing, but only to deny, i.e. to put away some ideal content as other than reality or to express our inability to recognise it as belonging to reality. The first step then towards ascertaining its import is to ask, *what* does it deny or pronounce unreal? what does it presuppose to be present before denial is possible?

It certainly does not presuppose an affirmation. Both fact and theory protest against such a view. We have not always judged a matter to be true before we deny it. And if an affirmation[1] of the same content is to subsist as a condition of the negation, it seems doubtful whether a negation would not always have to be self-contradictory. Although Sigwart's account of double negation suggests this view, yet he says explicitly[1] that it is an *attempted* judgment ('Ein versuchtes — Urtheil') against which negation is directed. And this seems so far to agree with experience. What is the nature of this attempted affirmation? 'Suggestion' and 'question' come into the mind as possible equivalents for it. It is difficult however to find a special significance for either the one or the other on purely logical ground. A question is closely related to a command, and has its differentia in being addressed to the will of another person. It is impossible in good faith to ask a question of oneself. The power of the metaphor by which men are said to question themselves rests on treating oneself as another person. A question is a demand to the will to reveal something known to the person whose will is appealed to; but if I know the answer, I need not ask myself; if I do not, I cannot ask myself.

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[1] Sigwart distinguishes 'affirmation', as the conscious opposite of negation, from 'Positive' judgment, as assertion without consciousness

A question then as such has not a logical differentia, and cannot be the logical presupposition of negative judgment. Still it may contain what we want. It is not merely an ideal content floating before the imagination, even if we were to grant that there are such floating ideas. It is an idea in some way tested by Reality. A suggestion seems to be the same. It is not a floating content; it is suggested *as* something, as, so to speak, a candidate for a place in a judgment already framed. That is to say, a question or suggestion as it is on logical ground, omitting any demand upon or incitement to another will, amounts at least to an idea whose content is Reality qualified in a certain way. Is this all? This would not suffice to explain the import of suggestion followed by affirmation or denial. It would not explain the significance of the decision even when negative. There is something more, and it is this. The content of the suggestion

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of possible negation. We shall see in the sequel the value of this distinction, which can only be taken as one of degree.

[1] Logik, vol. i, p. 150, and 194 (with assent to Bradley's Logic, p. 149 ff.). E. Trans. 119 and 149.

is taken within a whole in which we have an *interest* and which *is* referred to actual Reality. Every suggestion enters into a rudimentary disjunction. I mean by *rudimentary* disjunction an alternative whose limits are not made explicit. Where there is a question or suggestion, there is always a something—some general predicate—known to be true with reference to the matter before the mind, and an interest which we have in that something. It is within this something that the attempt to state it more precisely, the question or suggestion, falls. It is hazardous to obtrude analysis on the simpler stages of consciousness; but I do not see how we can have a suggestion followed by denial cheaper than this. We might even fall back on the principle which has been laid down in previous discussions, that any consciousness for which a continuous real world exists, sustains that world by a judgment. When a man first doubts and then decides, on such a question as whether the river which he sees before him is safe to ford, however simple the mental procees may be, there must be in some form a positive basis of the two or more alternatives as well as one suggested alternative. He must start with the fact that the river has depth, or current, which he must deal with in crossing, and within this fact the doubt 'too deep?' 'too swift?' has its meaning for him. In simple cases this embracing judgment of fact is hardly traceable except through the interest in the question. This interest, if looked at closely, betrays the nature of the alternative which the question involves. 'Too deep?' 'What then?' 'I cannot get home to-night,' i.e. the general fact is that the river is between me and home.

Negation then presupposes an idea *suggested* as true of Reality within a state of facts *judged to be* true of Reality and interesting to us in respect of the matter suggested; or in other words, an alternative judged to be true of Reality, but only so judged as one among a set of alternatives, and therefore, in itself alone, *problematically* judged true of reality—judged as a possibility, as one among a number of alternatives, or as subject to unexpressed conditions.

We are now able to decide the disputed question whether Affirmation is prior to Negation, or whether they are co-ordinate types of Judgment. Negation is not, as such, the denial of affirmative Judgment, and therefore does not presuppose the affirmation of that which is denied. So far, affirmation is not essentially prior to negation. On the other hand, Negation does presuppose some affirmation, that is the affirmation of a state of facts which, being judged true as a whole, carries with it the problematic affirmation, the affirmation as a possibility or enunciation as a conception in the world of meanings, of the idea 'suggested'. In this respect, therefore, affirmation is in the beginnings of knowledge, at any rate, prior [1] to Negation. The world must have positive content judged to be real as a condition of anything following from the removal of a positive suggestion. But I cannot believe that the consciousness of a positive world could in fact exist for an appreciable time without the development of negation.[2] Further, however, it is also true that in the beginnings of knowledge negation is a degree more remote from reality than is affirmation; [3] and this character of ideality clings to the negative form through its whole development, though without debarring it from the acquisition of objective value. The remoteness consists in this, that the suggestion which denial presupposes is, as we saw, not a mere floating content, but a suggested qualification of reality, in short 'a suggested affirmative relation'.[4] An affirmation can be, comparatively speaking, given as fact; a negation cannot, except in quite another sense, be given. It has to be made, and made by setting an ideal reality over against real reality and finding them incongruous. 'That fire is still burning' involves no doubt intellectual selection and is so far ideal, but 'that fire is not out' is one remove more ideal, because it has to bring up the idea of 'that fire being out' and test it by the perceived reality, and then only proceed to judge its exclusion to be a fact. We must not however exaggerate this difference. Affirmation itself, or even positive Judgment, cannot take place until the distinction between a mere idea and a fact of reality is recognised. And with this distinction the idea of negation *is* given. It might therefore be argued that Judgment, not to speak of affirmation, presupposes the idea of a negative relation;[1] just as negation presupposes that of an affirmative relation. The fact seems to be that affirmation presupposes the idea of negative relation in general, while negation presupposes *the idea of*[2] a corresponding affirmative relation in particular. This applies to the beginnings of knowledge. In complete thought we shall find the two more on a level.

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[1] In this sense it may be called, if we prefer to do so, 'positive' judgment.

[2] See above, Introduction, p. 22, on the *formal* implication of distinction in objectification.

[3] Cp. Bradley's Principles of Logic, p. 116.

[4] See Bradley, ib.

Thus it is true, especially in the beginnings of knowledge, that Affirmation is prior to Negation, both as one remove nearer to reality, and as supplying the reality within which alone Negation has a meaning. But it is no less true that Negation has from the first its essential place in knowledge; and as Reality becomes for us an articulated system, the value of negation approaches more and more nearly to that of affirmation, with which it finally becomes equivalent. This is however not to be understood in the sense that the import of negation disappears from knowledge; but in the sense that affirmation and negation alike become double-edged, each involving the other.

Bare denial and Infinite Judgment.

2. Negation then, in its primary shape, is the exclusion of a suggested qualification of reality. The bare expression of this import, reduced to its minimum, would be found in what has been called by a mis-translation of Aristotle, the infinite judgment.

The infinite judgment was a term applied by Kant (following, I presume, the tradition of formal logic) to judgments which had for their predicate what Aristotle called an 'indefinite' or 'undistinguished' name or predication;[3] i.e. such a phrase as 'not-man', 'not-good,' or the like. More important however than Aristotle's expression 'indefinite name' was his distinct verdict that such phrases were not names or predications. He gave them the title of indefinite or undistinguished names or predications, 'because they may be truly predicated of everything alike, whether existent or non-existent.'[1] It is plain that Aristotle's verdict is right, and that such names have no signification. They are 'undistinguished' because they are undistinguishing. It may therefore be observed in passing that to attempt to read all negation as affirmation of a negative name is an unmeaning device, though possibly guided by a feeling of a true ideal, viz. that negation, if it is to have a positive import, must involve an affirmative element. Only by this contrivance the affirmative element is ludicrously absent. We should be, as Mr. Bradley says, denying, and then affirming that we have denied.

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[1] Cp. Bradley, Principles of Logic, pp. 2, 110. I find some difficulty in reconciling these two passages.

[2] Not 'presupposes a corresponding affirmative relation as judged true.' That view we have rejected.

[3] ῥῆμα or ὄνομα ἀόριστον.

It is for this reason that the 'Infinite judgment' may be fairly represented by examples in which the denial, though undeniable, is unmeaning. 'Virtue is not square,' 'The soul is not red,' 'Man is not a stone.' These, *qua* negative, are fully on a level with 'A monkey is not-man,' 'A stone is not-Christian.' And so in illustrating the import of negation we may disregard the pseudo-affirmative character of the latter instances. Our interest in them is that, if strictly interpreted, they display to us the nullity of bare denial. 'Not-Christian' literally interpreted includes not only heathen humanity, but the fixed stars, the sea, and indeed, in Aristotle's words, 'everything whether existent or non-existent' except Christians. It refers to no one sphere in preference to another, and thus says nothing definite enough to be intelligible. The point being once established that negation *qua* negation has no significance, we may disregard the attempt to erect it into affirmation which draws our attention to this fact. And we may then safely take as instances of the infinite judgment, so far as its import is concerned, the judgments typified by 'Virtue is not square'. These show the true type of bare denial, for they are the only negative propositions in which usage does not irresistibly limit the sphere of the negation. And when the sphere is limited, the denial is no longer bare.

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[1] Ar. Περὶ Ἑρμ. 2. 3. Though the reading is doubtful in the application of these words to *οὐκ ὄνομα*, it is enough for our purpose that they are applied to *οὐ ῥῆμα*.

Thus it appears that bare denial, whether disguised as spurious affirmation, or taken as the mere exclusion of mere suggested predicates, amounts in the strict sense to nothing. The judgments by which it is typified are the exact counterpart of absolute tautology, and like such tautology, are not really judgments at all. Identity and difference are inseparable aspects of all that exists or can be thought; but in these two classes of would-be judgments identity and difference fall apart, and thereby the conditions of intelligible judgment are destroyed. Pure tautology aims at mere identity, and bare negation at mere difference. It will be found that any meaning which in practice we attach to an apparent tautology or an apparent bare negation is owing to the introduction of difference into the former, or of identity into the latter. 'Business is business' qualifies a certain class of affairs by the principles on which they ought to be conducted; 'The soul is not a machine' qualifies the soul, not by the mere exclusion of mechanical properties in favour perhaps of absolute nothingness, but by some positive characteristic of the soul which is incompatible with its being a machine.

Significant Denial.

3. We have seen that denial was not to be made into affirmation by the rough and ready method of the Infinite Judgment. But we started, on the other hand, from the postulate that denial, as a form of judgment, must be capable of contributing something positive to knowledge. Whether positive necessarily = affirmative is a problem that will solve itself as we come to understand the full nature of negation. I use it here simply as antithetical to 'nothing', or as an emphatic reiteration of 'something'. We must assume with Plato that knowledge is the knowledge of something; and if the nature of 'nothing', as e. g. the abstraction of empty thinking, can be known, then nothing is so far and in that sense something.

All significance then is in this sense *positive* significance, and significant negation must therefore convey something positive. What is it that it does convey? We shall find the answer if we look at that which all judgment has in common, viz. the *interest* or *bearing* of the judgment. What is it that we really mean or wish to predicate when we make an ordinary negative judgment? There is always, I may observe, something unreal in the analysis of isolated propositions. Apart from the context of a book or of a conversation, or from the precise standards which involve the fixed context of science, our interpretation of propositions into judgments is almost entirely arbitrary. On the other hand, it may be said that in a given context the 'bearing' which we ascribe to a proposition is not strictly within the limits of what the proposition enunciates, but is read by us into its meaning. It may be doubted however whether in actual living thought there is any judgment that is not an enthymeme, i. e. an argument with a suppressed premise or a suppressed conclusion. If we attempt to prune away from the judgment all the implied and suggested bearings of the proposition which conveys it, we shall find that we have whittled away the meaning which is the judgment itself. We must never forget the conclusion which we reached above, that the unity of the judgment does not exclude systematic multiplicity within it. The logical content employed in any given judgment is a many-sided although determinate idea, and is gripped and attached to its actual place in the logical mechanism now by some of its prominences and now by others. Some reservations will have to be made in this respect when we come to speak of judgments that deal with self-contained systems, a type which we have more or less anticipated in the individual and generic judgments. But in ordinary reflective judging we are constantly referred away and away along a series of grounds and consequences, and it is idle to attempt to reduce the judgment to simplicity. What we really *mean to mean* is only to be found in the explicit articulation of the whole system of fact which the interest of the moment covers; and all ordinary judgment toils after our interest in vain. But *prima facie* the positive judgment has an advantage in this respect over the negative. A positive content is at all events something; it is an instalment in satisfaction of our interest. If I say that the fire is burning in the dining-room, this judgment is no doubt compatible with various grounds and various consequences, and in the judgment as I mean it some particular ground and some particular consequence are probably included. Such a statement would not be made *à propos* of nothing, or if it were, it would be resented just as talking gibberish would be resented. There is some point or purpose to which it must be taken as contributing, and some reason—though possibly falling outside the content of the judgment—which serves as a ground for making it. But over and above all this there is, in the affirmative judgment, the positive logical content itself, which, though modifiable within very wide limits, yet cannot be modified beyond certain limits. The judgment may be in praise of some one's thoughtfulness, in condemnation of their extravagance, in contempt of their effeminacy, or in alarm at their carelessness. But there is a nucleus, not indeed fixed nor free from ideal selection and synthesis, yet not quite indefinitely variable and containing a positive element of appeal to normal perception.

In a negative judgment strictly interpreted as mere denial this nucleus is lacking, as the consideration of bare denial taught us. Therefore we have nothing left but those elements of meaning to which the interest of predication is the sole clue. Let us take a plain every-day judgment such as 'A. B. is not a dishonest man'. If we enunciated this proposition in the sense of the infinite judgment, meaning e.g. that A. B. is not a man at all, but a stone or a monkey, we should unquestionably be held to have violated the conventions of speech. The meaning of every judgment is to be looked for in the attribute to which is attached the interest that guides the selection of the content used in judging. But this attribute must obviously have definite relations, at any rate for some special purpose, to the content affirmed or denied. Whether it can precisely coincide with the content affirmed is a point to which I shall have to recur; but it cannot precisely coincide with the content denied, for if so, no result in which we had an interest would spring from the denial—the whole reason of our interest would be cancelled and be put away with the denial of the attribute on which it centred.

Now I may be interrupted at this point with the objection that this is exactly what is always happening in negation. When we have suspected a man or thing of having some attribute which interests us, and then find that we were mistaken, our interest in the individual may, and often does, fall dead at once. A man may be pointed out to us in a crowd as about to be our fellow-traveller on a difficult journey, and we may regard him with some interest on that account; but if we learn that it was a mistake, and that we have nothing to do with him, we shall probably after that regard him as 'nothing to us'. This is in fact almost a recurrence to the infinite judgment; for though we must know that he is a man, yet so long as he is not to be our fellow-traveller we do not care whether he is alive or dead; i.e. for any positive quality of his humanity. The judgment 'he is not coming with us' approaches then to a judgment of bare exclusion, the attribute in which we have interest being the attribute excluded.

In the first place, I think, we must to a great extent admit this contention; and simply refer it to a difference of degree. Negative judgment does begin with a phase inappreciably differing from the infinite judgment. But yet, of course, the question 'Coming with us?' must have originated in some such judgment as 'He is the sort of person who might be coming with us'; and it is within this fact judged true that the ultimate negation 'not coming' has its meaning. The interest is first awakened by the whole attribute 'such as to come', and could never have been aroused but for the presentation of such an attribute. If the negative judgment were really one of bare exclusion it would apply equally well to everything in the world, and no interest would have been aroused which could have led us to make it of a particular individual. But in the second place, it must be added that in as far as, in the alternative which formally arises within the attribute 'such as to come', all the interest practically falls on one side of the alternative, this is an actual defect of knowledge and morality, but has not, in the case supposed, been pushed far enough to prevent the insistance on the positive attribute 'capacity to come'.[1] And this attribute supplies in fact a positive content for the denial, though we by the hypothesis happen to be now indifferent to it. The denial really contains the judgment, 'The man is one who, we thought, might come, but is not coming.' The proximate case in the ascending scale of knowledge may be illustrated by supposing that instead of our interest dropping dead on hearing that he does not come with us, this negative judgment enunciates to us a regret that he is not coming, implying that we ascribe to him some positive quality which causes us to regret losing his company. In this case the positive bearing of the negation 'He is not coming with us' is primarily a *consequence* of the negation operating upon the judged content 'We should wish him to come with us'. This consequence supplies the denial with a positive import, and may be the only aspect of it prominent in the mind.

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[1] It is a commonplace satire to say of a man that the universe interests him only in as far as it is what his particular whim requires. What is not money, or total abstinence, or woman's emancipation, as the case

But there is something else to be considered. Every judgment without exception challenges the question 'Why is it so?' or at least 'How do you know it to be so?' the former question demanding the cause or real ground, the latter demanding the cause of knowledge or logical ground. These two kinds of ground run into one another, as we saw, and need not be distinguished for our present purpose. In denial, then, there must be *this* element also of positive import, the attribute which justifies the denial for us. It is plain further that in case of the non-existence of the immediate or apparent subject, this attribute may be judged directly of Reality, which is the ultimate subject in Negation as in all Judgment. 'The house on the marsh is not burnt down' may be true because there is no house on the marsh, and although reality—positive fact—excludes the burning down of any such house.[1]

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may be, is nothing to him. This is a good illustration of the moral and intellectual impotence indicated by any approach to bare negation. Hegel has quaintly compared the distinction between bare and significant denial to that between crime and civil dispute. If I steal, my act says 'This is not yours', without asserting that it is mine or any one's by right, i.e. it ignores the whole sphere of property or reasonable possession by alleging no ground for its denial. In a civil dispute I say 'This is not yours, because it is mine,' i.e. I assert rights of property, as you do; but I deny your right on the ground of mine.

[1] It may be objected that in such a case to say 'The house on the

Thus in Negation we have two positive elements which may be present together, or in various relations of prominence, or may wholly or partially coincide. The third element of import, the positive content explicitly employed in judging, falls away in the negative judgment, being replaced by a bare exclusion of an explicit content. This bare exclusion is what we discussed on its own merits in the second section of the present chapter, and found to amount in itself to nothing.[a] We are therefore referred for the meaning of significant negation to the positive ground, or positive consequence, of the exclusion which forms the outward and visible shape of negation. Thus we approach the solution of the problem how knowledge can take the form of ignorance—how 'what it is' can be known through knowing 'what it is not'.

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marsh is burnt down', is not so much false as unmeaning; in other words that the negative 'the house is not burnt down', has meaning only if there is a house, and presupposes or asserts that there is one. I have strong sympathy with this objection, which turns on the problem of a real distinction between subject and Predication within the Judgment. But an unmeaning judgment is clearly not true. The only doubt is whether its not being true justifies any negative except one which should brand it *as* unmeaning, e.g. 'There is no house to burn down.' The more hypothetical judgments are, the less they present this difficulty.

[a] I think that Mr. Keynes' objection (pp. 122–3) to my account of bare denial is sufficiently met by what has been said above, especially on pp. 281–2 and 285–6. I will only remark (1) that I count two elements of positive significance in the negative judgment viz. the ground and the consequence or positive interest and not the ground only; (2) that wholly to separate ground from a judgment is always to destroy in great part the meaning of the judgment—this is essential to a sound theory of inference; and (3) that our ready acceptance and interpretation of such judgments as his examples shows how far we are from bare negation when we accept them; for their natural meaning, which we accept, is conditioned by a whole system of context and positive interest which we presuppose as always present with us. Literally taken 'A.B. did not start by that train' may mean there was no A.B. in the world, or the man who started had not that name, or that A.B. is a mountain in South America, and therefore could not start by a train. How common it is for any careful speaker to say 'A.B. did not start—at least I looked and could not see him', thus supplying what we know the literal judgment wants. You may insist, 'but it means at least the denial;' but I insist 'It does not tell you what the denial means, except by assuming a context and positive interest, which take it beyond bare negation.' On the same passage in Mr. Keynes' book, see further, p. 297 below.

The primary analysis of the significant negative judgment presents therefore a close parallel to that of the hypothetical judgment. 'A is not B' may always be taken to = 'A is *x* which excludes (or which is implied in excluding) B'. Or in extreme cases we may have 'Reality is *x*, which excludes or is involved in excluding A B'. Just thus we saw that 'If A is B it is C' may always be interpreted into 'A is a factor in a real system *x* which given A B involves C'; or in an extreme case, 'Reality contains a system *x* such that, if A B were placed within it, C would result.' We examined at length the conditions under which such a relation as that enunciated in the hypothetical judgment could be made intelligible, and we found that a real system within which the separate terms should be interdependent was the bond of union which alone could justify such a reference of one thing to another. In the same way we saw that negation presupposes a real system—a system affirmed in judgment to be actual—as a condition of its intelligibility; and if it is replied that a system presupposes negation—for it presupposes difference—we must answer, first, that negation in its pure form as simple contradiction is the abstraction of difference, and *may* be later in origin than, or at least presupposes as coeval with it, the positive differences which all thought involves; and secondly, that in thought it is possible and indeed is the rule for factors to presuppose each other, and to grow into distinctness *pari passu*. In fact, Negation *is* simply the *logical, conscious* expression of difference.

Significant negation, then, like hypothesis, is intelligible within and with reference to a system judged to be actual. It is only within such a system that something can be made out of nothing by implying a positive ground or consequence in a bare denial. In other words, the essence of formal negation is to invest the contrary with the character of the contradictory, or to raise mere discrepancy or positive opposition to the level of the absolute or contradictory alternative which is the abstraction of difference. It is only contradictory negation which allows a conclusion to be formally drawn from the negative; contrary negation does not admit of this. It is only contrary negation which allows any import to be materially attached to the negative; contradictory negation does not admit of this. The fact that contrary negation can be given the force of contradiction, that a positive opposite can be known as a sole alternative, and that unless this is done knowledge remains inarticulate and chaotic, is simply the fact that Reality is a system. It is in considerations of this kind that we must look for a reconciliation of conflicting ideas as to the 'subjective' or 'objective' place of negation. We must ask in every case *what* negation? The negation of what and under what presuppositions? Without going further into extreme views about the objective import of negation, I am most anxious to persuade the reader that the fruitful question is not 'Can we conjure a meaning out of a bare denial?' but 'Why is it that in knowledge we cannot do without denial?' 'In your "system of differences",' it may be said, 'you put a significance into your negations, and then pride yourself on finding there what you put in.' This is true. What I want to insist on is the fact that this kind of significance cannot be put into anything but negation, and the light which this fact throws on the significance in question. Reality is a system, and you cannot have system apart from negation. This is the central fact from which all enquiry must start. The connecting link between difference, contradiction, and contrariety[1] is that difference becomes contradiction when taken as mere difference or as the abstraction of difference, that is, as expressed in a negative judgment which (like the infinite judgment) professes merely to exclude a given idea, or deny a given affirmation. If the denial were within a self-identical system it would carry a positive bearing. As *ex hypothesi* it has no positive bearing, it embodies mere difference without identity, or the abstraction of difference. Such a denial is the pure contradictory of the affirmation which it denies. On the other hand, difference becomes discrepancy or contrariety when not the formal abstraction of difference, but positive differents claim the same place, and the same place means the same relation to the same system. Such contrariety exists between 'A is B_1' and 'A is B_2'.

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[1] See below, II, chap. vii, on the 'Formal Postulates of Knowledge'.

Thus the articulate arrangement of differences under their systematic relations is the root of positive discrepancy. The system as determined by one relation excludes, under that relation, the system as determined by any other; and the system as a whole identifies any one of its *positive* relations with the mere difference from, i.e. the bare exclusion of, certain other positive relations. A is either B_1 or B_2. This is the combination of Contradiction and Contrariety.

It is not essential that the positive ground and positive consequence of a denial should be different. Under conditions of precise knowledge they must be the same. We shall see that where only two alternatives are possible, and they exclude one another, either of them is denied by the affirmation of the other, and affirmed by its negation. Under such conditions the denial of one alternative has the affirmation of the other both for its ground and for its consequent. 'He will either dissolve or resign' permits us to understand under the denial 'he will not resign' the affirmation 'he will dissolve' alike as its ground and as its consequent. It is plain that in actual knowledge there are degrees of this coalescence. The ground may be imperfect or extraneous; it may be replaced by any positive quality that excludes the predicate denied, i.e. in this case, excludes resignation. This positive quality might be even difficult to formulate precisely, and might run as close as possible to a bare denial; we might feel sure that 'he is not the man to resign'; which means that his general character as we understand it precludes the idea that he will resign. Such a general *ground* would be at once reinforced in its cogency and restricted in its result by the *consequence* of the denial, the necessities of the case being as supposed above. The two in their coalescence, 'he is a man who will not resign,' and 'not resigning, he must dissolve', would in that case form the complete analysis of the denial 'he will not resign'.

It has been necessary in this discussion to anticipate the account of disjunction which will be given in the next chapter. Perfect disjunction is of course a late form of knowledge. But it is an ideal inevitably involved in the nature of negation. All significant denial—all denial, that is, which rises above the level of the infinite judgment—corresponds to this ideal in two respects at least. Every denial has its meaning within an attribute or set of attributes judged to be real; and every denial affirms some positive matter which affects and is affected by its relation to such a comprehensive attribute. This may be illustrated in another way, which will also serve as a recapitulation of the stages of negation.

Negation is rooted in the fact of difference, but difference is not enough to warrant negation. Mortality is a difference within the identity of man, but we do not therefore deny that man is mortal. Significant negation begins, we said, when positive differents claim the same place in the same system—in the simplest case, when differents claim to be identical. Man is not mere mortality, i.e. does not coincide with mere mortality in the system of reality. As thus conflicting, which of course can only happen *in a judgment*, differents are contraries or opposites, and the assertion of one is the negation of the other. Now these contraries or opposites may be of any number. There is nothing to limit them. Any colour is the contrary or opposite of black, if asserted in the same relation; and so would any sound or taste be, *if asserted in the same relation*, which seems impossible for a sane man. If then we consider negation at this stage as embodying contrary opposition, what follows from it? 'This surface is not black' has indeed a ground, viz. that it is some particular other colour; but what about the consequent of the negation? There is none, except that the surface is *some* colour other than black, and therefore we lose by choosing the negative expression rather than the positive, and the significance of the negation *qua* negation is absent. What follows from the absence of a consequent as definite as the ground? Nothing less than this, that negation cannot be explained on the basis of mere exclusion of contraries. It is indeed possible to deny intelligibly on such a basis, in virtue of the general consequence of negation, but no reason can be assigned for in such a case preferring negation to affirmation. For in the case supposed we should be concluding from 'A is red', through 'A is *not* black', to 'A is of some colour other than black'—a manifest loss.

Negation can have no bearing, i.e. no interest or *raison d'être,* unless the contraries are limited so that something follows from the negation. In other words, Negation always *involves* Contradiction between contraries and not merely Contrariety. There may be any number of alternatives, but unless the number is limited or falls within some positive characteristic however vaguely known, nothing can follow from the denial either of one or of any number among them. Therefore though negation originates in difference, which it raises to contrariety by embodying it in definite relations, and though it does not arrive at formal contradiction between opposites till long processes of thought and language lie behind it, yet I am unable to comprehend how any negation can have interest apart from being taken within a positive whole, however vague, which is of the nature demanded by the relation of contradiction when established between opposites. Mere contradiction as between 'he is good' and 'he is not good' is given in the nature of negation from the first; and its development consists in filling this unmeaning form with significant opposites, so that from 'he is not good' we may be able to infer something more than that 'it is not true that he is good'.

Significant Negation then combines in itself the absolute Contradiction which was illustrated by the Infinite Judgment, and the Contrariety which arises between differents when referred to the same place in the same system. Without contrariety negation would have nothing that it could mean, but without contradiction it would not have in itself the power to mean anything.[a]

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[a] The above account was modelled so as to respect the distinction made in Formal Logic between Contraries, both of which may be false, and Contradictories, one of which must be true. But in common usage the relation between Contraries is called Contradiction, no less than that between Contradictories. And it is important to realise that Contradiction in the metaphysical sense, i.e. a deadlock which reason cannot acquiesce in, is rooted in what we defined as Contrariety, i.e. the unmediated reference of different predicates to the same subject. See Bradley, Mind, 72, 496, and Appearance, ed. 2, 562. I think that the above account of Contrariety and my account of Contradiction in the Companion to Plato's Republic, p. 149 ff., are in harmony with Mr. Bradley's statement. The ultimate nature of Contradiction is philosophically very important.

Opposition and conversion of Judgments.

4. The rules of the opposition and conversion of Judgments, which have come down to us almost as Aristotle formulated them, are founded on the classification of judgments according to 'quantity'. They therefore lay down the relations to one another of all classes of judgments which this principle recognises, in as far as the truth or falsehood of any one judgment affects the truth or falsehood of any other which deals with the same content. But when we attempt, as we have attempted above, to distinguish the *kinds* of truth which various types of judgment embody, then the relations between the various classes of judgment cease to be a matter of mechanical rule-of-thumb, although not hard to understand if we pay attention to the actual significance of the judgments with which we deal.

Opposition.

i. The principle from which we must set out is that everything which can be affirmed can also be denied. In some cases the denial will be confined to a shape closely corresponding to that of the affirmation, and in some there will be two kinds of denial which will fall apart.

Between Singular Judgments.

a. The Singular Judgment cannot be treated for the present purpose as a case of the universal. It has, as we saw, a universal character, but not in the sense of referring to an aggregate of individuals. If 'Caesar crossed the Rubicon' is true, 'Caesar did not cross the Rubicon' is false; and if the negation is false the affirmation is true. Thus the relation of Contrary Opposition, according to which the Universal Affirmative and the Universal Negative of formal logic may both be false, falls away in the case of a Singular Judgment in our sense of the term. In this type of judgment we have the simple relation of affirmation to formal negation which is contradictory, i. e. presents an absolute alternative. The reason of this we shall have further to consider in treating of Double Negation.

Between Plural Judgments.

β. The Judgments of Enumeration, Plural or Particular Judgments, including the Collective Judgment, present the relations towards each other with which we are familiar in the common scheme of opposition. The peculiarity of these relations is that in them an absolute alternative or contradictory opposition is only to be obtained by opposing judgments of different quantities. The reason of this is well brought out in a phrase which Aristotle employs in his account of opposition, where he states the contradictory of ' All are ' to be ' Not all are '—instead of using the expression of our text-books ' Some are not ' ; or when he says that the contradictory of a proposition which affirms a predicate universally, is one which asserts that of the same subject the predicate is not universally true. What is affirmed in a collective judgment is the homogeneity in a certain respect of an aggregate still regarded as an aggregate of enumeration, although endowed with sufficient unity to warrant itself as a completed whole. The denial of the judgment is the denial of this homogeneity, and is rather a consequence of than identical with the partial counter-enumeration which our logic-books take as its type. If, for instance, we assigned a number to the counter-enumeration, the contradiction would no longer be complete, and we should find ourselves in contrariety instead. To say that ' All these books are German ' and that ' Two of them are not ' cannot be a contradiction pure and simple ; for the falsehood of ' All these are ' is compatible with the truth of ' Two—three—or all of them are not '. This is enough to show that ' some not ', if we take it as a sign of counter-enumeration, is less safe in contradictory opposition than ' not all '.

As regards the ' contrary ' opposition of ' All are ' and ' None are ', the doctrine of formal logic is true so long as we confine ourselves to Judgments of Enumeration. ' None are ' asserts a complete counter-enumeration or its equivalent based on some other ground, and besides this assertion and its contrary there are as many alternatives as there are individuals in the aggregate, *minus* one.

The particular or incomplete collective judgments ' Some are ' and ' Some are not ' (sub-contraries) are of course compatible with each other so long as we do not determine them numerically ; and continue to be so then if we consider that incompleteness of enumeration debars us from all reference to a sum total. If on the other hand we permit the comparison with an assumed sum total, we pass at this point into calculation. The general conception under which we are enumerating always, it must be remembered, implies a total ; so that calculation lies very near to judgments of this type. 'Six men were killed and ten wounded (but not killed)' are judgments perfectly compatible with one another if the number of men on the ground was sixteen or more; but if there were only fifteen or fewer, then the two judgments at once rise into contraries. Both may then be false, but one must be. This however *is* calculation.

This account of the matter includes all that need be said of the Singular judgment if used in opposition to the Collective judgment. In such a case the Singular judgment takes the place and follows the rules of a judgment of Enumeration. It must however undergo a transformation, even it only implied, in taking on a relation to the basis of enumeration. 'How many Liberals voted against the Bill?' 'A. B. did, C. D. did,' &c., &c. These singular propositions are read off into enumerative judgments, 'One Liberal, two Liberals,' &c.

The further judgments which arise out of the Judgment of Enumeration and Measurement follow the characteristics of those forms of the true universal judgment to which, whether as generic or as hypothetical, they severally approximate.

Between Generic Judgments.

γ. I will now point out shortly the characteristics of these judgments themselves when placed in opposition. The tendency of the higher stages of knowledge is, as we saw in the last section, to fuse contrariety and contradiction into one. This is obvious, for instance, in the individual generic judgment, for the same reason as in the singular judgment itself. And even in the analogical judgment the tendency to fusion is strongly marked. The allegation of exceptions against a generic character, whether in form of expression positive or negative, must either be insignificant as when the exceptions are apparent and not real, or else tend by analogy to establish a contrary alternative or *positive contradictory*. In the former case the judgments which emphasise the exceptions must be taken to be not generic judgments at all, but mere enumerative judgments, which therefore cannot touch the essence of the generic judgments they appear to oppose. For we shall find the dominant principle in these relations to be that a judgment of one type α cannot deny a judgment of another type β; although the former may suggest a judgment of the type β which will constitute a full denial of the other judgment of that same type.[*] Under the head of such mere

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[*] Cp. Keynes 101, 122–3, and 259, and p. 369, below. See also Essentials, 116–17. Objection is taken to the views (1) that the Particular in categorical shape is unscientific; (2) that a judgment is in principle effectively denied only by another of its own type.

The governing principle of these views is the conception of the double nature of judgment, which makes every actual judgment a battle-ground between the existential and the necessary character. No form, I take it, is quite unambiguous (see Preface to ed 2), but the separate types of import do tend to find appropriate expression, and it seems only right in logical theory to insist on this tendency. And I think it a counsel of perfection for practical life.

Thus (1) if what you want of the categorical particular is to contradict the hypothetical or scientific universal, you are (p. 300, below) fighting a theoretical principle with unanalysed judgments of perception and enumeration. Do not nearly all popular errors rest on this practice? And you have an appropriate mode of denial in the modal particular, which is intended for the expression of incipient analysis, when it falls short of justifying a contrary universal.

If what you want of it is to assert existence, then you are falling back on the assertory or existential side of judgment, and letting the aspect of necessary connection slide into the background. 'But every judgment includes in some degree both.' Yes it does; and that is why we are always confusing them with each other. 'Est hoc, ergo propter hoc' is a more general formula than 'Post hoc— ', and expresses much the same fallacy. In Logic we might at least try to avoid both.

(2) If I were to say that a scientific judgment can only be denied by a scientific judgment and an existential by an existential, that might seem obvious, and would be all I want. A doctrinaire denies facts on the ground of laws ill-understood. An empiric denies laws on the ground of facts ill-conceived. M. Keynes says (p 124) that I admit that exceptions can overthrow a law, and the rest of my argument is *ignoratio elenchi*. But of course I mean exceptions interpreted in terms of a law. How am I to know that Volvox is not an animalcule, when Ehrenberg says it is, unless I can tell more or less to what kind it really belongs? I need not know what precise kind; but I must know 'some kind excluding animalcule.' And from that it is only a step of degree to knowing precisely what kind.

As to denying every judgment only by its own type, there is a peculiarity in the disjunction (cp. vol. ii, 194). It is (see p. 328) a unity of hypotheticals and a categorical, and, further, represents the whole system which itself gives meaning to a negation. So, taking the last point first, I should say that it is hardly natural to deny it, for in doing so you must presuppose another disjunction to give meaning to your negation. The place for negation is rather under than of a disjunction. The natural course would seem to be to let it alone and begin again. Cp. Essentials, p. 125, and below, 301.

But if you are able to specify and deny the categorical basis, e.g. to enumerative judgments would come all observations of artificial forms, mutilations, abortions, &c., in organic and spiritual beings; and all accidental juxtapositions rendered by judgments whose subjects have no connection with their predicates. The generic judgment 'Man is a creature with a sense of justice' is not invalidated by instances drawn from dead men, lunatics, or idiots; nor even, perhaps, from criminals, if there are such, whose conscience is obliterated by lifelong war with society. But I shall be told that a scientific law has no exceptions. This is just what I am maintaining; I am saying that exceptions are either apparent or real, and in the former case, that now before us, do not deny the law; in the latter proceed positively to indicate another law. In the second case, when the law is really impeached by the bearing of the exceptional instance, this bearing must have a positive import, which may not amount to a suggested law, but must be in the same region of essential individuality in which the characteristic that is denied has its import.

If I say that all exogenous trees are dicotyledonous, I am opposed by a real, not merely apparent, exception in the case of the Coniferae, which though exogenous have in many species more than two cotyledons. Here we have one Generic judgment (I use 'generic' in the logical sense which I have explained) opposed to another. It is not merely that here and there an aborted or mutilated instance is to be found in which a part of the plant is wanting; but that an enormous natural order with marked unity of habit, and in the strictest sense[1] sharing the characteristic on which the analogy is based, does not display the character required by analogy.

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say, in denying 'X is red or green', 'X is not a colour', that is one way of disposing of the whole disjunction; or if your objection is to its exclusiveness or exhaustiveness you can deny that one of the two special hypotheticals involved which alleges the necessity you desire to impeach. In all this categorical is opposed to categorical, and hypothetical or at least modal particular to hypothetical.

[1] Here an error is possible. The wood-formation of Coniferae, though proceeding from a cambium ring outside the old wood, has differences from that of Dicotyledonous Exogens. If these differences were of importance the exception might break down as not a case under the rule, Coniferae not being true Exogens.

Now I am not aware that any importance has been attached to determining the connection between exogenous wood-formation and the number of seed-leaves or cotyledons; but it is plain that the conflict of generic judgments so far as we have followed it does not destroy the idea of such a connection, which would be the effect of a mere contradiction, but suggests that it may be characteristic of plants with two *or more* seed-leaves. Supposing that monocotyledons were also found in some cases to form wood externally, the question would still arise whether any principle of development could be traced according to which the characteristics under discussion might arise together or owing to connected causes, as if e.g. there was a point at which the one natural kind approached very close to the other. If so, the supposed further exception would still lead to a positive principle or contrary, and not to an empty contradiction. Even if the first generic judgment were a sheer blunder and confusion, as has been the case from time to time with judgments propounded in science, it is scarcely possible to rectify the confusion except by substituting for it the true positive conceptions that arise out of the cases which overthrow it. An example in point is Ehrenberg's inclusion of a group of confervoid Algae (Volvox) under the class of Infusorial Animalculae, or again, probably, his alleged detection of highly-organised structure in the Infusoria proper. Enumerative exceptions are futile in such cases; what is needed is a re-interpretation of the character of the group as such. Such a re-interpretation is at once contrary and contradictory to the mistaken judgment which it corrects. But in the process of interpretation it may and perhaps must pass through a stage which may best be described when we are speaking of the hypothetical judgment.

Between Hypothetical Judgments

δ. The *contrary* of the Hypothetical Judgment is as usual a judgment of the same type. The complete contrary of 'If outdoor relief is refused the workhouses are crowded' would be 'If outdoor relief is refused the workhouses (*caeteris paribus*) are not crowded'. This denial means that the condition expressed in the hypothesis 'If . . refused' is not merely inoperative to cause pressure on the workhouse, but is actively operative to decrease that pressure. I insert the limitation *caeteris paribus* merely to secure the judgment being taken as truly hypothetical, because in such concrete matter as this there is a tendency to interpret the judgment as collective in the sense that 'Every case of a is a case of β'; so that it may be objected to on the score of accidental instances. In abstract matter, e.g. in geometry, where the hypothetical judgment has an unambiguous import, these objections are understood to be inadmissible. Parallel lines are taken *qua* parallel when it is said that they do not meet, and so forth.

This contrary is the form of negation to which all precise thought aspires. If the condition is irrelevant and wholly unconnected, then indeed the entire type of knowledge to which we have aspired is a delusion and a snare, and the mere contradictory which will express our ignorance must be found in a judgment of a lower type. But if the condition is relevant it must operate somehow, and we can only choose between one view of its operation and another. Such a choice is expressed by embodying mere contraries in an absolute alternative or contradictory; by considering 'If A is B it is not C' as the contradictory of 'If A is B it is C'; as it is the *only* way of denying it *by a hypothetical judgment.*

But if we are to destroy the hypothetical judgment itself as an expression of ground and consequence, we must aim our negation precisely at its form. The enumerative particular would not help us here. When we have said that 'If (i. e. in so far as) a man is good he is wise', it is idle to reply that some good men are not wise. This is to attack an abstract principle with unanalysed examples. What we must say in order to deny the above-mentioned abstract judgment is something of this kind: 'If,' or 'Though a man is good, yet it does not follow that he is wise,' i.e. 'Though a man is good, yet he need not be wise'. The particle 'though' introduces the condition as a supposal, but by its adversative force prepares us for a denial that it has any connection with the consequent.

This same form, which may be called the modal particular, is the appropriate contradictory to a generic judgment which has to be altogether surrendered and cannot be corrected by a positive contrary. Its meaning is however not as clear as that of the Generic or Hypothetical judgments. For it does not assert a positive relation, but drops down into an unanalysed quality of exclusion, and thus into the confused concrete of phenomena. It may therefore mean that the supposed condition is inoperative and irrelevant, or it may mean that it is weakly operative and liable to be overcome by normal counteracting conditions, or that it is operative as asserted in the hypothetical judgment, but liable occasionally to be overcome by exceptional conditions. This last is a common meaning, but is not the true negation of the opposed abstract hypothetical judgment, and ought to be discarded from science, though there is no means of relieving the modal particular negation from it. How then are we to treat the correlative case, in which we deny 'If A is B it is not C' by 'Though A be B yet it may be C'? This seems to confirm our view that the character of mere negation is incompatible with that of the hypothetical judgment. For the former of these two contradictories is of course the same judgment which we have already considered in the light of a contrary and *positive* negation; and the latter seems therefore to retain under its affirmative form the essential character of mere denial. 'Though A is B yet it does not follow that it is not C' expresses the only sense in which the affirmative modal particular is a true or mere contradictory to the negative hypothetical judgment.[1] This sense is not in itself satisfactory if taken as the import of possibility, and real possibility demands a nearer advance towards the affirmative contrary—a recognition of some real operative condition making for the connection alleged possible. That this is so only confirms the suggestion made above that in significant negation contrary and contradictory tend to become one.

I may anticipate the case of Disjunction so far as it here concerns us by saying that the denial *of* a disjunctive judgment, though formally possible, is not a problem that naturally arises in logic. The disjunction is the presupposition and the goal of negation as an *organon* of knowledge. By denying it as a whole we sweep away the fabric of knowledge relative to the matter in hand, and must begin over again from the beginning, as there is no sphere left within which anything can arise from the denial of the disjunction. To deny a disjunction in this sense we should not trouble ourselves with the alternatives, but analyse them into their common basis and deny that. An impersonal judgment, as expressing wholly inarticulate knowledge, is the appropriate form for such denial, *unless* we are denying one disjunction under another. 'The soul is neither square nor round' may be denied by 'The soul is not in space'; for this denial has a positive ground and consequence founded on an implied disjunction, 'either in space or intellectual.' But if we are presented with a number of alternatives about a matter which seems to us to have no basis in reality nor relation to actual knowledge at all, then we may reply with the impersonal negative as the form of thought most suited to mere absence of positive content. 'The disembodied spirit in its earthly presence is revealed either by contact or by signs.' To this an entire unbeliever would probably answer, 'There is no earthly presence of disembodied spirits,' and here he approaches, not wholly by his own fault, the infinite judgment. He has merely said that reality is without the matter alleged, and his saying has no positive import beyond what arises from the imputation to reality of a character—whatever that may be—not necessarily incompatible with, but rather undistinguished by, the presence of disembodied spirits.

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[1] See below, on Privation and Affirmation.

Specific denials of a disjunction on the ground of incompleteness or superfluity[1] are of course either under a further disjunction which the denial tends to make explicit, or in the second case, *prima facie*, under the disjunction to be denied itself, with the result of showing that a different one is needed.

Double Negation.

ii. In treating of the contradictory relation, that namely between two judgments of which one simply denies the other, so forming an absolute alternative, we saw the results that spring from *double negation*. We appealed to double negation for instance in the case of the singular judgment, such as 'Caesar crossed the Rubicon'. This, with its simple denial 'Caesar did not cross the Rubicon', forms an absolute alternative, although the denial has a meaning and is not a mere infinite judgment. So far as this is the case, the opposition between the two is at once contrary and contradictory.

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[1] See following chapter on this case.

Do the two cases of inference from falsity in contradictory opposition stand on the same footing? The one is from the falsity of the affirmative, the other from the falsity of the negative. The former amounts to single, and the latter to double negation.

The explanation of single denial which has been given above may be briefly restated here. The abstraction of difference, taken in respect of its contrast with identity, and so as mere non-identity or otherness, is employed in negation as the vehicle of a positive contrary, which contrary is thus invested with the full alternative force that belongs to otherness as such when contrasted in the abstract with sameness as such. As regards the history of early thought on this question, of course we are not to look for determinate abstractions in primitive minds. But primitive minds probably are abstract, though they do not deal with abstractions. We must look for the germ of contradictory negation in mere repugnance or repulsion, which, although a positive state, has a peculiar aspect of negativity to which the inarticulate abstractness of the primitive mind lends an aspect rather of contradiction than of contrariety. Every one who has watched children must have noticed the remonstrant ' No ' without any *expressed* content, which is a sign of aversion to something done or suggested. The absence of *indicated* reference to any particular matter is often surprising, and impresses an observer by the difficulty of finding either the bearing or the ground of the negation. Here it is rather the positive contrary that is undeveloped and latent than the mere rejection or contradictory. Therefore I cannot but think that the absolute alternative—mere generalised otherness or rejection—makes itself explicit by the side of the positive contrary at a very early stage of thought.

Thus though I do not take every negation to be necessarily aimed against an affirmation of the same content, yet it seems to me that the pair of judgments which form a contradictory opposition embody an ultimate fact of knowledge. Single negation is in form the substitution of *mere* difference, or nothingness, for the combined identity and difference which alone have meaning. This form is the basis of the alternative in contradiction. It is the alternative antithesis between something and nothing. That 'nothing' is furnished with a meaning *in hunc effectum* does not appear from or affect the form of contradiction. Single negation, then—the passage from the falsity of 'A is B' to the truth of 'A is not B'— must be regarded on its formal side as the abstraction of a universal characteristic of knowledge. It means that A is, under the conditions and for the purpose of the judgment in question, in a relation of *pure* otherness to B.

The case of double negation—the passage from the falsity of 'A is not B' to the truth of 'A is B'—is in my judgment accounted for by the preceding remarks. The true problem, to my mind, is not how negation should be the absolute alternative of the corresponding affirmation, but rather how, being in its exterior form and vehicle such an alternative, it should become possessed of positive intelligible import. The fundamental nature of negation, thus understood, may no doubt be embodied in the principle of Excluded Third or Excluded Middle, which asserts that of two contradictory enunciations one must be true and the other false. Thus stated, the principle is merely formal, because the question immediately arises, 'What are contradictory assertions?' The definition might be made plainer by substituting for the phrase in question an expression such as 'the assertion and denial of the same content'; but no definition can relieve us from the task, which I have attempted to perform in this chapter, of explaining what a mere denial of an assertion really is. Excluded Middle is thus merely the abstract case of Contradiction or simple negation, and the proof of the principle lies in the analysis of negation.

I may illustrate this view, in the case of double negation, by two conceptions that deserve attention. The affirmation that admittedly results from double negation may be ascribed to the reappearance of an original affirmation at which the first negation must have been aimed, so soon as that first negation is cancelled by the second. I take this view to be true in substance, but false in the fact which its expression postulates. It was pointed out above that not every negation presupposes an affirmation. Sigwart, in treating of double negation in his second edition, correctly postulates as the condition of denial only an *attempted* affirmation. But it is true, and this is what Sigwart throughout meant to convey, that every negation bears on its face the nature of an alternative, so that, though we may not in fact have proceeded to it by denying an explicit affirmation of the same content, we are yet able to go from the negation of one member to the establishment of another. It is one thing to say that every negation *is* preceded by a corresponding affirmation,—for we may not have judged on the subject at all,—but quite another to say that every negation bears on its face that, if we judge, it is the sole alternative to the corresponding affirmation. This principle appears to me to be of the essence of the matter. It is this that gives double negation its distinctive precision and emphasis.

In opposition to Sigwart's idea, erroneously expressed as I admit it to be, of re-establishing an original affirmation, Mr. Bradley has maintained that the warrant of double negation simply consists in this, that in order to deny a negation we must already be in possession of the corresponding affirmation. We can only, he contends, deny A is not B on the ground that, within our knowledge, A is B. This allegation is made with good reason. We can indeed deny A is B on the ground that A is *x* or *y*, each of which excludes B; but we cannot assert A is B on the ground that A is not *x* or *y*, each of which excludes B. We cannot deny the consequent not-B on the strength of denying the antecedent *x* or *y*. The old rule of the hypothetical judgment, 'Affirm the antecedent or deny the consequent,' forbids this.

But on looking closer we shall observe that this impossibility is based on the imperfect view of the hypothetical judgment which assimilates its rules to those of the judgment of enumeration. This view disregards the possibility of a connection at once synthetic (i.e. not tautologous) and pure (i.e. free from irrelevancy). For in this case the denial of the condition is the denial of the consequent; and it is this which has been before us throughout as the essential and ideal meaning of judgment. This assertion of a pure connection between condition and consequent becomes, in the case of a negative consequent which is now before us, identical with our position that all intelligible negation takes its meaning from contrariety, though its form may be that of contradiction. To say that in 'If A is *x* it is not B' we cannot go from 'not *x*' to 'not not-B' (i.e. to B) is to say that there cannot be contradictory opposition between contraries.[a]

Contradictory opposition between contraries[b] can of course exist only if the possibilities are limited by means of a disjunctive judgment, and in this case Mr. Bradley admits that the double negation may be got at otherwise than by the corresponding affirmation. If we divide all Liberals into Unionist Liberals and Gladstonian Liberals we can go from 'That Liberal is not Unionist' to 'He is not not Gladstonian'. So too in a pure nexus with a negative consequent: 'If powder is slow-burning (and in that case only), it does not strain the gun unequally.' Under this judgment, if we know that the powder is not slow-burning, we are able to say that it fails in the quality of not straining the gun unequally. We can unquestionably get this inference whenever we approach the knowledge of a pure ground involving a reciprocal character in the hypothetical judgment, which makes it, if either member is negative, include the same cases as a disjunction.[c] The pure ground and the limitation of cases are merely different aspects of the same form of knowledge. The ground is the fundamental and operative character by which a system imposes certain relations on its parts; the limitation is the external and formal consequence of these relations, which may be mimicked for the purposes of common logic by an arbitrary or conventional restriction of alternatives.

My only difference from Mr. Bradley consists therefore in the view which I have maintained throughout, that apart from some limitation there is no intelligible negation—nothing but the infinite judgment, and therefore in strict logic no negation at all. Under such a limitation we can always go from denial of a positive quality to a positive result which we may as a matter of both theory and of fact approach from the side of double negation, although of course the identity of double negation with affirmation is in a reflective stage of culture too immediate to admit of the two being really distinguished. The reason for which I am anxious to insist on this not very practical distinction will appear more clearly when I come to speak of induction. It is, in brief, the importance of the negative instance; that is, of approaching any positive content from the side of its limit, of the exact boundary at which it ceases and some other content begins. For this boundary is a negation by denying which we not only affirm the content that lies within it, but affirm it in its conditions and genesis, at least for knowledge. 'If a nation has no true art it is not religious' is a judgment that gives the analysis of a group of 'negative instances', which analysis passes into an affirmation supported by those instances, in the form 'A nation which has true art is not not religious'. The conclusion thus obtained, 'This nation (having true art) is religious' may be *bonâ fide* arrived at through the double negation I have described, and may be at first unsupported by the direct observation 'This nation is religious'.

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[a] The negative character of not-B might raise the question of two negative premisses (Muller in 'Mind' lxxx). But *assuming reciprocity*, the hypothetical major has disjunctive force, and that adds a new factor. See reference in note c.

[b] See p. 293, note *a*, above. The Contradictory opposition here spoken of is not Contradiction in the wide sense, but the relation between Contradictories as recognized by Formal Logic, cp. p. 333, note, below.

[c] See p. 329, note, below.

Conversion.

iii. Conversion is usually treated with opposition under the head of immediate inference. It is primarily a transition from one grammatical form to another which introduces no new elements into the content. Whether, or in what cases, it really involves inference, i.e. a passage from one *judgment* to another warranted by the first, is the main question which arises in treating of it, and of course includes the problem *what* inference, if any, is involved.

The unity of the judgment, it will be remembered,[1] does not exclude a considerable measure of diversity. It is often a mere chance whether a range of affirmative thought is condensed into one proposition or comminuted into several. And inference is working through the whole of such a range as the judgment gains depth and width, and defines its edges. Thus, if we mean to say anything definite about the point at which we pass from one judgment into 'another', we must look at what is *bonâ fide* implied as the essential import of the judgment-forms that we discuss, and not at the actual transitions which attend our interpretation of any proposition, and which vary with mental endowment in the interpreter, and with the lucidity of expression of the given sentence. That is to say, we must distinguish interpretative inference from substantial inference—transition within the judgment from transition between two judgments.

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[1] See above, chap. i.

The question may seem indeed to settle itself at once. The unity of judgment is determined *prima facie* by the unity of the Content judged. A new subject or a new predication, i.e. one not related as part within whole to that of a given judgment, is needed to constitute another judgment. But in immediate inference there is no new content—at any rate no new positive content. Can there then be a new judgment?

The real interest of Conversion lies in the discussion which it provokes of the precise relations imposed upon its content by any given judgment, and of the boundaries which separate the *bonâ fide* meanings of the various judgment-forms. In arriving at these relations or at this meaning we use interpretative inference; it is only when we find ourselves able to go from relation to relation by re-applying the same form of judgment, or from meaning to meaning by passing from a judgment of one type to a judgment of another, that we are really employing substantial inference. It does not much matter, however, where we elect to draw the line of transition from judgment to judgment, so long as we understand the connection of the implications concerned.

Simple Conversion of Singular.

a. I will begin with an accidental case of simple conversion, not recognised in the text-books because the content of predication in singular judgment is not necessarily of singular reference. 'The Duke of Cambridge is Commander-in-chief.' I think it is beyond question that this proposition might be met with 'The Commander-in-chief is the Duke of Cambridge' in a tone which would give it the force of a criticism or retort, although its content would be warranted true by the proposition as first enunciated. I am not prepared to abolish the distinction of subject and predicate within the judgment, and if we retain the distinction, then the mere transposition of the two makes a difference. We understand the subject primarily as a designation and the predicate primarily as an attribution. The idea of concrete individuality clings to the subject, and that of specially selected determination to the predicate. The second of the above propositions might certainly be understood to mean that the qualities of the office were limited in the relation in question by those of the individual, and not those of the individual adapted to those of the office.

If it is replied that this may be possible enough, but that really one who commits himself to the former proposition has bound himself to know and judge the concrete synthesis of qualities which the predication constitutes along with its subject, I cannot deny that this is so. And I am therefore content to rank such inference as is illustrated by the above transition under the head of interpretative inference, i. e. inference that falls within the logical unity of the judgment as *bonâ fide* expressed by either propositional form. It is true that in some simply convertible judgments, e.g. 'A = B in weight', the order of terms can make no possible difference of import. But the reason of this is not that the proposition is simply convertible, but that the content is of a highly abstract character which annuls all individuality, and thus destroys any significance that might attach to the difference between subject and predication. A similar character belongs in some degree to all quantitative judgments.

Conversion by Limitation.

β. The Universal Affirmative Judgment (under which, in ordinary logic-books, the Singular Judgment is comprehended) is not admitted to be convertible *simply*, i.e. by mere transposition of subject and predicate, but is supposed to be convertible 'by limitation', i.e. by transposition together with reduction to the level of a 'particular' judgment.

With reference to the first part of this rule, the whole course of our investigation of judgment is a comment on the degree and *rationale* of its truth. It is equally certain that the prohibition of simple conversion is warranted by common usage, and that the 'pure case' (which does not mean mere tautology) is an ideal operative throughout the judging activity. I am confident that Quantification of the Predicate and the Equational Logic owe much of their success to their recognition of this ideal, though their forcing it upon the ordinary judgment by truncating the meaning of the latter is a blundering *anticipatio naturae.*

The second part of the above rule, the 'limitation' of the converse, has different values as applied to different classes of judgment which correspond to the so-called 'universal affirmative'. If we take a judgment of the collective type and argue e.g. that because 'All houses in this street have gardens' therefore 'Some houses which have gardens are in this street' we do not seem to gain anything by the re-arrangement. And we certainly lose something, for we cannot recover the original judgment by re-conversion of the particular thus obtained. Of course re-conversion can only give 'Some houses in the street have gardens'. But we know, to begin with, that all the individual houses in the street unite with their other attributes that of having gardens. We seem, then, only to have advanced to a doubt of what we knew. There is however a shade of difference suggested, as in the singular judgment examined above, by the mere transposition of subject and predicate. The denomination of the individuals is less emphasised than the content enunciated of it. The former fills the place of pointing with the finger to an object of perception, the latter that of the significant ideas by which the perceptive judgment qualifies such an object. But this transition seems to fall within the interpretative inference, and not to amount to substantive inference.

In the case of the generic judgment of either type the import of the change becomes more emphatic. 'The dahlia is one of the Compositae' tells me that this regular flower, apparently a mass of petals like a garden rose or peony or hollyhock, is really an aggregate of little florets like those of the daisy or dandelion.[1] In short, this judgment distinguishes the dahlia from flowers externally not unlike it, but having no structural affinity with it whatever. 'The Compositae include the dahlia,' or 'Some Compositae are dahlias,' presents us with the typical or generalised structure of a Composite flower determined and differentiated by the peculiarities of a dahlia. The rose or peony would in this case never come into our heads; we should be occupied with some such individual as the daisy or the thistle, and the object of the judgment would practically be to distinguish the dahlia from other Compositae, not to distinguish it from flowers which are in no way akin to it. The one judgment qualifies the external appearance of a dahlia by the internal structure of the Compositae; the other qualifies the diagrammatic [illegible]ype of a composite plant, as admitting, among others, of the specific peculiarities of the dahlia.

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[1] In the double dahlia—the form till lately commonest in gardens—every floret is developed into a one-sided corolla like those of the florets which form the ray of the common daisy.

I have but little doubt that th[illegible]s account represents the meaning of the two propositions *in fact*. But it may be said that we have no *right* to separate them, and that we ought to demand the explicit recognition of both these determinate affinities as essential to [illegible]e meaning of the generic universal j[illegible]ment itself; and it may even be insisted that if the second [illegible]osition contains more than the first in any respect—e.g. [illegible] more concrete appreciation of diversities of composite [illegible]re—th[illegible] it cannot be wa[illegible]ated by the first. This [illegible]ggestion would make sh[illegible]t work of all conversion wh[illegible]r beyond the rank of grammatical change, unless—πλέον ἥμισυ πάντος—mere selection can make the old into new, or unless some principle is appealed to that goes beyond the judgment itself. The former part of the objection may be met by saying that undoubtedly the original judgment *may* be made with the whole significance of the two propositions; but that when so made it is somewhat artificial, and that the distinction of a specific adaptation within the Composite type is naturally a different process from the qualification of a given shape by the abstraction of that type. I may put the antithesis thus: the generic judgment, which we took as the *convertend*, if completed into identity, would determine the composite type by limitations restricting it to the case of the dahlia; while the particular, in this case the *converse*, if similarly completed, would extend the range of species by disjunctive enumeration till they expressed the whole content of the natural order of Compositae.

Modal Conversion.

γ. This conversion of a generic judgment may be taken either as interpretative or as substantial Inference. Tested by common usage it is, I incline to think, a substantial transformation, resting on some such principle as the possibility of treating every content in turn as quasi-individual. Tested by the logical ideal, it is a mere phase in the interpretation of the judgment, which involves reciprocal determination in the affirmation of a nexus. In either case it contains the only fundamental principle on which conversion can really proceed; viz. that every content can be exhibited as a quasi-individual element in a system. This is, for instance, the principle of which 'modal' conversion is merely a corollary. In the typical example employed above modality has no *real* application; modality only appears where the disjunction is one of ignorance. The particular judgment 'The type of the Compositae includes among its alternative modifications that of the dahlia' *may* be read into 'A composite may be a dahlia', i.e. the fundamental structure C is a real element compatible with the modification D. But this is a mere consequence of feigning ignorance when we have knowledge; in order to get the possibility we have to imagine that conditions are unknown which in fact are distinctly known and enter into the content of the judgment.

The pure Hypothetical Judgment differs from the Generic by disregarding individuality, and therefore the above principle is disguised in converting a Hypothetical, so that we appear in this case to obtain pure modal conversion. 'If arsenic is taken in such and such a quantity, it will cause death with such and such symptoms', which gives the converse 'Death with such and such symptoms *at least may* have been caused by arsenic'. This is a motived possibility or real possibility, not a mere possibility, and forms a substantial conclusion and one warranted by the convertend. But this result is still rather a corollary from the converse, than the converse itself. The example just quoted is one hardly deserving hypothetical form. It involves no traceable modification such as to set up a clear nexus between antecedent and consequence. Thus we cannot assign any precise conditions under which the consequent is related to the given antecedent, and the possibility that such death may result from such poison is merely a fact *plus* unknown alternatives. In a true hypothetical there is more than this. Grant that it is not simply convertible, still we can bring the possibility home to the nature of the consequent. 'If straight lines are parallel, they do not tend to meet.' The real converse of this is, 'Straight lines *in the same plane* which do not tend to meet are parallel.' But the limiting condition is not given in the convertend. Nor do I mean that we have a right actually to state it in the converse, but yet we must think of the converse where such a condition can be suggested as exhibiting a real if undefined attribute of the content which is now the subject, in virtue of which this content includes the case mentioned in the antecedent of the convertend. The affinity between generic judgments and the more valuable hypotheticals is very close, and we lose all hold on the generic element in judgment if we insist on reducing a definitely determinable content to one real alternative in an unknown number of unknown ones. I cordially agree therefore with Mr. Bradley's distinction between *real* and *mere* possibility, and only insist that the true content even of a modal converse is the positive nature in virtue of which the subject-content is variously determinable, and not a mere conjunction of attributes *plus* other unknown conjunctions.

Simple Conversion of Universal Negative.

δ. We are now to speak shortly of conversion in which the negative is employed. Every negation rests, as we have seen, in its purely formal aspect, on the ultimate or absolute disjunction. But it does not follow from this that a real process of substantial inference takes place where negation intervenes.

I begin with the simple conversion: 'A is not B,' therefore 'B is not A'; 'No negroes have straight hair,' 'No straight-haired man is a negro.' I should certainly prefer representing this transition as a true process of argument to illustrating it by a diagram of two separate circles, because by the latter means we destroy all idea of the structure of the judgment. But yet I cannot think that in our present stage of reflection the argument if any is more than interpretative, i. e. more than we are always using in arriving at the full meaning of any sentence. We can formally trace out the process of inference, but in using it we are like an engine running free and doing no work. No doubt the formal steps are, beginning from the ultimate disjunction between 'is' and 'is not', to say 'No negroes have straight hair'; 'Straight-hairèd men are either negroes or not'; 'If negroes, then they are both straight-haired and not straight-haired'; 'Therefore it is false that they are negroes', 'therefore they are not negroes.' But all this is given in the meaning and practice of any civilised language; and though it might be possible to stumble in using this converse,[1] that is no more than may happen in reading any sentence however simple. The elementary meaning is simply that the two contents are in the relation of abstract otherness to each other, and refuse to be brought together in the modified or concrete otherness which subsists within the affirmative judgment.

Contra-position. ε. *Contraposition* seems to contain a more remote conclusion. It goes from A is B, to Not-B is not A, e.g. from 'Every negro has woolly hair' to 'A man who has not woolly hair is not a negro'. I remark on this process, i. that the skeleton argument with symbolic letters seems far more remote and obscure than the intelligible example; ii. that we are much hampered in the apprehension of our inference by being forbidden to conclude that 'Not-A is not-B', which the ideal of judgment demands as the expression of the negative instance, 'Just not-A is just not-B,' e.g.—the affirmative being 'True freedom is virtue'—'If you fail to produce freedom, you fail so far and for that reason to produce virtue'. And, iii. substantial inference is less likely here, for the unity of the judgment is much greater in affirmation than in negation; and when we have just, so to speak, dipped the object *a* in the colour *b*, it seems idle to ask on the basis of that judgment whether what is not of the colour *b* can be the object *a*. We are in fact no longer dealing with *a* and *b*, but with *a b as including all a*. We may say of course, 'Oh yes! not-*b* is not-*a*, because if not-*b* is *a* then it is *b*.' But this remark is made ready to our hand; we have just qualified *a* by *b* and therefore are at least entitled to '*a* is not not-*b*', which brings back our problem to the last head, that of simple conversion.

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[1] As e.g. if any one tried to argue that 'All not-straight-haired are negroes'.

In short, then, in both these cases we employ the absolute alternative; but this is given in the *form* of negation or contradiction, and needing no true limitation of alternatives, can only rank as a formal principle of intelligible thought and speech, not as a real addition to content of inference. Transition by help of such a formal principle I call Interpretative Inference.

Privation. Affirmation and Exclusion.

5. The external shape of negation belongs as we saw to ignorance, and significant negation is knowledge disguised as ignorance. For bare denial would be devoid of meaning. Now this external shape of negation seems really appropriate where positive knowledge fails, i.e. in the region of what has been called Privation,[1]—the mere absence of positive determination. But this region is also the region within which there falls the limitation of knowledge, a matter of the most serious and positive import. Where knowledge simply fails us, and consequently we seem to have nothing left but privation or bare denial, how are we to pronounce on any suggested possibility, α. by way of affirmation, β. by way of exclusion?

Privation and Real Possibility.

The case α is the more complicated of the two. The privation or bare denial is in this case the bare denial of an incompatibility with reality, i.e. of an impossibility. *Exclusion*, as we have seen, must rest on a positive quality, a ground of negation. Here, *ex hypothesi*, we can find no *exclusion* of the impossibility. We are supposed to know simply nothing, either *pro* or *con*, about the positive matter whose possibility is in question. Are we therefore bound to admit that it is not

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[1] 'Privation,' *Privatio*, used by Sigwart as equivalent to Aristotle's στέρησις, στερητικός, and as contrasted with opposition, ἐναντιοτής, and distinct from negation, ἀπόφασις. Στέρησις seems to be applied to any negative enunciation, ἀποφάσις only to the denial of an affirmation; see Aristotle's Organon, 38 b, 13, with Waitz's note. Unluckily the distinction of privative and negative terms as given e.g. in Whately's Logic has just the reverse meaning to that of privation and negation in Sigwart and Bradley. 'Privative' in Whately indicates a positive opposite, and 'negative' a mere absence. The association with active 'deprivation' implying a *loss* appears to be the cause of this usage.

not-possible, and, as a consequence of this double negation, that it is possible? If we are led to do this—and the trick is often attempted, especially in popular theology—we feel that we have been cheated. A possibility, in the usual sense of the term, is something. We are foolish if we do not keep it in view and let it influence our deliberations in any way which its nature demands. Yet this something has here *ex hypothesi* been created out of nothing. But in knowledge at least nothing can come of nothing, and we are trapped in a contradiction.

What we have to remember is that our denial of the exclusion of the positive content is or approximates[1] to a bare denial; or in other words, rests upon no positive ground. We do not *exclude* the impossibility; we only fail to find it. And therefore our denial is meaningless or nearly meaningless, and amounts to nothing. In other words, the possibility which we are asked to infer from mere not-impossibility has as so inferred no foundation in positive reality. A real possibility of any result consists in something given as actual, which, under conditions of known nature and not known to be impossible, would give that result. A gun forms a real possibility of shooting, if there is no reason to suppose that cartridges are not to be had; an acorn is a real possibility of an oak, if we know of nothing to hinder its being planted and growing. But in the case supposed we have nothing like this; we have simply nothing—a failure to find incompatibility. Therefore we ought, strictly speaking, to conclude not that it is possible, which is an affirmation, nor even that it is not impossible, which borrows the form of intelligible denial, and therefore presupposes a positive ground of denial, but simply that we do not know it to be possible. This conclusion gives its true value to the form of bare denial by making the ground of negation what in the case of ignorance it really is, an actual state of our own minds which excludes the knowledge in question.

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[1] 'Approximates,' for a suggestion, if intelligible, contains *some* ground in its mere conceivability, and thus affords material for intelligible support or refutation. But by far the larger element in the importunity of many suggestions is drawn from the fallacy of inferring from non-impossibility to real possibility.

Instances of assertion resting mainly on this confusion are to be found e.g. in expositions of the so-called modern Buddhism, the elaborate dogmatic fabric of which is chiefly protected from criticism by the impossibility of discovering any ground on which it may be taken to rest, and against which, therefore, a positive objection can be raised.

To put the point in other words, in the case before us it is not true that double negation is equivalent to affirmation. For the double negation in question is founded neither on the affirmation itself, nor on the denial of a specific alternative to the affirmation. If there were such a specific alternative to the affirmation, it could only be denied on some positive ground, and such a denial would not be the bare denial of which *ex hypothesi* we are treating.

Or again, without appealing to the difference between privation, i.e. bare denial, and exclusion—a confusion between which processes however is the root of the fallacy—we may simply lay it down that a real possibility is something actual, and that a bare denial affirms nothing as actual, and therefore a bare denial cannot affirm a real possibility.

And it must be added with reference to remote suggestions generally, that a failure to demonstrate impossibility can almost always be secured by a high degree of remoteness or of abstractness in the suggestion itself. Thus, if accepted as really possible because not demonstrably impossible, such a suggestion would profit by its own wrong. Reality cannot, for us, contradict a suggestion that has no point of contact with reality. Things in themselves, according to the popular notion of the Kantian doctrine, are the content of such a suggestion. They made no claim to affect knowable Reality, and therefore knowable Reality can present no quality which excludes them. As a rule, to disprove the grounds on which a fact is advanced is not the same thing as disproving the alleged fact; denying the antecedent does not amount to denying the consequent. But in the case of unverifiable suggestions the grounds which are implied in the suggestion are for a given state of knowledge the sole grounds conceivable, or at least the grounds which can be stated are capable of exhaustion, and the disproof of them may be taken as for that state of knowledge disproving the suggestion. It must be remembered that denying the antecedent does deny the consequent *qua consequent of the antecedent denied.* At any rate it is clear that in such cases as those under discussion the failure to prove impossibility which arises from the emptiness of the suggestions themselves must not be taken as amounting to the establishment of real possibility.

Privation and Impossibility.

β. We cannot go from bare denial, or privation, of an impossibility, to real possibility, because *privation* of impossibility does not involve positive affirmation as the *exclusion* of impossibility does. It may seem therefore that we have decided in the negative by anticipation the question whether privation or bare denial can ever justify exclusion. We denied the claim of privation to establish real possibility, on the ground that it could not exclude, as one form of the ground that it could not affirm. Thus when we omit the incompatibility or impossibility, and consider merely the exclusion of any positive content, it seems that we cannot predicate exclusion on the strength of privation. Up to a certain point common sense and experience support this result. Gold has never been found in Northumberland, but that alone does not prove that it never can be found there, unless the geological formation forbids, i.e. is a positive ground of exclusion.

Still, the two transitions, from privation of impossibility to affirmation of real possibility, and from privation of actuality to exclusion of possibility or of actuality, do not stand on quite the same level. The former is explicitly a passage from denial to affirmation, or, as we saw, from nothing to something. The latter retains its negative form and *vehicle* unchanged, and leaves the change of its *ground* and *bearing*, the two other elements of meaning,[1] to be moulded by the context. Any mere denial or privation when expressed intelligibly is given a ground and bearing with reference to our cognitive state, as we saw in dealing with the instance, 'We do not know that there are things in themselves.' What is a bare denial with reference to reality is a positive affirmation about our knowledge. We have agreed that we cannot transform 'We do not know it to be impossible' into 'It is possible'. Can we transform 'We do not know it to exist' into 'It does not exist'?

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[1] See above, p. 289.

It is plain, I think, that many beliefs become abortive and cease to be regarded because we become convinced that reality does not constrain us to accept them. And it seems an unworthy shirking of a theoretical difficulty to treat these beliefs as shelved but not denied. If, whether in serious speculation or in grave practical deliberation, some idea cognate with the matters then before the mind has ceased to exercise the slightest bias on thought or on action, it seems idle to say that upon such an idea we have suspended judgment as upon something that may be true in itself but is nothing for us. It seems clear that upon such an idea—I may instance the Swedenborgian hierarchy of spiritual beings—we have in fact sate in judgment and have condemned it as unreal. But I admit that the speculative expression of our relation to such conceptions meets with serious difficulty from the necessity of basing denial upon a positive ground. On what positive ground can I base a denial that there are exactly seven heavens, or that there are just seven orders of superior spirits? I unquestionably do deny it, that is to say, 'I do not believe it.' The habitual use of such phrases as this,[1] which refer grammatically to a fact of my intellectual state, but actually serve as negations of something ascribed to reality, bears witness to the connection which I am attempting to point out.

Incompatibility in the ordinary sense depends on the system of reality. Differents which claim the same place are incompatible, and, in short, everything is incompatible with reality which, while not conforming to our ideal system which stands for reality, is yet without the power to modify it. But, as I pointed out above, all this falls to the ground where the system does not extend. Where 'we do not know' in the sense of having no knowledge—not merely in the sense of lacking complete knowledge—we can say nothing, and *ex nihilo nil*. 'Then,' it may be said, 'neither acceptance *nor rejection*.' I cannot follow this. Knowledge is positive, and acceptance and rejection are not co-ordinate alternatives. We

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[1] Compare *οὔ φημι*, which means 'I deny', or our common phrase 'I don't think that'—which is really equivalent to 'I think that—not.'

doubt, where we have a basis of fact to go upon, and presumptions that appeal to that basis; but where we have nothing to go upon we cannot doubt.

The only conclusion that I see open to us is of this nature. Where privation seems to warrant exclusion, we must look for the positive ground of exclusion in the determinate completeness of our ideal Reality. Such a ground may be hard to state, and may amount in positive content to little more than our experience of the persistency of the privation. But its nature must be that the ideal Reality by its organised completeness excludes the matter which attempts to introduce itself. This must not be taken to mean that we are nearly omniscient. It means that the general plan and growth of our knowledge is such as to afford no basis of attachment for the proposed accretion, although for this very reason we are unable to specify any definite antagonism between the content suggested and the positive contents already accepted as part of Reality. Reality is not especially incompatible with seven heavens; it could be so only if we accepted some kind of heaven as a reality and were prepared, on the basis of our knowledge about it, to reject either the particular number seven, or the application of number to such a subject at all. What we really have to say is that we do seem able to trace, however imperfectly, something like a development of the sensuous into the spiritual world; and that the main lines of this development appear to have a completeness of their own, without growing out into a duplicate and quasi-material world.

It is not enough to destroy the grounds on which a suggestion is explicitly based, unless we can show that they form not merely the sole ground alleged, but the sole ground possible. And in a region beyond our knowledge this *ex hypothesi* cannot be done. It is often possible to show by what logical fallacy or by what psychological tendency a suggestion was generated; but this is not a logical refutation. It may however grow into a refutation if, besides the tendency which caused the error, we can exhibit the satisfaction which reality offers to the rational necessity embodied in that tendency. We thus not only destroy the *raison d'être* of the error, but show a presumption that there is an excluding ground.

Exclusion by Privation then rests on a conviction, won by persistent lack of affirmation, of the *true* negative limit and external contour of knowledge, which limit, *qua* the true limit, must be held true of reality. A privation cannot ultimately be referred to our knowledge only. If persistent in the history of thought and justified by the tendencies of knowledge, it must sooner or later be taken as true of reality..

At best, we must remember, negation is always negative. The last step from the positive ground to its formal expression by means of denial, retains the form of privation, i. e. of ignorance. This is what the old saying means, 'you cannot prove a negative.' The negation is not after all quite the same as the positive opposite latent under the negation. You cannot prove that parallels never meet. In order to do so, you would have, like the Irishman, to ' be there when it did *not* happen '. You can only prove that they always do this, that, or the other which in virtue of your geometrical experience you take as equivalent to not meeting. That is to say *assuming* your geometrical system to be *ad hoc* exhaustive, then your failure to see, on the basis of that system, how parallels can meet becomes knowledge though it retains the form of ignorance. It expresses a limit or outline essential to geometrical science. Thus the cases of persistent privation and of true positive exclusion (genuine denial, not bare denial) differ simply by the nature of the positive ground which underlies them respectively. In privation this ground is general, drawn from the character and tendencies of Reality; in true exclusion it is special, drawn from a system within which the alleged reality would fall. It would seem fair to concede to the former somewhat more and to the latter somewhat less finality than common theory recognises.

CHAPTER VIII

DISJUNCTION AND THE STATEMENT OF CHANCES

The Disjunctive Judgment.

1. THE Disjunctive Judgment combines in an explicit form the characteristics of the Generic and of the pure Hypothetical Judgments.

Its Genesis.

i. It is needless however, and would be artificial, to lay down rules for the precise mode of transition from these judgments to the complete Disjunction. The whole assertory state within which the simpler forms of judgment, at any rate from Comparison upwards, have their import, is from the first of a disjunctive nature. Reflection may therefore be stimulated to the explicit formulation of this type of knowledge by very various occasions. But the common ground which must operate in all these occasions is the discovery of differences into each of which the identity underlying all of them enters as a whole, and in all of which taken together its manifestations are exhausted. Every difference has the former of these characters in some point of view. A conjunctive judgment, or conjunction of judgments with an identical subject, can always be made disjunctive by wilful abstraction. A diamond is carbon, and crystalline, and very hard, and highly refractive. This is a conjunctive judgment or set of judgments.[1] But if we limit the underlying identity, the nature of the stone, by the several conditions under which it exhibits these several predicates, then each of these predicates may be regarded as not conjoined with but exclusive of the other attributes enumerated. A diamond may be considered either merely as an element, or merely as a transparent substance exhibiting crystalline structure, or in its power of scratching other hard substances, or in its effect upon light. This is disjunction—arbitrary and subjective, if we please, but still disjunction. Any distinguishable attributes may be regarded as reciprocally exclusive by our simply refusing to attend to more than one of them at once. *x* may be both *a* and *b*, but qua *a* it is not *b*. But in another respect also the above instance of disjunction is bad and arbitrary. It makes no serious attempt to exhaust the attributes. We stop enumerating them simply because we do not care to go on.

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[1] See above, chap. 1, on the Judgment in Time.

It is clear that a disjunctive judgment may originate with a conjunction of judgments like the above, which turn out to have assignable relations to one another, or, again, with the discovery of an error in a generic judgment ; e. g. '*Cereus* is a night-flowering plant.' 'No : *Cereus-grandiflorus* is a night-flowering plant, but there are a hundred species of *Cereus*, and not all are night-flowering.' Such considerations would force upon us the disjunction, 'Cereus blossoms either at night, or in the early morning, or &c., &c.' Again, the attempt to convert a generic judgment naturally leads to a disjunction.[1] And so does the challenge thrown down by the specific condition and consequent of a pure hypothetical judgment. 'If we catch the train this morning we reach London to-night.' In presence of such an assertion it is not in human nature to abstain from asking, 'And if we miss the train, what then ?' so as to make explicit the disjunction, 'We either catch the train or miss it ;' and probably some such further consequence as, 'We either get to London to-night, or have to sleep at Crewe.'

All these are merely different ways of giving utterance to the interest which attaches to some pervading identity and compels us to pursue it throughout its modifications. Such an interest, as we have seen, environs every genuine judgment, and makes it an element in a system. And in proportion as such a system is made explicit, negation enters into knowledge. For in every system the parts have an aspect of negative relation to one another.

Thus the immediate occasion on which we form a disjunctive judgment may vary. But the characteristics of true disjunction do not vary. By true disjunction I mean a judgment in which alternatives falling under a single identity are enumerated, and are known in virtue of some pervading principle to be reciprocally exclusive, and to be exhaustive.

[1] See above, chap. vii, on the Conversion of generic judgments.

This is the disjunction of which I shall treat, being convinced that what seem to be different kinds of it are in fact nothing but imperfect examples.

Imperfect Disjunctions.

ii. I will mention three of these.

Not exclusive.

α. First, there is the so-called disjunction in which the alternatives are exhaustively enumerated, but are not taken to exclude one another. 'He is either knave or fool' does not, it is said, exclude the possibility that he may be both. On the question of the genuineness of these disjunctions I must refer to Mr. Bradley's detailed discussion,[1] which appears to me to show decisively that we never *really* mean to take into consideration under our judgment the conjunction of the alternatives specified in our 'Either'—'or'. Without following him into the study of grammatical details, I content myself with remarking that any Disjunction in which the alternatives are not reciprocally exclusive must of necessity fail to be exhaustive—the case or cases in which any of them are conjoined being *casus omissi*. If, for the purpose of the disjunction, such a case may be reckoned under one of the other alternatives, then indeed the disjunction may be reckoned to be perfect; but then the case of conjunction does not rank *as* conjunction, but under one or other of its component elements. Thus, 'To go by train you must have either a first, second, or third class ticket.' A man may of course buy all three, if he pleases; but the possession of them does not constitute a fourth case of liberty to go by the train. He goes by the train in virtue of one or other, though he may change carriages at every station if it amuses him. The conjunction of all three tickets forms no separate alternative as a particular way of going by train, and therefore is rightly disregarded in the disjunction. It is not indeed implied that 'If he has a first-class ticket he has not a second or third', but it is implied that 'If he goes in virtue of a first-class ticket he does not go in virtue of a second or third'.

Disjunctions of Ignorance.

β. Secondly, there are the troublesome cases often taken as the true instances of Disjunction, which may be called 'Disjunctions of ignorance'. The essence of these is that they refer to an individual (actual or supposed) and not to an individuality, and consequently express doubt or indecision rather than knowledge.

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[1] Principles of Logic, Book I, chap. iv.

'A triangle is either isosceles, scalene, or equilateral.' A triangle here *can* mean any individual triangle you may pitch upon—'any given triangle'. And with respect to such an individual triangle the disjunction says that it must belong to one of the three kinds mentioned, but that we do not know to which. Lotze, for instance, appears to accept this expression of indecision as the final type of disjunction But it seems obvious that this uncertainty is purely dramatic or fictitious, and is a mere corollary from the true disjunction, which is, 'A three-sided plane figure as such must have all its sides equal, or two only equal, or all unequal.' Or we may take a case where the doubt is real, as often in common life; but here also it is a mere application of or inference from the true disjunction of knowledge. 'Being an Oxford man, he is either a University College man or a Balliol man, or &c.' This judgment, which is a real expression of doubt or ignorance, is based of course on the positive knowledge that the conditions of University life require the student as such (generically) to attach himself to some one of the corporate bodies enumerated in the judgment. With disjunctions of this type we must class the commonest of all expressions of doubt or ignorance. 'He is either angry or jealous.' 'He has either measles or scarlet fever.' These, like the above, differ not in principle but only in perfection from the ideal disjunction. What operates is something we know, and know to contain the specified alternatives. We do not however specify our knowledge in detail--it may consist in a content hard to define—and we merely point to the concrete individual, in whom it is embodied and from whom it takes its interest. About this individual, as his complete state goes beyond our knowledge, the judgment takes the shape of doubt, just as in the case of 'any given triangle'. The higher logical form may be imposed on the lower content; this is a possibility which follows from the nature of reason, and which makes abstract distinction appear in some aspects so valueless. Every thinkable content has in miniature all the characteristics of reason, and in the abstract you can hardly say anything of the self-conscious mind which is not also true of a protococcus or of a pebble. The precise modes and degrees in which the content fulfils the spirit of its abstract form are what we must keep our eyes on in a true concrete science. I cannot admit then that the mere inference from our disjunctive knowledge respecting that which an individuality implies, to a doubt respecting a given or supposed individual, has a claim to rank as a genuine species of disjunction.

Disjunctions referred to point of time.

γ. It is a similar vice to account for the exclusiveness of a disjunction by referring it to a point of time. No judgment whatever refers to an atomic point of time; and no universal judgment refers to any time except that implied in the content of the judgment itself. The time at which the predication happens to be made has nothing to do with the import of any judgment except in as far as it is taken into the content by reference direct or indirect to present perception.[1] The denial of coexistence in time, which appears in some disjunctions to be the principal meaning, is a corollary from the nature of the disjoined contents, not a result of the present tense employed as a vehicle of predication. 'A railway signal shows to the same side either a red, or a green, or a white light.' Now of course this judgment informs us that at any given moment of time we shall only see one of the three lights. But to interpret the judgment as if it essentially referred either to the moment at which it is made or to 'any given moment' is a fallacy on all-fours with that pointed out above of interpreting 'The triangle' as = 'any given triangle'. The judgment means that the nature of a railway signal is to show one light to the exclusion of another and the other to the exclusion of the one. From the nature of the case they exclude one another in time and in spatial direction. But the present of predication is coextensive, in its reference, with the nature of the signal, and does not refer especially or exclusively to the moment at which the judgment happens to be pronounced, nor even hypothetically to 'any given moment'.

We must clear out of the way, then, the above disjunctions of ignorance, or dramatic disjunctions, and consider exclusively the perfect disjunction as a form of knowledge. One more peculiar type, indeed, will come before us later as the statement of chances.

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[1] See above, chap. v, on Time in singular judgments.

Logical affiliation of true Disjunction.

iii. A true or ideal disjunction is a generic judgment whose content is developed or inter-related by the aid of hypotheticals.

The Generic Judgment.

a. It is a generic judgment because it deals with an individuality, a content which is a whole or system in itself. So far, like the generic judgment, it is quasi-categorical. The subject is such as can be real, and the judgment assumes its reality. But the predication made of this real and quasi-individual subject is peculiar, and has points of analogy both with the negative and with the pure hypothetical judgment. The individuality is exhibited in the different forms which it is capable of assuming as a whole, and which consequently it cannot unite in itself under a single set of conditions. If the individuality is considered as essentially affected by time, these forms may be successive; if it is a generic or specific character, they may coexist in space and time with each other; all that is necessary is that the subject-content should enter as a whole into each of the enumerated forms. What, then, is directly predicated of the subject-content? I see no theoretical reason to deny that the 'either—or' considered as the articulate analysis of a universal[1] system of attributes can be intelligibly and categorically predicated of it. Every predication includes differences, and an extended present, and therefore predicates as facts elements which can be regarded as reciprocally exclusive. In fact, every universal is a synthesis of such elements. But undoubtedly there will always be positive qualities which are the condensed or summarised expression of the total analysis, and may present themselves as its ground, being thus in disjunction as the positive ground or bearing in negation, and as the underlying quality in the hypothetical judgment. 'The triangle is either scalene, isosceles, or equilateral' contains as this condensed relation, or synthesis of differences, 'a plane figure bounded by three sides, which may have any relative length so long as any two of them are together greater than the third.' Such an attribute might be called the categorical or positive basis of the disjunction;[a] but it is an illusion to suppose that a basis or ground is *necessarily* more real or more primary than its consequences, and that therefore the implied predication in disjunction is more categorical than the explicit 'either—or'. The ground is to its consequences as whole to part; but if the consequences are fully stated in a connected system this distinction falls away, and in disjunction such full statement is the ideal.

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[1] 'Universal' not in the sense of abstract or generalised, but in the sense of a concrete identity containing differences.

The disjunction is therefore the only judgment-form that in strict theory can stand alone. All connection is within a system; and only that judgment is self-sufficing which affirms at once the system and the connections within it. I do not say that every disjunction is thus ultimately self-dependent, but relatively to a number of hypotheticals which have their truth within it every true disjunction has a substantive character. Thus the disjunction which lays down the nature and kinds of the triangle contains the ground and basis of all the hypothetical judgments which expound the properties of that figure. In other words, if completed and made explicit, any one of those hypotheticals would result in that disjunction; which however itself falls within the ultimate judgment that would expound the nature and modes of space. The above then is the generic or substantive element in disjunction.

The Hypothetical Judgment.

β. But we need in addition the reciprocal relations between the forms which constitute the explicit development of this predicated universal. For these relations we must go to the hypothetical judgment, and to the hypothetical in a very late and artificial form, viz. that in which the negation of one content is known as the ground of the affirmation of another content, and the affirmation of the one content is known as the ground of the negation of the other content. The perception of the relations which these two types of hypothetical judgment embody is essential to the exhaustiveness of the disjunction and to the reciprocal exclusiveness of its members. In order to know that the alternatives enumerated are reciprocally exclusive, we must be able to say (using as an illustration the simplest case in which there are only the two alternatives B and C), 'If A is B, it is not C.' And in order to be sure that no possible alternative is omitted we must be able to say (in the same example) that 'if A is not B, it is C'. According to a rule of Conversion, or rather of Inference, accepted for the case of Hypothetical Judgments ('Deny the consequent') the equivalent or converse judgments, 'If A is C it is not B' and 'if A is not C it is B' are involved in the two corresponding judgments above mentioned. If this Conversion or Inference is disputed, then we must say that all four cases, 'If A is B—,' 'If A is C—,' 'If A is not B—,' 'If A is not C—,' must be perceived independently before the predication of disjoined alternatives is justifiable.[1]

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[a] See Appendix to this chapter.

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[1] I insert here some details which are legitimate matter of curiosity, but would needlessly overload the text.

i. It might be urged, on the analogy of the argument employed above (p. 324)—that a disjunction which is thoroughly exhaustive cannot but have its members reciprocally exclusive—that the hypothetical which *prima facie* secures exhaustiveness (If A is not B it is C) ought to affirm the *reciprocal* exclusiveness of the antecedent and consequent, i.e. to exclude the case B C. If it did so, on the other hand, it would become at a blow equal to the disjunction 'A is either B or C,' and would include in itself the case 'If A is B'—with its converse. In other words, it would become a *reciprocal* judgment, correlative to a definitory affirmation, and as such would admit of conversion or inference *by denial of the antecedent*; just as if we were to infer from 'A is B' that 'not-A is not-B'. This, as we have seen all through, is the ideal of the judgment; and a hypothetical judgment with negative condition or negative consequent, that fulfils this ideal, coincides already with the disjunction. But usage does not warrant the ascription to the hypothetical 'If A is not B it is C' of the meaning 'If A is not B without-C it is C without B'. As commonly employed, therefore, it lays down a certain outer limit, but does not exhaust the subdivisions within the limit. This is just the point of contrast between the hypothetical judgment in its ordinary signification and the complete disjunction. But there is a certain tendency on the part of the former to advance towards the latter. It is plain that the reasons which induce us to give prominence to the alternatives mentioned as the only ones to be specially considered may readily transform themselves into reasons why only the alternatives mentioned *can* be considered, or, perhaps, can exist. We have such reasons *just* warranting a disjunction in the instance given above (p. 324), where the case formed by the combination of the alternatives considered exists, so to speak, in fact but not for the law.

ii. It may be worth while to point out that disjunctions with more

The reference to these two or four hypothetical judgments has its value in elucidating the nature of the system which a true disjunction embodies. It exhibits in the plainest light the indispensable function of negation in articulated knowledge, and the positive import with which in virtue of that function the negative is invested. We have already seen the nature of this import in the analysis of the significant negation, where, however, the positive ground and consequence of denial were matters of tacit understanding and inference from context. In explicit disjunction, on the other hand, we find them after they have been developed independently and distinctly in the hypothetical judgment, and affirmed as actual attributes within a system that is alleged to exist in reality.

When are parts disjunctively related?

iv.* But, it may reasonably be objected, it is not in every system that the parts are disjunctively related. As a rule the parts of a system are predicable in conjunction and not in disjunction.

Apart from the case of intentional abstraction by which any conjunction can be turned into a disjunction—for in the last resort within every system every part involves the whole nature of the system—this criticism is just. A human

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than two alternatives must be treated, as regards the hypothetical judgments involved, as a succession of dichotomies. The hypothetical judgments of each type involved in such a disjunction would therefore be equal in number to the disjoined members, i.e. each alternative must be made in turn the positive and negative condition of an hypothesis, with a compound consequent, the disjunctive nature of which cannot appear in the hypothetical judgment. A is either B, C, D, or E. Then we have the negative conditions, 'If A is not B, it is within C D E;' 'If A is not C, it is within B D E;' 'If A is not D, it is within C B E;' 'If A is not E, it is within C B D.' The positive conditions will correspond severally to the above negative conditions.

Here we see a second defect of the hypothetical as compared to the disjunctive judgment. It can only handle one reciprocal relation at a time, and cannot master a whole system of such relations in a single view. In the above analysis the hypothetical judgment does not enable me to express more than a single contradictory relation, as between a particular A and its not-A. The idea of a pervading contradictory relation, characterising *any* one part as against all the others, cannot be expressed in any one hypothetical judgment. By saying 'If A is not B, it is *either* C, D, *or* F' we should be pressing the hypothetical type beyond its powers.

body is made up of trunk, limbs, and head; not of *either* trunk *or* limbs *or* head. The government of a civilised nation consists of the legislative *and* the executive power, not of the legislative *or* the executive. A genus, again, may be said to be identifiable with *all* its species, not merely with *either* this *or* that, though here we are on more doubtful ground. Triangles are isosceles, scalene, *and* equilateral. Men are white, black, *and* yellow. We could hardly say however that '*the triangle is* isosceles, scalene, and equilateral', or that 'man is white, black, and yellow'. The difference between conjunctions of the kind here brought forward, and true *disjunctions*, is formally speaking a difference of the aspect in which a real system is regarded, but materially, therefore, has an intimate dependence on the actual nature of the real system in question, which may be such as to throw one aspect or another prominently forward. Every universal may have its differences conjunctively enumerated, whether they are in time or in space, or merely distinct in thought. But in so far as the universal itself enters as a whole into each difference, which it can do in very different degrees, so far each difference, if imposed as a condition on the universal, excludes all the other differences. A man's having a hand does not interfere with his having a foot. But a man's having a feeling in his hand does begin to make a claim on the universal, the man himself, which is to a certain limited extent incompatible with his having a feeling in his foot or elsewhere. And when we come to consider such acute interest or feeling as occasions the absorption of the whole mind in the perceiving or suffering member, then it is true to say, 'The man perceives or feels *either* with eye *or* with ear *or* hand *or* foot,' as the case may be. So again if we think of a triangle as a mere abstract generality that describes a heap of various figures, we may say that it includes, or that—in a collective judgment—its individual instances *are*, this, that, and the other. But if we think of it according to its complete conception as an individuality that must necessarily take individual shape, and if we follow the process by which such shape must be determined, then we can only express our insight by the use of the disjunctive 'Either—or'.

The conception of the whole as conditioned by one of its parts takes the place of that imaginary reference to an atomic point of time which has been supposed to be of the essence of disjunction. 'A moving object is either here or there' means 'if here, not there' and 'if not here, then there'. It does not mean 'Within the indivisible moment in which I am judging a moving object can be in one place only'. For I cannot judge in an indivisible moment, nor can I refer to a present that is an indivisible moment. In any extended time a moving object traverses space, and our 'present' is always an extended time. And so the disjunction if referred to our 'present' time would not be true, and the moving object would be, like Sir Boyle Roche's bird, 'in two places at once.'

Is the disjunction reducible to Hypotheticals?

v. When we have understood the nature of disjunction there is not much profit in asking whether the disjunctive judgment can be 'reduced' to two or more hypothetical judgments. The mere fact that the hypotheticals in question[1] are separate judgments, and that the disjunction is a single judgment, is enough to show that we have in the latter a *combination* of unity with reciprocal exclusion which we have not in the former. I have endeavoured to express this unity by representing the disjunction as a combination of the generic and the hypothetical judgments. But it must be remembered that at best we are dealing with grammatical types which are only the symbols of states of knowledge; and it is most probable that any one who is able to make the two complementary hypothetical judgments 'If A is B it is not C' and 'If A is not B it is C', supplies out of his own mind the systematic relation which the two taken together involve, in a judgment equivalent to 'A is either B or C'.

Not to dwell longer than I can help on formal points, I merely remark in addition to what was said above,[2] that in any case these hypotheticals themselves presuppose *the* ultimate or formal disjunction, 'A is either B or not-B,' by their introduction of a negative relation into knowledge; and that further, if we wish to take the two hypotheticals above mentioned as implying the two which follow from them by denying the consequent, we are once more relying on this formal disjunction, which is essentially involved in such a transition.[1] The material importance of the whole question lies in the process by which the *form* of disjunction, in itself on a level with or consisting in the Law of Excluded Middle, i. e. of contradictory opposition,[2] acquires the material significance of Disjunction between positive contraries. I have tried to show above that these two elements, the bare rejection and the positive contrary, are probably to be regarded as distinguishable from the first, but as tending to coalesce, and not as later and earlier phases respectively of the same movement. Or if, in history, earlier and later, then the later, the abstract formal negation or bare rejection, is to the earlier, the actual choice between positive alternatives, as a separable outgrowth which consciously reunites with it in the region of reflective intelligence. No anthropological doctrine can affect—though it may elucidate—the above logical analysis of the relation between the negative and its material import as made explicit in the judgment.

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[1] See footnote, p. 329.

[2] p. 329.

The statement of Chances.

2. The statement of chances is a case of the Disjunctive Judgment.

Limits of the problem in present work.

i. The title which I place at the head of this section indicates the limits of the question which I propose to treat in it. The calculus of chances, like all mathematical reasoning, has at its root an inference that can be expressed in ordinary language. In treating of inference it will be incumbent upon us to discuss the differentia which separates calculation from ordinary reasoning; and we shall find the outward and visible sign of this differentia to consist in the enormous abbreviation of reasoning processes by their condensation into the import of recognised symbols. It is a further question in virtue of what peculiar nature an inferential process can submit to such an abbreviation, and also to what extent the abbreviation has the effect of substituting something else, e. g. application of a rule-of-thumb,[1] for the reasoning process so 'abbreviated'. But the value of any such abbreviation must ultimately rest in a logical sense upon the reasoning which it represents, and this reasoning must be in its nature explicable in language like any other reasoning.[2] Thus in the calculus of probabilities, though I am obliged from mathematical incompetence to omit much that might be of interest to an expert even from a logical point of view, yet the principle of the statement of chances is not a matter of technical method, but of fundamental postulates of knowledge. And also, no doubt, it illustrates the necessity by which totalities of a certain degree and kind of abstraction become subject to numerical manipulation.

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[1] See the account of Contraposition, above, chap. vii; the process by 'denying the consequent' is essentially the same with this.

[2] It is an unlucky confusion that the so-called law of Contradiction only explains Contrary opposition, and that it is the law of Excluded Middle that lays down the principle of Contradictory opposition. See Book II, chap. vii.

Affiliation of abstract disjunction.

ii. The statement of chances rests upon a species of disjunctive judgment, but not on what we have spoken of as the true or real disjunction—which might also be called, in contrast with that which we are about to discuss, the concrete disjunction. The concrete disjunction, in as far as it reaches its ideal, embodies differences that are distinct and individual modifications of the underlying system, and provides in the nature of the common subject a complete account of the conditions which determine it to each of these differences. In using such a disjunction we know precisely how and why the whole or real subject must enter into each of the differences which constitute it. And though it may be said that we do not or need not know when or how far each *condition* involved is or can be *real*, yet we must know something of the relation of such conditions to the reality of the system which they affect (because their reality is partly relative to its reality), and at least there is no sort of reason for supposing that the reality of these known conditions is to be taken as an equal amount in the case of all the several alternatives. The variety of the world and of all reality throws the whole presumption the other way. The idea of equally grounded alternatives is a negative idea, and can only exist by a defect of knowledge, or by an abstraction from what we know.

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[1] I have in my mind as an instance the use of tables of logarithms. It does not appear formally essential that any one who uses them should understand the reason of the rules he applies, i.e. (I presume) the nature of Indices

[2] A *practical* reservation must here be made in considering the higher mathematical processes from this point of view, because they may presuppose a number of stages consisting of subordinate processes, and be inexplicable apart from these latter. But to explain the whole complication in ordinary language might involve a lengthiness that would make it harder to follow such an explanation than to master the proper mathematical language.

The abstract disjunction, on which alone a statement of chances can be based, cannot be a system of alternatives whose conditions and relations are thoroughly understood. It is rather allied to what was spoken of above as the disjunction of ignorance. The affiliation to the hypothetical judgment is indeed the same in all disjunctions that are formally perfect, i. e. both exclusive and exhaustive. But on its other side the abstract disjunction does not, like the concrete, descend from the generic judgment with its penetrating and dominating individuality, but rather from a judgment of enumeration such as the collective judgment, with its homogeneous parts which do not occupy individual or distinctive relations to the containing identity. It is true that to give meaning to any disjunction, or to a statement of chances founded upon it, the parts or members of the whole must be distinct as well as homogeneous. But the distinction is in this case mere distinction, interesting as a result, like the differences between the six sides of a die, but on the side of its relation to the whole not rooted in any known heterogeneous modifications of that whole. In other words, the *number* of parts, or the fact that each is *one among so many*, is the primary fact, and their *nature* is secondary. We may illustrate this by contrasting an example of a concrete with one of an abstract disjunction. 'The constitution of a modern nation,' it may be said, 'is necessarily either democratic or plutocratic.' Here the fact that the species assigned are two in number is of no importance. No one would think of trying to infer anything from it as to how many nations were likely to be democratic and how many to be plutocratic. The whole weight of the judgment rests upon the component elements implied in 'modern nation', and upon the development of those elements which the judgment indicates to be necessary. But if we take such a judgment as 'A die must turn up one of its six sides' we here regard the individuality of the several sides as indifferent with respect to the probability of their recurrence, though not with respect to its results. The important matter is the *number* of the alternatives. For either we are unable to estimate the operative causes which determine one alternative rather than another, or we wilfully abstract our attention from them for the sake of falling back on a more general process of estimation. We are to suppose then that as the basis of our statement of chances, we have before us such a formal disjunction as the above, closely akin to the judgments which arise in the process of enumeration, but with the addition of those known relations between the enumerated parts which are embodied in the hypothetical judgments with negative antecedent and consequent respectively.

Essence of the statement of chances.

iii. We are then in a position to enter upon a process which I can only describe as taking stock of our knowledge by arithmetical methods. We know that the die has six sides and no more, and that as the result of a single throw it must turn up one of them. We know that we do not know of any cause operative in the nature of the die or in the conditions of the throw that should favour any particular side, nor of any grounded presumption whatever in favour of any one side in particular. In the case of the die, which herein differs from many cases of the statement of chances, we may say that we *know that there is not* any permanent operative cause either in or outside the die that can favour one event in particular. Therefore—and here is the all-important step which really constitutes the statement of chances—we go from 'no known inequality of the grounds for affirmation'[1] to 'equality, *so far as follows from this knowledge*, of the grounds for affirmation' of the several alternatives. Having made this step, i. e. having placed the grounds for each alternative on a level as equal, we are of course free to treat them as units, and a ratio expressing the relation of each to all follows as a matter of course. Each alternative counts for one, and none for more than one. In other words, the ground for the affirmation of each, assuming the reality of the common subject, is represented by a fraction of which 1 is the numerator and the total number of alternatives the denominator. We do not attempt to say what the ground is, but we say that, by the terms of the disjunction, it is one out of so many equal and reciprocally exclusive grounds. This transition from indifference of formulated knowledge to equality of grounded affirmation, and so to the relation of units within a sum total or fractional parts within unity, is the logical foundation on which the statement of chances rests.

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[1] Judgment about the future, *if* we judge, is of course affirmation as much as judgment about the past. But it is not essential to the statement of chances to refer to the future. They can be stated on given premises about any event, although its real issue be known.

Applications of the statement as calculus.

iv. All its developments are applications of this principle. *a*. For instance, it may happen that alternatives which are separate units in respect of the ground for affirming their reality are identical in respect of that result of which the chances are being stated. It follows that the chances of this result are represented by as many units out of the sum total as there are equal alternatives which produce it. In other words, the chances of the common consequence of a number of alternatives whose chances are known is the sum of the chances of those alternatives. If you bet with me that you will throw a six first throw with a single die, then all five alternatives from one to five inclusive have annexed to them the consequence that I win the bet. The total number of alternatives being six, I have thus $\frac{5}{6}$ of the chances in my favour, or the chances against you are 5 to 1.

Alternatives and Results.

The further processes of the calculus must always reduce themselves to obtaining a correct enumeration of the equal alternatives, and a correct estimate of the number of those equal alternatives which have annexed to them the result whose chances are to be stated.

Physical alternatives.

β. The combined chances of independent events illustrate both these principles very simply.

To find the physically different throws that are possible with *two* dice, we must take into account which is which of the two dice. It is obviously possible in a single throw for any side of the one die to concur with any particular side of the other die, i. e. writing down the six sides of the one die as headings, you will have to write down all six sides of the other as possible cases under each of these headings. This relation is of course expressed numerically by multiplying the whole number of sides of one die by the whole number of those of the other: $6 \times 6 = 36$. This gives the correct enumeration of the alternatives that are physically possible, the chance of each in case of a single throw being $\frac{1}{36}$. Generalising this process, we may say that in two or more independent sets of alternatives the chance of the concurrence of two or more particular events, as many as there are sets concerned, is determined by the product of the numbers of alternatives forming each separate set multiplied together. That is to say, the chance of any particular concurrence of events, consisting of one out of each independent set, is 1 divided by the product in question.

Interesting results.

γ. But to return to the example of the two dice, it may be that the 36 possible concurrences would not all count as different, because e. g. the throw 2 with the die *a* and 1 with the die *b* may be treated as the same with the throw 1 with *a* and 2 with *b*. Therefore the results which are the alternatives according to this mode of counting have not all of them the same chances in their favour, i. e. do not severally contain the same number of physically distinct alternatives. Each of the six throws 1 and 1, 2 and 2 &c. up to double sixes inclusive corresponds to one physical alternative only out of the possible 36, and therefore has only the chance represented by $\frac{1}{36}$. On the other hand, each of the fifteen 'throws' 1 and 2, 1 and 3 to 1 and 6 inclusive, 2 and 3, 2 and 4 &c. to 5 and 6 inclusive, is a result annexed to two physical alternatives (1 and 2 or 2 and 1, &c.), and therefore counts as two equal cases or units, and has a chance represented by the sum of the chances of these equal cases, viz. by $\frac{2}{36}$ instead of $\frac{1}{36}$. These 15 'throws' then, answering to two actual alternatives apiece, exhaust the 30 real cases that remain after deducting the six doublets, and the whole 36 alternatives are thus accounted for. Or again, if we take into account merely the number of points thrown at each throw, without regard to their distribution between the two dice, we get six combinations of throws that will give seven points, five for eight points and six points respectively, and so on to one combination for two points and twelve points respectively. The chances of throwing 7 are therefore $\frac{6}{36}$, and of throwing the other numbers $\frac{5}{36}$, $\frac{4}{36}$, and so on down to $\frac{1}{36}$ respectively.

What does statement of chances represent?

v. Logicians are not agreed as to the proper description of that which is expressed by the ratio that embodies a statement of chances. Their disagreement arises more from the subtlety of the distinctions involved, which makes description difficult, than from a substantial difference of opinion as to the relation between reality and the cognitive act in question. It is not unnatural, for example, to say that the ratio expresses our subjective expectation. But this is an obvious slip, because the whole process of the statement is undertaken in order to correct and control our subjective expectation, and is futile unless it does so. The complete counterpart of this idea would consist in maintaining that the ratio expressed an actual behaviour on the part of real things. I do not know that this suggestion has ever been made in this extreme form. Something of the same kind however is commonly believed with respect to the realisation of chances in a series, which I shall speak of directly.

The ratio of chance seems really to express the amount of ground, *which is afforded by the knowledge formulated in the disjunctive basis of the calculation*, for affirming the reality of the result whose chances are in question, on the assumption that the general case, which forms the subject, is realised. Instead of 'amount of ground' it would be more usual to say 'degree of probability'. And by avoiding the expression probability we do not really escape the tautology which it would introduce. For the idea of a measurable amount of logical ground, like the idea of a measurable degree of probability, is only intelligible with reference to the statement of chances.

'Expectation' sounds more like a term which might elucidate the definition, and if we say that the statement of chances represents *the expectation which is justified by the premise*, we might not be far wrong. But I do not feel sure that anything can be meant by a degree of expectation except the mood, whatever that may be, which is founded upon a statement of chances. So that the chance would not come from measuring the expectation, but the expectation from measuring the chance. Even if we identify expectation with judgment—a proceeding which is more than doubtful—we cannot say that the chances necessarily represent the degree of certainty *with* which we judge [a] in a statement of chances; because the statement, giving due numerical weight to every equally-grounded alternative, asserts in the proper ratio the reality of *all* the alternatives, and is in this aspect a judgment with its conditions made fully explicit, and therefore necessary or apodeictic. Not that a statement of chances is usually or ever true in fact; but this divergence from fact, or abstract character, affects the categorical aspect of the whole disjunctive basis of the statement, the truth of which basis is in no way affirmed or impeached by the ratio of chance founded upon it. The chance that one side of a 6-sided die will turn up is $\frac{1}{6}$; but this says nothing about the certainty that there is a 6-sided die. Paradoxically enough, the statement of chances seems to measure, if anything, the degree of certainty of a problematic judgment made without knowledge of or in abstraction from the statement, as to the probability of a single alternative out of a number without reference to the remaining alternatives. Not that a statement of chances can govern the meaning of a judgment made in ignorance of it, and it is indeed hard to see any meaning in degree of certainty apart from measurement by the enumeration which gives a ratio. But on the other hand, the moment we have the ratio, we have with it the *whole* consequence of the assumed reality, and the judgment which asserts the ratio is mere arithmetical necessity, the nature of the assumed reality being given. The judgment that 'the odds against any one side of a (6-sided) die turning up at one throw are

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[a] It is rather 'the degree of certainty *which* we judge,' i. e. affirm. It is beside the point to discuss the probability of the truth of the judgment which states the chances. It is the degree of certainty of a particular alternative happening, not that of the judgment which states the degree of certainty, that is represented by the chances. This is the view of the text in the end, and I let the discussion stand, because it illustrates a possible error.

5 to 1' has not a probability as 5 to 1, but on the premises is necessary. But it determines what I *ought* to mean, when I say, in ignorance of or not considering the calculation, 'it is unlikely that a six will turn up the first throw.' If I reflect and say, 'It is unlikely (with an unlikelihood of 5 to 1) that &c.,' then my judgment has ceased to be problematic, and has become necessary, i. e. the conditions of its probability are analysed and made explicit. Thus probability as a character of judgment disappears when measured.

The ratio of chance then expresses the amount of ground for affirming, that follows from the knowledge formulated as the disjunctive basis of the reckoning. We may possess knowledge that does not conform to the conditions of the statement of chances, or at least that is not relevant to the special disjunction which we are able to employ. Such knowledge may cause us to distrust the reckoning slightly or wholly as it affects some particular case. I may think for instance that the calculated risk of being run down by a cab in the streets of London does not apply to one man who is in the prime of life and habitually alert, or to another who is bedridden and never goes into the streets at all. But none the less the probability of cab accidents for each individual of the population *on the data that are taken* is a mere question of calculation and can only have one correct answer. My private notion that I have supplementary data which ought to be considered, or that a more careful distinction should be made between classes of persons in the data, does not in the least affect the probability which flows from the premises in any particular calculation.

The calculus of chances, in short, bears the character of the judgments from which it is derived. Like the pure hypothetical judgment, and the greater part of the abstract judgments derived from the judgment of enumeration, it affirms of Reality indirectly and conditionally. Its truth is a truth of necessity, a consequence that follows from a selected or fancied character when taken as real. Such a consequence is not subjective or arbitrary. Given the premises, it can only be drawn in one way, and every other result from those premises is wrong. But yet it does not express actual concrete fact. It expresses a truth necessitated by the nature of Reality, but not as it stands embodying a fact of Reality. It is simply an arithmetical consequence of a highly abstract disjunctive enumeration.

Chance and actual series.

vi. In every statement of chances we admit our partial ignorance. If this were not so, the statement would involve a flat contradiction. For our grounds for affirming reality are equal in the case of all the alternatives, and yet our statement of chances is based on the assumption that only one of them can be realised.

Fallacies relating to series.

a. But if we do not bear in mind the proportion of ignorance which enters into our data, we are tempted into two fallacious attitudes.

In the first place, we consider ourselves justified in being astonished at the realisation of the alternative which has very few chances in its favour.

And in the second place, we palliate the apparent contradiction between equal grounds for reality and unequal realisation by affirming that the statement of chances has genuine truth only in an actual series which realises all the equal alternatives equally. To realise all the equal alternatives equally is of course the same thing as to realise all the interesting results[1] in the ratio prescribed by the statement of chances.

First, then, we have very small ground for being surprised at the actual occurrence of that alternative which had fewest chances in its favour; and absolutely none for being surprised at the occurrence of a marked or interesting alternative which has against it enormous odds, but only the same as against every alternative which can possibly occur. In the former case we are cherishing a private presumption that the knowledge embodied in our premises represented the actual operative causes which determined the realisation of one or another alternative, and this is *ex hypothesi* not the fact. In the latter case we are misled by a special interest into comparing, as if they were cases of which the chances should be equal, cases which are not 'equal alternatives', but 'interesting results' comprising unequal numbers of equal alternatives, viz. on the one side a single case which is in some way remarkable, e.g. a hand at whist consisting solely of trumps, and on the other side all other possible hands whatever, which we implicitly contrast, as a single case, with the opposite 'interesting result'. We are therefore surprised at the immense adverse odds in spite of which this result has been realised, not reflecting that there are precisely the same adverse odds against any one of the alternatives which occur in everyday experience, though not, of course, against all of them together.

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[1] See above, p. 337.

And secondly, the realisation of the ideal alternatives in a series of real cases is confessedly a fiction unless we stop the series at an arbitrary limit—say, for instance, the actual limit of individual objects or events in question in space or time—and even within this limit the series is not what we want.

There are under this head of an actual series two possibilities which *prima facie* at least must be distinguished. We may have to deal with a natural or deductive cycle of alternatives, or with an arbitrary or inductive cycle.

In what may be called a natural cycle the alternatives are ideal cases that follow obviously from the nature of the general subject; and are distinct from the real cases, the actual instances or events, which may or may not continue to present themselves beyond the one real case which is postulated in the statement of chances. The sides of a die or of a coin furnish ideal cases; the throws of a die or tosses of a coin are real cases. The natural cycle is the lowest number of actual events in which the ideal alternatives could be all realised equally; i.e. six throws in the case of the die, two tosses in the cases of the coin. Beyond this natural cycle there is nothing to suggest a limit to the series of real cases. We have therefore to ask, in considering the verification of natural cycles by experience, whether the real cases correspond to ideal cases (i) within every natural cycle that is observed; and (ii) in the series as a whole continued without limit.

In what, on the other hand, may be called an arbitrary cycle the 'ideal alternatives' are derived from the enumeration of real cases. The population of Great Britain at a given date would be such a collection of real cases, and that these real cases are identical with the equal ideal alternatives is shown by the fact that their number forms the denominator in the fraction that states the chances. If e. g. the population were 30,000,000, and 600,000 people died in the year, the chance, assuming constancy of the average,[1] that any one individual taken at random would die within the following year would be $\frac{600000}{30000000}$, viz. $\frac{1}{50}$. The analysis of such a statement of chances appears to me not quite simple, and I doubt whether Mr. Venn, in maintaining that chance essentially refers to series, has identified its elements rightly. The individual human beings composing the population in question, in whatever order we choose to take them (say, in the accidental order of our enumeration), must correspond, I think, both to the real case[2]—the throw of a die or toss of a coin—and also to the ideal cases—to the sides[3] of the die or of the coin considered as possible alternatives. And the two general cases of dying or not dying within the year cannot, I think, correspond to the equal ideal cases or possible alternatives, but must be treated as those combinations of alternatives which arise when several possible ‘ ways in which the event may happen ’ have an identical consequence in which we are interested. Or, in short, we might put it thus, following in part Mr. Venn's interesting discussion :—English humanity is the ‘ event ’ ; each individual is ‘ a way in which the event may happen ’ ;[4] dying and not-dying or male and female[5] are general consequences or results each of which emerges from a large number of ‘ ways in which the event may happen ’. Thus, as is right in theory, the ratio of chances is determined by the number of ways in which the event may happen, which in these arbitrary cycles = the number of cases in which the event does happen. It is a consequence of the inductive character of the cycle that this number of ways has no obvious and necessary meaning, but is a mere inference from the number of times that the event does happen. It is as if there was a die of unknown structure thrown 30,000,000 times, and exhibiting a white side 29,400,000 times and a black side 600,000 times. If then we chose to assume that we had before us a die with 600,000 black sides and 29,400,000 white sides and to estimate all further chances on that basis, we should be in an analogous position to that which we adopt when we calculate chance on the basis of a ratio observed *de facto* in a cycle of cases. The chances of white and black respectively are then $\frac{49}{50}$ and $\frac{1}{50}$ on the basis of this real but arbitrary cycle, regarded as the foundation of an ideal cycle of alternatives identical in content with itself.

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[1] Or following what was said above of the independence of calculation and real event, we may neglect the constancy of averages, and say, referring to the same year for which the enumeration is taken, ‘ the chance that any particular individual will have died in that year.’ On the same premises, the chance is the same after the event as before it.

[2] This is what Mr. Venn describes (Logic of Chance, chap. i, sect. 6 ; chap. iii, sect. 33), in the ordinary language of the theory, as the ‘ event ’, and in his own language as the collection of attributes.

[3] This is what Mr. Venn describes either as ‘ a way in which the event may happen ’, or as the ‘ occasional attribute ’.

[4] Or a particular modification of the collection of attributes by occasional attributes.

[5] Mr. Venn, Logic of Chance, chap. i, sect. 6. Male and female are not the ‘ ways in which the event may happen ’. This would give the chance of each as $\frac{1}{2}$.

Now the arbitrary cycle itself, the 300,000,000 individuals and 600,000 deaths, being given as real, there is no question of its correspondence with a pre-established ratio. If we ask whether the ratio of $\frac{1}{50}$ which it prescribes is realised in experience, we can only mean to enquire whether the distribution of the deaths is regular within the observed cycle, or whether the same ratio prevails outside this real cycle, which latter question may be followed up if we please by the same question as before about the regularity of occurrence. I am not quite sure that in speaking of the uniformity which attends large numbers of instances our writers always remember that such uniformity demands the comparison of two or more cycles. The ratio of results within a single group, even if embracing all hitherto observed instances, is in the absence of an antecedent rule a mere fact and not a realisation of anything. Every ratio is definite, and any two numbers have a ratio, so that it is a truism to speak of a definite ratio as prevailing between classes of cases in a single group.[1]

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[1] Logic of Chance, i. 6. The distinctive characteristic of probability is that occasional attributes as distinguished from permanent ones are found on extended examination to exist *in a certain definite proportion of the whole number of cases*. The italics are Mr. Venn's. How could they exist in anything but a definite proportion ?

Within a single large group if we speak of adherence to a ratio, we can only mean regularity of occurrence. And in the case of individuals that are not events in time the significance of such regularity and its existence must depend simply and solely on the order in which we consider and enumerate them.

Thus the two kinds of cycles or groups of instances seem to be quite differently situated with respect to empirical verification. i. A natural cycle from the first corresponds or does not correspond to its antecedent law, within the limits which that law spontaneously prescribes. *If* we throw all six sides of a die in every six throws there can be no doubt that up to the point when we break off the series is an empirical verification of the chances as we stated them. It shows that the unknown causes operate equally, and thus produce the result, which we anticipated by neglecting them. But in treating of actual experience we may practically disregard this kind of correspondence, which is not common in fact and at all events could hardly repeat itself through an extended number of observations.

ii. We are thus referred to the second question which we mentioned as capable of being proposed respecting a natural cycle, viz. whether the law which it presupposes (e. g. every side to turn up once in six throws of a die) is realised in the long run. It must be observed that to speak of realisation in the whole set of actual cases, whatever their number may be, cannot furnish a standard in this question, because this number is constantly varying and is very different in different subject-matters. There is nothing between realisation in every natural cycle and realisation in an infinite series if we keep clear of causal presumptions which do not belong to the reckoning of chances pure and simple. There is no doubt that the law presupposed by a natural cycle may be realised in a great and increasing number of observations, and that inferences may with the aid of causal presumptions be drawn from this realisation. But for all that, it is simply nonsense to speak as if the true and only true realisation of a ratio of chance was in the series of real instances continued *ad infinitum*. Mr. Venn, who is consistent in regarding this as the solution of the antithesis between equality of ground and inequality of reality, denies, as I understand him, that the formula $\frac{1}{6}$ [1] has any meaning as applied to a single real throw of a die, except by association with the idea of a series in which *all* sides should equally be exhibited. Here we come into the province of fiction. There is no reason, in the cases before us, that such a series should be a fact at all. And in these and in all other cases alike it is impossible that the infinite series could be a fact. And yet, if not a fact, it fails to solve the antithesis as a solution of which it is propounded. It is not in fact possible to go on trying for ever, and it is not in theory true that supposing we did go on trying for ever (abstracting from the contradiction involved) every alternative must be realised according to the ratio. The ratio may be justly erected according to our grounds of knowledge, even if some of the alternatives are absolutely impossible and therefore could never occur although, *per impossibile*, the series of trials should be prolonged to infinity.

In the case of the arbitrary cycle the answer to the question is still less favourable. The primary interest of the arbitrary cycle is just in that statement of chances affecting individual real cases which is suggested not to be the true meaning of the ratio. There is in this case no antecedent law between which and the real cycle a correspondence could be observed. The real cycle itself is and prescribes the law. And although cycles of real events that fall outside it, or minor cycles within it, can be tested and compared with it in respect of the ratios they display—as we compare the ratios of deaths to population in successive years—yet it is not easy to say on what ground the first cycle that we happen to observe should furnish a rule to which subsequent cycles are expected to conform. The whole group of observed events or individuals is a comprehensive fact, within which the ratio of the ways in which they happen or of their classes is also simply a fact. Any comparison which we may make of the ratio exhibited in minor cycles within this entire group has only the interest of the comparison between different groups of actual occurrences or individuals.[1] On the assumption that the operative but unknown causes are not changing progressively, or that we can allow for their progressive change, we may no doubt expect to find that a ratio which we elicited from case 1 to 1000 will hold good for cases 1001 to 2000. But if what we want is merely the serial form, we have it already in cases 1 to 1000; while if what we want is the multiplication of observations, why should the 2000 cases—a wholly arbitrary number—be especially satisfying? The fact is, that when we have the serial form given to begin with, as in these arbitrary cycles, or groups of instances limited only by our ceasing to enumerate them, we see that it omits just that peculiar transition which is the essence of the statement of chance. It only presents us with this transition in so far as, surrendering its serial form, it becomes the basis of a fractional expression which summarises our knowledge, drawn from the series, with reference to some instance or instances whether within or external to itself. This criticism applies to the serial form as such. The equal realisation of alternatives considered as equal (i. e. apart from regularity and irregularity, which are equality and inequality as judged by minor cycles) destroys the peculiar relation of equal knowledge to unequal fact, which is the ground of chance.

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[1] Mr. Venn seems to have been influenced by considerations such as those of p. 340-1, above. It is true, as there shown, that the judgment which measures probability loses *ipso facto* that isolated reference to a particular alternative which marks the genuine, or at least the natural problematic judgment. In every statement of chance we have an apodeictic judgment involving the entire content of a disjunction in the bearings of its members upon one another (as condition and consequent). So far Mr. Venn and I are together. The question therefore reduces itself to that of the purport with which the reference to the remaining alternatives is charged, whether it depends on the idea of a completed series, or can be explained by the (assumed) equal claim which each alternative makes on reality in virtue of the (assumed) equality of their grounds. The latter view seems to me to be demanded by the nature of the abstraction on which the whole idea of stating chances depends.

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[1] Mr. Venn, Logic of Chance, ii. 8, seems thoroughly to accept this result, and to conceive that *all* probability, even in what I have called natural cycles (in his phrase '*a priori*' probability), is at bottom this and no more. I cannot but think that if probability in a specific case means anything, it must, even though dependent on an arbitrary cycle, be stated as above in the form of a natural cycle.

On the infinite series, or approximation in the long run, I can say no more than I have said above, and others have said before me. The thing is simply a fiction, and the reference of the realisation of a ratio to it proves, if anything, that it is ultimately necessary to admit that chance is independent of a real series.

Causal Inference from Series.

β. The true bearing of a series on the verification or corroboration of a ratio expressing probability must consist in its relation to the causal presumptions which dominate our judgments about reality. All judgments that deal with fact assume, though they may not explicitly assert, causation. Statements of chance do not proceed by following causation into its ramifications; we should thus have concrete knowledge and not equality of alternatives. But when the results of experience coincide with the predictions of the calculus, this suggests to us *not* that we knew the right causes or any causes at all, but that the actual causes at work have a character compatible with the results which we obtained through the indifference of ignorance. If, on the other hand, the results of experience deviate widely, so far as experience goes, from the ratio suggested by the calculus, then *though this deviation can never amount to a flat contradiction*, yet it suggests an arrangement of causes incompatible with the results which were generated by the indifference of ignorance. If we cast six twenty times running with the same die, we have no right to say that this theoretically speaking contradicts the ratio of chance (unless we take as a standard, which no one would ever do, the natural cycle of six throws), for in 120 throws the balance might be restored. In other words, no sequence is impossible in such a case, nor is one more improbable than another, but of course any one sequence is immensely improbable beforehand as against the whole remaining mass of possible sequences. And in any marked or so to speak identifiable sequence this improbability strikes us as though it had occurred in face of some enormous probability in favour of some *one* other sequence, all the less identifiable sequences counting as if one in number, though $n-1$ in probability. This is a partial account of our surprise; but as has been well explained by Sigwart, there is in it the further element that whereas the twenty consecutive sixes are on the assumptions from which we started but one among an enormous number of equal possibilities, they happen to be of such a nature that on another assumption, incompatible with those, they would follow with absolute or all but absolute certainty. We know that if the die is cogged it will always turn up the same side, or to speak generally, if there is present an operative cause which necessarily produces one alternative, that alternative will always be produced. This suggests the comparison between one ratio, that with which we started, which gives a very minute probability for the result found in practice; and another ratio, formed on a different assumption, which gives the observed result with something like certainty. Nothing binds us *absolutely* to either, but it is plain that *so far as experience has gone* the probability is with the latter in the proportion in which it gives the result observed with greater probability than does the former. It must be carefully remembered that here, as all through this discussion, we are dealing with hypothetical judgments only. The probability I speak of is only on the data taken into account. If I am playing with a most respectable friend who says he has got the die from a good shop, I may prefer to believe in the reality of a peculiar case rather than in a fraud.

Observed and calculated series *may* coincide.

γ. I do not see that it is inevitable, as has been maintained, that an observed series *must* deviate from the calculated ratio, as it passes through fragments of a fresh cycle. Of course its coincidence with the ratio will not be demonstrable while any cases are wanting to finish the cycle; but if we shrink from saying that the observed numbers can coincide with the calculated numbers in a half cycle or less, we must not, I think, say that they deviate, unless coincidence at the next natural cycle is already impossible. Five complete cycles of sides and three different sides in thirty-three throws of a die surely form a case which should be distinguished from the same five cycles of sides plus three repetitions of the same side.

The series a reality for some purposes.

δ. In speaking of the truth of chances based on statistical averages, we may illustrate what has been said by the different positions of an insurance office and an individual customer of the office. To the individual whose expectation of life is in question the chance of life gives but little information, at all events so long as it is large. Whether he gains or loses by insuring his life is for him practically a mere uncertainty. He knows what it is reasonable to expect on the general data of reasoning, but he has no sort of ground for being surprised if it does not happen. For the office, on the other hand, so long as the averages are constant, the fate of individuals is wholly indifferent except in so far as they are more lightly or more heavily insured. If the office could be sure that in each class of customers (ranking them by the amount of their insurances) the average of deaths would be maintained at the same figure, it would make no sort of difference to it who in particular died. Thus it is true that in a real cycle the ratio of chance *may* in a certain sense become a fact. What is not true is that in becoming a fact it remains a chance,[1] and that if it fails to be realised in the short run it must be realised in the long run.

Probability of judgment in the absence of knowledge.

vii. Before passing to the subject of modality I will mention an interesting point in the theory of chance, which is cognate to the above discussions on Privation, Affirmation, and Exclusion.[2] What statement of chances expresses the attitude which we ought to adopt towards an affirmation in the absence of all knowledge? The accepted answer appears to be $\frac{1}{2}$, 'for if we make it less than this we incline to believe it rather false than true,' or putting I suppose the same ground into mathematical language, 'If we grant that the probability may have any value between 0 and 1, and that every separate value is equally likely, then n and $1-n$ are equally likely, and the average is always $\frac{1}{2}$.' I am not prepared to deny this conclusion, which of course follows from its data, but I think that it may be instructive to discuss these data, which appear to me somewhat superficial. It appears that the symbol $\frac{0}{0}$ has also been proposed, on the ground that 'the *a priori* [formal?] probability derived from absolute ignorance has no effect upon the force of a subsequently admitted [real ?] probability.'[1]

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[1] Cp. Adam Smith's attack on lotteries. You may see, he said, how much the chances are against you, by the fact that if you take all the tickets you are sure to lose.

[2] Chap. vii, supra. See Jevons' Principles of Science, p. 212.

It cannot but strike the looker-on that these two suggested values $\frac{0}{0}$ and $\frac{1}{2}$ seem to correspond with the conceptions of non-impossibility and of real possibility respectively; and that to take probability as having the value $\frac{1}{2}$ in the absence of all knowledge[2] is analogous to conjuring a positive favourable presumption out of an absence of counter-presumptions. The question is, according to the analysis of chance which has been stated above, whether the two alternatives 'true' and 'false' are *sole* and *equal* alternatives. Interpreting absence of knowledge as Jevons interprets it, to include entire ignorance of the meaning of an enunciation, I do not see that they are *sole* alternatives. 'If I ask the reader to assign the odds that a Platythliptic coefficient is positive, he would hardly see his way to doing so unless he regards them as equal.'[3] But to a reader who does not know what the words mean—and this I suppose is what Jevons intended—there is no judgment conveyed. The alternative 'unmeaning' must then be allowed for in addition to 'true' and 'false'. This, it may be said, makes nonsense of the problem. The unmeaning is not a judgment, and the problem is only about judgments. Granted; but then I must not include the very large division of the unmeaning in the statement of chance without recognising it as a separate alternative. And if I am to exclude it altogether I must either be given a sentence which I am able to recognise as a judgment, or the problem must refer to any judgment as such without considering whether I know beforehand what it is. The latter case is not that in question, and could be treated perhaps better through statistics of error in the sciences, than by deduction from the alternatives 'true' and 'false'. It includes in the reckoning all judgments known to be true, whereas the present problem says nothing of these, but only of judgments on which being presented to me I am unable to return a verdict based upon positive grounds.

The real question then is this. Given a judgment which I can understand, but which I have no positive ground either to affirm or deny, what are the chances in favour of its truth and falsehood respectively? The conditions of this problem cannot of course be actually realised, because to understand the meaning of a judgment theoretically involves some consciousness of *pro's* and *con's*. Yet there are in the world so many almost arbitrary judgments, that the question has some importance. Truth and falsehood *are* in this case, the case of intelligible judgments, *sole* alternatives, but I cannot think that they are, under the supposed conditions, *equal* alternatives. I cannot think, that is, that every separate value of probability between 0 and 1 is equally likely. For the judgment being *a* form or indeed *the* form of knowledge, the hypothesis of ignorance, in this case 'absolute ignorance', on which the statement of chances is erected, has a peculiar relation to the content of such a statement when that content is the judgment. If we knew there was a certain motion below a certain limit of velocity, but had no further clue to the velocity of the motion, it might be true I suppose that every degree of velocity below that limit was equally probable. But if we know that there is a judgment made, or proposed to be made, and have no clue to any degree of positive probability in its favour, then for us the zero of probability is the fact, and if we were to make the judgment in question it would in our mouth be false even if in reality true. Thus, on the basis of my individual knowledge, such a judgment *qua* judgment is by the hypothesis *prima facie* false. But my knowledge is not all reality, and therefore I dare not say that falsity holds the field as an absolute certainty. The possibility however drawn from the mere difference between my knowledge and all reality, is an unmotived possibility; for there is at least no antecedent likelihood that my knowledge is always wrong. And I am not entitled to raise this unmotived possibility into an alternative having equal grounds with the *prima facie* falsity which follows from the hypothesis.

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[1] Bishop Terrot, quoted in Jevons, l. c.

[2] Cp. Appearance and Reality, ed. 2, p. 505, note.

[3] Jevons, l. c.

I am not prepared to suggest any way of representing these chances in numbers. Without equally grounded alternatives we cannot state chances, and I do not see where in this case these are to come from. If one read 'doctrines' for judgments, so as to restrict the question to matters of some depth and importance, one might obtain interesting enumerations out of the history of science bearing on such relations as those of false anticipations compared with true discoveries. But it would all amount to very little. I only desired to point out that the suggested symbols $\frac{0}{0}$ and $\frac{1}{2}$ seemed to lie in the track of the fallacy discussed above.[1] To say that objecting to a judgment we do not know to be true is as unreasonable as accepting a judgment we do not know to be true—and to say that truth and falsehood have a chance of $\frac{1}{2}$ each is to say this—appears to me to be a sophism in the vein of Sir Anthony Absolute.[2] If you have no reason for accepting a judgment, you must decline to accept it. If you only decline provisionally, and say that in future, or to the knowledge of wiser minds, the judgment in question may perhaps be proved true, then you unquestionably are cherishing some distinct though general presumption in favour of the judgment, and it is not one of those whose chances 'in the absence of knowledge' we are discussing. We do not treat really arbitrary suggestions with so much respect. It seems to me monstrous to say that half the equal grounds are for truth and half for falsehood when the fact is that you *have* no ground to think the thing true, and *require* none to think it false.

This brings me to one further distinction. The reader ought to reply, 'You do need a positive ground of denial in order to deny, and in stating the chances as $\frac{0}{0}$ you are denying, which *ex hypothesi* you have no right to do.' But I suppose that by the symbol $\frac{0}{0}$ we do not so much deny the judgment as refuse to state the chances. It is only the logical interpretation of this refusal that brings something like a denial into the matter. You cannot obtain a denial out of a pure privation, i.e. a mere profession of ignorance, but then in view of the positive mass and far-reaching presumptions of knowledge as a whole, no privation however *complete*[1] can be quite *pure*,[1] i. e. quite free from positive grounds of denial. A complete and persistent privation must always, as I have tried to show above, verge upon an exclusion. But if we had at command a direct and positive ground of denial, then I imagine that we should not restrict ourselves to refusing to state the chances, i. e. to ${}^{0}_{0}$, but should employ the symbol 0. Or, in order to indicate that a case has no chances in its favour because all the possible chances are absorbed by another case which is certain and which excludes the former, we ought I should think to use the expression ${}^{0}_{1}$, i. e. negation grounded in positive certainty, which is the remainder of ${}^{1}{}_{1}{}^{1}$, and so represents the total certainty available for the two cases as entirely absorbed by the one. Thus we should have three symbols representing ideas which we ought not to confuse, ${}^{0}_{1}$ for demonstrable impossibility, ${}^{0}_{0}$ for absolute ignorance (privation alike of real possibility and of impossibility), $\frac{1}{2}$ for a conflict of proofs, such that truth and falsehood are equally grounded alternatives, which in presence of absolute ignorance is not the case.

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[1] See chap. vii, p 316.

[2] 'Absolute. "Sure, Sir, this is not very reasonable, to summon my affection for a lady I know nothing of."

'Sir Anth. "I am sure, Sir, 'tis more unreasonable in you to object to a lady you know nothing of."'

Sir Anthony wishes to represent the chances of attachment and non-attachment to any unknown lady as $\frac{1}{2}$ each, or even as more than $\frac{1}{2}$ in favour of attachment. This is really not a bad parallel to the view criticised in the text.

APPENDIX TO CHAPTER VIII

ON SOME RECENT DISCUSSIONS OF DISJUNCTION

Are disjunct members exclusive?

Mr. G. R. T. Ross (Mind, N. S., 48) and Mr. Keynes (pp. 277 ff.) have recently supported the view that members under a disjunction ought not to be considered as reciprocally exclusive. I will comment on some of the arguments that have been used, and will try to explain my own position further.

Is there really anything to argue about? If the whole question is one of usage, and that is arbitrary, it would concern grammar rather than Logic. But I think that three points of interest for Logic are involved: *a* the sort of guidance which a logical instinct can follow in common speech; *β* an application of the theory of incompatibility, contradiction, and negation; γ a substantial though not formal derivation of the exclusive from the exhaustive character.

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[1] By a 'complete' privation I mean one in which we know absolutely nothing *in favour* of the matter of which we deny all knowledge, while I call a privation 'pure' in as far as we know nothing positive *against* the matter of which we deny all knowledge. And my suggestion is that, looking at knowledge as it really exists, wherever we have the former case it is almost impossible to have the latter.

Choice involves incompatibles.

α One must not expect to find a definite theory governing usage. If a moral is to be drawn from it at all, one must show some simple character in the matter in question which could serve as a guide through feeling and habit. And I think Mr. Ross has helped me to put this in form, though his argument is directed against my view. His phrase, 'the nature of the predicates,' is suggestive. I believe that usage favours 'Either—or' when predicates are coming which are felt to be incompatible. They are the particles consecrated to choice, and choice means incompatibles, i. e. that you cannot have it both ways. Choice is always a special attitude, whether in action or in judgment. When we feel it coming we adjust ourselves to the situation by a special form of speech. No doubt the area of choice—the space within which a right choice must fall—is of interest to us. This is the exhaustiveness of the 'Either—or' = 'within these cases the right choice must fall'. But the primary character, that which determines the very existence of a need for choice at all, is the presence of incompatible alternatives—the exclusiveness of the disjunct members.

'Alternatives,' whatever else they may be, are surely incompatible. No one was ever heard to say in presence of a pair or set of alternatives, 'I choose them both or all,' unless he means that it was an error to propose them as alternatives. And it seems to me plainly erroneous to speak of an alternative proposition while suggesting that the alternants may both be true (Keynes, 277, 279). If you may take both, you are surely not in presence of an alternative.

A conjunction cannot include two operative alternatives.

β Now incompatibility has degrees. And I wish next to point out that there is a certain marked degree of it which is present in the cases under any disjunction whatever, even on the hypothesis that exhaustiveness is the primary assertion which is intended. Take, for example, Mr. Ross's comment (loc. cit.) on the argument (Bradley's Logic, p. 124; above, p. 324), that if the alternatives are not exclusive the disjunction cannot be exhaustive, the case of their being conjoined being a *casus omissus.* 'We might ask how it is that *bc* can be exclusive of *b* and *c* when as a matter of fact it includes them.' For the possibility that *bc* may really go under *b* or *c* (either *b* or *c* being inoperative in it) I refer to my argument on p. 324 with the example of the railway tickets. In instances of that type it is needless to mention *bc*; but this is because it already falls under *b* or *c*, and therefore the cases are already both exclusive and exhaustive. (This disposes of two of Mr. Keynes's cases on p. 279—the candidate for an appointment, and the peer. The membership of more than one University, in the first case, and the possession of more than one title, in the second, are not special cases of eligibility and of peerage. In either instance only one can operate, and the possession of more than one constitutes no separate case under the Disjunction.)

But if *bc* is to be a distinct case under the disjunction, then we have an interesting point. *bc* then cannot include the cases *b* and *c*, and for this reason. Each of these cases, *qua* case of the disjunction, is proposed as capable of meeting the requirements of the situation by itself and therefore completely. Take it as a cause (Keynes, p. 278); (it might also be an explanation or a consequent; this makes no difference). 'He has either used bad textbooks, or has been badly taught.' Each of these suggestions is propounded as stating an adequate cause of the existing situation. The mention of it as a separate case makes this certain, even if we do not deny the possibility of the fact contained in it being combined with another fact, such as bad education. In the case put (the case *b*), the boy's ignorance, or whatever is the matter, is accounted for by bad textbooks. Now the case *bc* cannot include this case *b*, though it may include part of the fact suggested in it. In *b*, bad textbooks are *the cause.* In *bc* bad textbooks are not the cause, but a co-operant cause. You cannot take two causes out of a 'plurality of causes', and treat them as co-operant, without completely changing the operation of each. It is a flat contradiction to predicate conjointly, without special explanation, two predicates which are capable of being predicated alone. When my builder gives me three explanations of the same overcharge, it is not possible for me to accept them all, and this follows simply from the fact that each claims to be sufficient taken by itself. The facts alleged in them may in some way be combined, but the three sufficient explanations cannot possibly be true together, though the builder is inclined, like my critics, to say so. *b* and *c* are each exclusive of *b c*. So with the well-worn case of rogue and fool. If *X* is both rogue and fool, that excludes the cases of simply rogue and simply fool. The man is different in every fibre of his being. So if 'the witness is perjured or the prisoner is guilty' (Keynes, 279) you mean to say that you see two possible explanations of the witness's evidence, constituting incompatible cases. You can choose that which seems most probable; but they are rival explanations, and you cannot, having regard to the facts before you, accept them both. The witness may be perjured and the prisoner nevertheless be guilty, but that must be on further grounds, not present in the facts so far to be explained.

Incompatibility of cases, then, is the primary meaning of the words 'Either—or'. I will analyse Mr. Ross's curious example (Mind, loc. cit.), pp. 492–3. The point of it is that if disjunctions are both exclusive and exhaustive, '*A* is either not-*b* or not-*c*,' and '*A* is either *b* or *c*', mean exactly the same thing; e. g. if you say 'a man who says he has seen a ghost is either not sane or not truthful' you can go from that to 'a man, etc., is either sane or truthful'.

I note first, that this result is sound in the main. It looks absurd, because contrast calls attention to the positive aspect of the terms. Pronounced *avec intention*, it would be an example of the phrases in which mere precision constitutes effective irony, 'He is *either* sane, *or* truthful,' i. e. certainly not both. I think that the effect of precision here really proves the exclusiveness of the disjunction to be essential.

And secondly the instance with Mr. Ross's comment itself shows clearly that what we primarily mean is the incompatibility of the *positive* terms. In a word, we mean to say 'He cannot be both sane and truthful'. But there is a curious point in the argument which might obscure this fact, and I will take it to pieces.

Starting from the *negative* terms, not-sane and not-truthful, what we want to assert is primarily exhaustiveness. But the reason of this is that exhaustiveness as between them is the same as exclusiveness between the two positives.

X is either not sane or not truthful, implies, (1) If *X* is not (not-sane) he is (not-truthful).

The type is, If *A* is not *b*, it is *c*, the hypothetical which ensures *exhaustiveness.* This is clearly the right analogy, expressing the meaning of the original statement, and keeping the negative terms as such; and I think, from his arrangement of the propositions on p. 492, Mr. Ross recognises this.

But, being 'simplified' by cancelling the double negative (whether it is a true double negative is questionable), this proposition results in

'If *X* is sane, he is not truthful,' a proposition of the type, If *A* is *b* it is not *c*, which is the hypothetical that secures not exhaustiveness but exclusiveness. And this is undoubtedly what the speaker meant primarily to assert, viz. exclusiveness (incompatibility) between the positive terms, though this took the form of exhaustiveness of the negative terms. Again,

(2) If *X* is (not-sane) he is not (not-truthful). This is of the type, If *A* is *b* it is not *c*, the hypothetical which secures *exclusiveness.*

This, 'simplified' as above, gives

If *X* is not sane, he is truthful, of the type, If *a* is not *b*, it is *c*, the hypothetical which secures not exclusiveness but exhaustiveness.

This judgment, perhaps, as Mr. Ross says, we do not want, and the disjunction 'Either sane or truthful' is excessive by that factor. And it arises, as he implies, from assuming exclusiveness. But the point is that the exclusiveness assumed is between the negative terms, and we have seen that this is the same thing as exhaustiveness of the positive terms. But it is this exhaustiveness which is the superfluous element. What we asserted from the beginning and meant to assert all along, was incompatibility, exclusiveness of the positive terms, and it was only because we saw it meant this that we asserted exhaustiveness of the negative terms. What we meant to say was simply 'He cannot be both sane and truthful'; and the 'Either—or' saddled us with an unnecessary element, not in the assertion of exclusiveness of the positives, which was the whole point of our saying, but in the assertion of their exhaustiveness, which is more than we needed to say. Taking the positive terms, then, which are what we are really concerned with, the argument proves my point, that a disjunctive proposition is primarily concerned to affirm the incompatibility of the cases set out under it.

Now to predicate without distinction or reconciliation (e. g. without explaining how and in what degree they co-operate) separate predicates of the same subject is the definition of contradiction. And therefore it is plain that the cases set out under a disjunction, and *prima facie* meant to be separate and sufficient for the situation, are not predicable conjointly without special explanation and defence of their relations in the conjoint case, which, if the several factors of it are really operative, cannot possibly include the other full cases as they are when separately predicated. The appearance to the contrary has been shown to mean that the alleged conjoint case satisfies the disjunction in respect of one of its elements and no more.

The whole significance of the theory of negation lies in showing how these independent cases, incompatible when you offer to predicate them conjointly, become consistent when arranged as the import of a disjunctive system (see p. 289 above). Every significant negative takes its meaning from such an arrangement implied or expressed.

Exclusiveness follows from independence. Other criticisms.

γ. I am convinced, then, in the first place that 'Either—or' implies choice, that choice implies incompatible alternatives, and that incompatibility or mutual exclusiveness of the cases (otherwise than as the content of an ordered system) is the primary meaning of disjunction. But in the second place, supposing it is maintained that the primary meaning is 'one or other at least and, it may be, both', I submit that the above-mentioned considerations suffice to establish exclusiveness as a logical consequence of the disjunctive attitude, even if it is a consequence not always foreseen (as in the case of my builder). If you assert 'either this or that', even intending to concede the possibility of both together, you clearly mean that this or that by themselves are at least possible independent cases. You mean that each is a possible cause or explanation or course independently of any other.

But if so, the nature of incompatibility or contradiction determines the sheer impossibility of predicating two such cases together. The joint predication can never include two cases of the disjunction which have been offered as independent. We have seen the two modes in which the appearance of such a joint case may arise, and how in each of them we really have no joint case of *b* and *c* at all.

I pass to other criticisms of my view of Disjunction.

Mr. Keynes (p. 283) criticises a statement in Essentials, p. 124, that the proposition, 'The signal shows either red or green,' asserts it to have 'some colour', which is not asserted by either of the hypotheticals entering into it. Mr. Keynes objects that 'if not red, then green', plainly makes this assertion. The passage in Essentials is obscure from its brevity. I refer for my meaning to p. 328 above and the following pages. The disjunction unites the two characteristic hypotheticals and the system within which they are true and significant in a single survey and affirmation. You cannot represent an assertion like this by two separate judgments, each of them partial and hypothetical (cf. Essentials, 116, and Bradley, Logic, 121–2).

On my general view of the logical rank of Disjunction, to which Mr. Keynes (p. 283) objects that it gives an unreal importance to classification, see besides 328–9, Vol. II, p. 194 ff. These passages contain, besides an estimate of Disjunction, a special reference to the subordinate importance of classification. It is not classification, but the system of reconciled incompatibles in which every universal when made explicit is seen to consist, that I hold to be best exhibited in Disjunction. It depends on the principle that in every system the nature of the whole, differently conditioned, is in every disjunct member. This seems to me a fundamental character determining the explicit formulation of every universal, and therefore necessarily prominent in logical theory.

Mr. Ross's argument that from a simple constructive dilemma you can only get a conclusion of an exhaustive, and not one of an exclusive nature (p. 495), seems to me to tell us nothing about the disjunction. The reason of the failure is plain. According to the ordinary convention you can get no inference from *A* is *d* (under the proposition if *A* is *b* it is *d*), nor from *A* is not *c* (under the proposition if *A* is *c* it is *f*). You have barred the disjunction from transmitting its characters. If you recognised reciprocal hypotheticals you could go from *A* is *d* to *A* is *b*, from *A* is *b* (under the disjunction *A* is either *b* or *c*) to *A* is not *c*, and from *A* is not *c* to *A* is not *f*.

The instance of the man at the top of the burning house (at p. 495) seems to me to be spoilt by considering whether the man's view is true or not. All we are concerned with, surely, is what his view is, as expressed in the proposition ascribed to him, 'If I jump from the window I shall break my leg, and if I do not I shall be burned to death.' Whether he is right or not is a thing no one can possibly tell. But that the whole motive for formulating such a proposition is the intention to make a choice between alternatives taken as incompatible, is surely undeniable.

I think, as I said above, that Mr. Ross is right in saying that it is the nature of the predicates which makes them incompatible. But it seems to me that this is the guide of our instinctive usage of 'either—or', that when we feel a choice between incompatibles before us we confront it with that form of thought, although we may not always realize that it debars us from conjoint predication, in the strict sense, of the cases which are offered to our choice as severally independent and sufficient solutions of the situation. I use 'the situation' as the most general term for the fact or idea with which we are confronted, and which forms the disjunctive heading under which the alternatives of our affirmation or action have to fall.

CHAPTER IX

MODALITY

Kant's view fundamentally just.

1. I PROPOSE to conclude the discussion of the judgment with a short treatment of Modality. For Modality, if it exists at all, is simply the degree in which individual judgments participate in the certainty of that permanent and all-embracing judgment by which the individual intelligence sustains those qualifications of the Real which for it constitute Reality. Our account of Modality must therefore resolve itself into a recapitulation of the principal types of judgment, having for its object to bring together in a single view certain of their characteristics which have already been noticed. The question before us is whether and in what sense there are degrees of logical certainty ; not merely of habitual conviction, or of readiness to act on a belief, which are psychological and not logical, but of that characteristic which forms the differentia of judgment, and which may be described as *logical* assertiveness. This logical assertiveness itself indeed includes a psychical or psychological element which must be carefully distinguished from the purely logical or rational element of assertiveness.

One preliminary difficulty meets us on two sides. We find Kant [1] maintaining that modality affects only the copula in judgment, and that therefore, though a measure of assertiveness, it is indifferent to the content affirmed. And we find it maintained against Kant that modality has no reference to the copula in judgment, nor, consequently, to the assertiveness of assertion, but is a peculiarity of the content affirmed which does not affect the essential act of affirmation.[2] Both of these views, it will be remarked, by separating the assertiveness of assertion from the content asserted, represent judgment as an arbitrary and irrational activity. It is not surprising that in the 'Grammar of Assent' ecclesiastical interest should have thrown itself zealously on the side of such a conception.

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[1] Kant, Kritik der r. V., p. 97 (Hartenstein), 'Von der log. Function im Urtheilen.'

[2] Sigwart and Bradley certainly agree in this. Lotze seems to say it of the old Modality, in which '"It must be so" counted as apodeictic', but does not distinctly say it of Modality as understood by himself (and subsequently by the other moderns above mentioned), which requires

The view which I have attempted to explain in the present discussion is incompatible with both of the above-mentioned ideas. Every judgment, as we have seen, unites in it two elements of certainty; the formal or psychological element, which consists in a reference of its content in virtue of a perceived identity to the individual's personal world of perception and experience. and the material or logical element, which consists in the attachment of this content by rational necessity to the organised nature which the Real possesses as already qualified by the individual's knowledge. The former element corresponds to memory and the latter to reasoning. All reasoning is in the medium of memory, but memory as such does not involve reasoning. The two activities however are one in origin and in *ultimate* nature.

Now the psychological element of certainty does not vary. The unity of a content with the individual perceptive self admits of no degrees. If it appears to vary in degree, we are remembering *by* something, i.e. we are eking out memory by reasoning. Memory as such is dumb in presence of questions or comparison of grounds. It tells us nothing beyond the mere content which it recalls. But material or logical certainty depends on reasoning, and is therefore capable of more and less, and is the chief element of the assertiveness of judgments. It follows that modality *is*, as Kant said it was *not*, a characteristic of the content affirmed, but is also, for that very reason, as the moderns say it is *not*, a measure of the assertiveness of assertion. Whether modality must be said to affect the 'copula' at all, or the copula only, depends on what we mean by the copula. If the copula is formal and empty, an unexplained act of reference like that of memory,

hypothetical or disjunctive form for apodeictic judgment. In his suggestion as to Modality Lotze was of course anticipated by Hegel. Sigwart and Bradley deny the superior logical certainty of the apodeictic judgment, and Sigwart even disparages it on the ground of its mediate character. Here, at any rate, he commits a gross blunder.

then modality does not affect it, or rather it does not affect modality. If, on the contrary, the copula is the act by which grounded necessity is recognised, then it is the essence of modality. The formal and indifferent copula of traditional logic *is* the psychological copula, and when treated as the logical copula becomes a logical copula indifferent to the logical content,[1] which is absurd.

As a matter of organic principle, therefore, I shall follow Kant, although by what I must regard as a confusion, he refers modality to the form of judgment and not to the content, and although his 'problematic judgment' is rightly pronounced by later writers to be no judgment at all. His summary runs thus: 'Because, then, in this aspect [i. e. in modality] everything incorporates itself with the intelligence by degrees, so that one begins by judging problematically, and subsequently takes the matter to be true assertorically, and ultimately affirms it as inseparably united with the intelligence, i. e. as necessary and apodeictic, it follows that we may confer on the three functions of modality the further appellation of so many moments of thought as such.'[2] Here as throughout philosophy it has been the task of later writers to realise in the concrete a conception enunciated by Kant, but by him only half liberated from the formulae of obsolete conventions. Hegel's analysis of the hypothetical and disjunctive judgment, adopted by Lotze and subsequent writers, is the realization of Kant's idea of modality as progressive incorporation with the understanding.

The Problematic Judgment.

2. In order to begin at the beginning we must start from Kant's 'problematic' judgment, which, *as he describes it*, is not a judgment at all. The problematic judgment according to him expresses mere logical possibility,[3] not 'objective'

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[1] Lotze does not fall into this trap. But he seems only to avoid it at the cost of separating off a particular fragment of content to be identified with the copula, at least when he is explaining the traditional view. But as against this the traditional view was right. The copula must have to do with all of the content or with none. In Lotze's own view of Apodeictic Judgment it has to do with all.

[2] Nearly equivalent to what I have often spoken of as non-impossibility. See below.

[3] Kritik der r. Vernunft, p. 97 (Hartenstein's ed.).

possibility, and he gives as examples of it the antecedent and consequent in hypothetical judgments, or the isolated members of a disjunction.

Nature of Suggestion or Question.

i. But such elements of thought are of course by themselves not judgments, i. e. as Kant himself says, they are not assertory. It may be doubted however whether such elements of thought exist by themselves at all, and whether they do not of necessity enter into some judgments. Their nature is no doubt the same as that of the genuine[1] Question, which Sigwart has the merit of having discussed. I cannot think however that his psychological expressions help us to grasp the logical differentia of question or doubt. It is nothing to say that the idea merely floats[2] before the mind. It must, in order to rank as a definite doubt or question, make some specific claim, be a candidate for some definite place, in the permanent judgment of consciousness. There is a nearer approach to something intelligible in the suggestion that a problematic judgment is a judgment about oneself, saying that in the matter before us we are unable to judge. But this, though often true in fact, is an evasion of the theoretical difficulty. There must be some definite logical situation in virtue of which we say this. When we judge that a judgment is merely possible we must judge that its content has mere possibility. The reduction of judgment of possibility to possibility of judgment is an attempt to take refuge in psychology from a logical difficulty.

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[1] The question which expresses a real doubt or ignorance; not one to which we know the answer, but ask in order to force the interlocutor to give it. This latter is an imperative, as Sigwart says (vol. i, 231). But then the former hardly perhaps has the differentia of the Question, which *is* just this imperative significance. So the 'genuine' Question is perhaps not a Question at all, but only a state of knowledge. *What* state of knowledge? Obviously one that presses for an advance, so probably a disjunction of ignorance. Kant's problematic judgment includes all supposition *qua* supposition, although known to be false or known to be true. But I do not think that Sigwart's complaint of a want of distinction between these cases and that of genuine doubt is well founded, because a supposition *qua* supposition is considered by an act of abstraction apart from the relations which constitute its falsity or certainty. Therefore, as subject to this abstraction, any supposition may rank as a doubtful judgment.

[2] 'In der Schwebe.' Sigwart, Logik, i, 230; E. Tr. i. 177.

Although then the mere idea of a judgment, floating before the mind, is not, even if it exists, a judgment at all and therefore not a problematic judgment, yet every genuine question and every judgment made subject to a ‘ possibly ’ or ‘ perhaps ’ represents a peculiar logical situation, and not merely a psychological incompleteness of the act of judgment. It is not true as a rule that we begin with floating ideas and advance from them to judgments. I doubt if such a beginning is possible ;[a] it is certainly not normal. Our thought consists in the continuous modification of judgments—I had almost said, of a single judgment. The question, considered as a state of knowledge, is a disjunctive or hypothetical judgment used as the premise of an inference. A groundless question is as unreal as the ‘ infinite ’ judgment. I cannot ask ‘ Are you going home ? ’ except on the basis ‘ You are possibly or probably going home ’, which means when analysed either ‘ If x is the case you are going home ’, or else ‘ Either you are going home, or x is the case ’. The question does not mean ‘ I have a floating idea in my head of you as going home,[1] and want to know if I am to refer this to reality ’. It means rather, ‘ I judge true of reality a definite situation in which some conditions of your going home are included ; I want to qualify reality by this situation more precisely defined ’ or to qualify a further element of Reality, viz. yourself, by it. Thus Kant is not so far wrong in identifying a problematic judgment, or, as we may call it, a genuine question, with an isolated member of a complex enunciation ; but it is only as a corollary from the complex thought or as an inability to make its outline precise, and not as a mere isolated member of it, that the problematic expression can be a judgment at all. For in its isolation, *ex hypothesi*, it is not referred to reality.

Problematic and Apodeictic Judgment.

ii. Possibility, then, as Mr. Bradley has told us, is a species of necessity, and it seems to follow from this that the problematic judgment is a form of the apodeictic judgment, and that any series of gradations in which the two have separate places—such gradations as Kant laid down—must be in contradiction with the nature of the case. If the judgment of possibility is the first form under which matters of knowledge attach themselves to the understanding, then it can hardly be a species of the final form, and ought to be verifiable in early thought.

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[a] Ultimately, there are no floating ideas. See above, pp. 4. 70, and Bradley, Mind, 60, 445 ff.

[1] The idea is not ‘ *of* you as going home ’ but ‘ *that* you may be going home ’ ; i. e. something *is judged*, which may result in your going.

We must begin by admitting the difficulty indicated by the last objection. All judgment, I have said, is in one respect assertory. It is probable that very early thought may present no other aspect. The distinction between memory and intelligence is a late distinction. The old man in Homer[1] who 'knows ten thousand things' cannot but remind us of the schoolboy whose friend 'knows an awful lot'. Whatever is in the mind, such expressions seem to suggest, ranks alike as knowledge. The *asseveration*, indeed, may be supposed to begin as soon as man feels the danger of deceit; and this form of speech recognises a distinction in degrees of certainty, and attempts to raise one matter of knowledge to the standard of another. But on the whole, the distinction between memory and intelligence, and therefore that between mere reference to perceived reality and systematically grounded insight, would probably be found a vanishing distinction if we could examine the earliest phases of the human mind. Possibility, mere assertion, and necessity, as they exist for the civilised mind, are based upon differences that concern the material logical or systematic element in judgment. We have to remember however that logical facts exist long before the technical names for them, and we must not limit the existence of modalities by that of words like possibility and necessity, but only by that of speech-elements bearing the power of 'may', 'must', 'shall', or 'would that'. And we must add that we cannot see how judgment should exist apart from all sense of rational necessity. A mere instinctive identification with reality, wholly without why or wherefore, is rather a theoretical limit below which judgment cannot be taken to exist, than a historical phase of the judging faculty.

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[1] Homer is of course not primitive, but poetry is very conservative, and Homer is full of ideas which are derived from very early strata in the mind's formation.

The next difficulty is that possibility appears on the view taken above to be a species of necessity, and yet to be prior to necessity. In order to explain this we must refer back to the doctrine of opposition and conversion. Strictly and properly, a judgment can only be denied by another judgment of the same nature; a singular by a singular judgment, a generic by a generic, or a hypothetical by a hypothetical. But a very strong implication of denial can be conveyed by a judgment which being of a different type from that contradicted denies the right of this latter judgment to the type which it has assumed. If, however, the denial is to be prosecuted in earnest, the judgment of the lower type must be capable of maintaining itself on the level of that which it assails. Possibility, if not derivative or calculated, represents such a first effort at denial, directed against a necessary judgment, and may or may not go on to assume an explicitly necessary form.

Particular Judgment. Exception and Instance.

a. Thus possibility as prior to necessity follows the meanings and development of the Modal Particular, which, owing to the continuity in the evolution of thought, on which we have so often insisted, has its roots far back in the quasi-singular or particular enumerative judgment. That is to say, the consciousness of possibility begins—I do not mean in every case, but in its most rudimentary logical form—with a rule and an exception or with a positive instance suggesting a rule. And it passes into a further type with the essentially negative, or again with the essentially positive *modal* particular. The latter, indeed, the positive modal particular, is hardly intelligible apart from the explicit recognition of necessity. But in accordance with a principle to which we shall have to recur it is quite customary for thought to employ a derivative and secondary judgment which is dependent upon a primary principle remembered as a rule. Many judgments of possibility—those current in systematic thought—are after this fashion corollaries from judgments of necessity, or rather from the fact that certain judgments of necessity are accepted as true. I now proceed to illustrate the development just described.

Possibility is at first negative. Bare negation indeed is nothing, but possibility in its simplest case comes very near bare negation. Such a possibility when veiled under the equivocal form of the particular judgment rests on an *exception.* We must not suppose that the possibility is positive because the exception is a positive case. The content of a mere exception as related to the rule which it impeaches is purely negative, i.e. only contradicts, and suggests no contrary principle. Suppose we have a half-collective and half-generic judgment like 'All English railways are well managed'. Suppose that then we come upon an English railway which is not well managed, and embody our observation in the judgment 'There is an English railway which is not well managed'. This may readily be interpreted as a judgment of negative possibility, amounting to no more than this, 'It is not true that English railways are in every case well managed,' or in other words, 'It is not impossible for an English railway to be other than well managed.' The principle is just the same if the rule is negative, e.g. 'No crows are white.' An exception would contradict this, but only by establishing little more than mere possibility (not-impossibility) that a crow may be white. 'There is a case in which a crow is white;' 'It is not true that no crows are white.'

The *instance* makes the step from negative to positive possibility. Like the exception, it implies a generalisation, at least incipient, but it supports this generalisation instead of contradicting it. Supposing, what is always ultimately the case, that the *exception* is a latent *instance*, the judgment which expresses the exception will change, in coming to express an instance, from a mere contradictory to a contrary which is also contradictory. Let the original rule be 'No secular education can be spiritual' and the exception be 'In the case of literary education we have a secular education which is spiritual', with the negative result 'It is not true that no secular education can be spiritual'. Then when we go on to treat the exception as an instance[1] we get a result hardly differing from the former in words, but charged with the material distinction that we now see reason to think that 'secular education as such may also be spiritual', i.e. It is possible that—or 'There is real ground for supposing that secular education, &c., &c'.

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[1] Exception presupposes rule, and rule presupposes positive instance, it may be said, so that our idea of negative possibility coming first gets us into a circle. The fact alleged is true; a positive rule must come from somewhere, and probably from a positive instance. But prior to the idea of denying the rule the generalisation is direct or naïve, and does not pass through the stages of modality except on the embryo scale on which it also implies negation.

Negative and Positive Possibility.

β. When we advance to the modal particular we have the same distinction in a purer form. The modal particular has been treated above as the converse by limitation[1] of a hypothetical judgment, and at all events may always be treated as the contradictory of another hypothetical judgment. 'If A is B it is C' becomes when converted by limitation 'If A is C it may be B', which latter is *at least* the contradictory of 'If A is C it is not B'. Where the modal particular really originates by the conversion of a hypothetical judgment, it is of course an inference or corollary from a principle with reference to another and opposite principle. But as usual the actual or historical modes of initiation of the judgment are one thing, and its logical essence another. Obviously the modal particular may be generated either by inference from explicit principles, or by the suggestion of rules through instances.

In any case the judgments 'A may possibly be B' and 'A may possibly not be B' have two degrees of meaning analogous to those of the exception and the instance. They may be *mere* contradictories of the hypothetical judgment to which they are respectively opposed, or they may be contradictories growing into contraries. If they are *mere* contradictories, corresponding to *exceptions*, then the judgment 'A may be B' merely means to overthrow the principle that 'A cannot be B'; that is to say it asserts that if or though A is, yet it does not follow, from that, that B is not. It is easy to give the corresponding significance to 'A may not be B'. But precisely the same judgments 'A may be B' and 'A may not be B' are capable of corresponding to the *instance*, and their meaning then is that there is some positive connection between an unspecified condition x, which is fairly conceivable of A, and B or the negation of B as may happen. Under these circumstances, even though we do not know that Ax itself is actually found in experience, yet we have ground for saying that there is a rational connection or antagonism between A and B. Many degrees of these connections are to be met with. If x were the entire ground of B, and we knew x to be true of A, then we should no longer have possibility but certainty. But if x is part of the ground of B, and we know x to be true of A, then we have a degree of real possibility varying with the relation of x to B. Or again, if x is not the ground but a consequent of B, and we know categorically that A is x, we have in effect an inference, from the hypothetical 'If A is B it is x' converted by limitation into 'If A is x it may be B'. I will give a concrete example.

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[1] If the hypothetical judgment has a negative consequent, it is of course needless to limit it in converting.

Plants (A) may possibly possess sentience (B). Taken as the mere guess of an unscientific mind—prepared to say the same no less of stones and metals, air or water—such a judgment would represent a mere negative possibility, or in other words, it would express no more than the fact that having considered the universal judgment 'Plants are not sentient' the individual mind happens to see in it no sequence of reason and consequent, and therefore pronounces that 'There is no ground for asserting that plants are not sentient'.

But the same judgment 'Plants (A) may possibly have sentience (B)' is capable of conveying a more positive meaning. It may rest on the conviction that 'If plants (A) have irritability (x) they must have sentience (B)'. Assuming ignorance as to actual observations of irritability in plants we nevertheless have then a certain congruity[1] to go upon in saying that A may, in a sense that has a certain basis of reality, be B. But still more strongly, if we could say 'If plants (A) have sentience (B) they must have irritability (x)', *being able to supplement this with the fact* 'There are plants (A) which have irritability (x)', we should be able to affirm There are positive grounds for maintaining that plants A have sentience B'.

Possibility may therefore mean (i) the inability to make a certain hypothetical judgment, so long as we bear in mind that *mere* privation or inability is a limit which the judgment must not *actually* reach if it is to retain significance. This possibility is non-impossibility. Or it may mean (ii) the inference from a hypothetical judgment, whether explicit or suggested in instances, which (*a*) assigns an intelligible condition (or makes us believe that there is such a condition), which would if *real* establish the consequent whose possibility, assuming the reality of the subject of judgment, is in question, or (*b*) assigns a certain logical consequent to the attribute or event in question considered as a logical antecedent, which consequent is known to be real. This logical consequent may be either an effect with alternative causes, or a consequent with alternative reasons. The ideal or reciprocal form of the hypothetical judgment excludes mere possibility and therefore does not concern us here.

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[1] We should not e.g. attach any such weight to the judgment that if a plant had poetic genius it must have sentience. I claim throughout that congruity is essential and not accidental in supposition. If we go outside the system of fact which is our basis of supposal, we get results analogous to the 'infinite judgment'.

Essence of the Problematic Judgment.

γ. It is plain from the above examples that the terms 'possible', 'probable', 'may', 'might', and 'must', stand for more or less reflective estimates of certain kinds of knowledge. The essence of the problematic judgment is the substitution of such an estimate for the concrete steps of inference really involved in an affirmation. Possibility results in referring to reality, *without transition*, but *subject to an estimate*, what is only connected with it by transitions. When the whole transition is made explicit, the allegation of possibility is superseded.[1] The judgment which has all its conditions and reservations fully assigned to it is of the apodeictic order; possibility arises from effecting the reference to reality apart from the conditions. The idea of 'possibility' is our substitute for the omitted conditions. Obviously such an idea may emanate from all degrees of confused perception or of reflection. We may be silent about the conditions either because we cannot clearly grasp them, or because we are explicitly abstracting from them. But an estimated indirectness of transition there must be if we are to judge problematically. Disjunction can be treated in the same way, owing to the hypothetical inter-relations of its members, and thus the statement of chances is a clear and extreme case of the estimate in question. Its essence is to burden the reference to reality of one alternative with a hindrance drawn from the number of other alternatives. All possibility indicates a similar tendency. But the statement of chances has measured its own cloudiness and made certain of its own uncertainty.[1] It is therefore no longer problematical. It supplies a definite predication of a hindrance to reality, and not a hindrance to predication.

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[1] Cp. p. 341, supra.

Thus the true problematic judgment is a judgment with a peculiar and reflective content, which interferes with its assertiveness. It is a hypothetical or disjunctive judgment in disguise. All judgment whatever is within a real system, but the problematic judgment has its relations to its real system peculiarly obscured or neglected. In the question, for instance, we only make explicit a part of the intellectual state, ultimately affirmative, on which our desire for further knowledge depends. A question indeed often vanishes when we insist on its being clearly put.

Assertory Judgment.

3. The other forms of modality may be briefly dismissed. Every judgment may be called assertory, as we saw, in virtue of its psychological reference to self-feeling. If any judgments are to be called assertory in a strictly logical and material sense, they must of course be the singular judgments which depend on union of attributes within the concrete subject of the judgment, and not on their necessary connection in a larger subject falling outside the judgment. Such judgments are even *logically* assertory in as far as the concrete subject is merely individual as a synthesis of differences not connected by abstract necessity. So far on the other hand as it displays individual *character* and lends itself to analogical affirmation, it stands for the present purpose on the same footing with the subjects of necessary and apodeictic judgments. The assertory judgment has a higher degree of assertiveness than the problematical judgment as such, because

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[1] This is the reason why the 'Thermometer of Probability' (see De Morgan, Budget of Paradoxes, p. 416) would not be of very general application. What it measures is the ratio of the whole number of equal alternatives arising on a certain condition to the number of desired alternatives so arising. But in concrete knowledge we have no security of finding equal alternatives.

its reference to reality though not apodeictic is direct and open. It would be impossible to maintain Kant's view of a progressive incorporation of contents with the understanding, if he meant that every trivial judgment, say, of perception, was preceded by a recognition and estimate of uncertainty. But all experience supports his contention that imperfect judgments in which only one element is clear, and in which this clear element is attached to reality through others which are not clear, belong to a less complete phase of knowledge than judgments in which the reference is clear and complete. If the problematic judgment arises by intentional abstraction from precise knowledge, this makes no difference. We are returning on purpose to an imperfect form of judgment from a more perfect one, in order to exhibit a net result which the more explicit form will not display. We must stand by the result which we thus obtain. We cannot eat our cake and have it.

Of course where an instance indicates a possibility, the assertory affirmation of the instance and the problematic affirmation of the possibility do not refer to the same content. 'This Drosera shows irritability' is a singular judgment of perception. The problematic judgment 'There are conditions though unknown in detail under which the nature of a plant develops irritability', i.e. 'A plant as such may have irritability' arises from the analysis of the above instance into a general suggestion. Such an analysis is probably concurrent with the perception of individual identity on which the Singular judgment rests, but the two are not identical. Imperfect insight into necessary connection may affect the same content which is being erected into a thing, but is not one with this process of erection. Assertory assertion and problematic assertion may be and must be conjoined in every problematic judgment, but they refer to different elements in the content affirmed, the former to the system as a concrete real whole, the latter to some element of the system as related by abstraction to the other elements. When the latter aspect of the judgment is dropped or superseded we have the assertory affirmation pure and simple. The assertory judgment has no degrees of assertiveness except in so far as in virtue of its specific content the problematic or apodeictic judgments inevitably show themselves within it.

The Apodeictic Judgment.

4. Judgments of apodeictic character, i.e. hypothetical and disjunctive judgments, lay claim in virtue of their form to a higher degree of assertiveness than either problematic or assertory judgments. The reason of this is that their form has for its differentia the exact exposition of the transitions, conditions, or alternatives subject to which the judgment is true of reality. By such exact exposition the content either becomes an articulated system, or at least reveals itself as fitted to take a place in such a system. The former is the ideal of the disjunctive, the latter that of the hypothetical judgment. Reality considered as absolutely known is of course *ex hypothesi* taken to be absolutely asserted. But Reality is not by any sane person considered to be absolutely, i.e. completely and precisely known. Degrees of certainty in apodeictic affirmation arise from the consciousness—a logical consciousness made explicit in the structure of judgments—that the individual's knowledge is but imperfectly identified with the ideal judgment which would qualify Reality by the complete content of Reality.

The Hypothetical Judgment.

i. In the hypothetical judgment, as we have seen, though important elements are made explicit, yet the connection which is affirmed implies an underlying reality which is not expressed in the content of the judgment. Therefore the hypothetical judgment is subject to two elements of uncertainty, viz. its own reference to the limited reality the affirmation of which it implies, a reference which may be partial or confused, and further the relation of that affirmed real system to the content of Reality as a whole. 'The British Parliament is able to alter a statute affecting its own duration.' This is rather a generic than a hypothetical judgment; but for the present purpose these two types must rank as very closely akin. About such a judgment there is first the question whether it really follows, or how necessarily it follows, from the facts which we are prepared to affirm of the reality known as the British Constitution, and secondly, what opening these facts themselves, as compared with the greater reality of our entire experience on the ground of which *they* in turn are affirmed, leave for error or for unseen modification. Any disjunctive judgment, as compared [1] with a given hypothetical judgment of which it may be regarded as a development, makes this underlying real system explicit, and therefore has no source of uncertainty but that of a failure in the necessity by which this system itself is attached to Reality as a whole. But of course the disjunctions which we commonly use are for the most part systems within known systems, and therefore stand on the same logical level of certainty with hypothetical judgments. Such e.g. is the disjunction which expresses the number and nature of the conic sections, resting as it does on the ultimate real system which we take to be the nature of space. Only disjunctions that embody a complete and coherent sphere of knowledge, such e.g. as the nature of space, have the character which has just been ascribed to the ideal disjunction. But even with such a disjunction we still have the difficulty in the background, 'Does the real system which we affirm'—in this example the nature of space—'*really* emanate as a necessity of knowledge from the whole reality which is forced upon us by experience?' It appears to me to be quite idle to maintain that all judgments, or even that all necessary or apodeictic judgments, are on a level in this respect. Ultimately, we may imagine, nothing can be rightly known without knowing all else rightly, so that every isolated fact and principle of knowledge would be implied in, say, the existence of morality or in the existence of an intellectual world. But as knowledge is in fact constituted its parts are fragmentary and incoherent, and there is much that we affirm upon only a partial or limited necessity, while much again is so incorporated with the whole fabric of our real world that we feel bound to maintain the former if we would not fall into hopeless contradiction with the latter.[1] The assertiveness of affirmation is not indeed measurable outside the calculus, but it is capable of being perceived and to some extent compared. It is not a mere feeling, but an insight into connections. It is absurd to maintain that in affirming the whereabouts of a friend, or the harmonious or other effect of a combination of colours, or the continuity or non-continuity of matter or of space, I am pledging my intellectual existence to the same degree as when I affirm the relations of the multiplication table, or (subject to the requisite interpretations) the law of causation, or the existence of moral purposes.

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[1] It is absolutely necessary, when we attempt to compare judgment forms in respect of their essential import, to select instances which belong to one and the same progression. A hypothetical judgment drawn from an advanced phase of science has at once more content and more precision than a disjunctive judgment drawn from everyday experience. To judge the capacities of the two forms by instances so selected would be like judging the powers of civilisation and savagery by comparing a civilised infant without allowance for age to an adult savage.

The uncertainty which may attach to apodeictic judgments arises then from the same cause as the uncertainty of problematic judgments, but the cause is operative in a different mode and in a slighter degree. We always *feel* certain when we judge, for all judgments involve the same psychological identification with self-feeling. But we *know* that this certainty is conditional on our expressing Reality with precision and completeness, and this we are well aware that we never do. Of course, to begin with, the apodeictic form of judgment is no guarantee, any more than any other, against falsehood or frivolity. There may be no underlying real system at all, or that which is taken to be referred to may in no way justify the sequence erected on it. Any false generalisation is an instance of this. Or again, the necessary transition may be quite incompletely set forth, so that the judgment sinks *ipso facto* into a problematic judgment in spite of its apodeictic form. Such are judgments in which a remote cause or consequence taken as a sign is substituted for the ground of the sequence, so that the judgment though not untrue in fact has the appearance of a riddle. 'If the stick will beat the dog the old woman will get home to-night.' Here we have no indication of the real arrangement on which the sequence rests, and the condition, so far as can be seen without copious supplementation from ulterior knowledge, is irrelevant and equivalent to an unknown condition, implying as it does a number of unknown conditions. This is the explanation of the examples to which Sigwart appeals as showing that apodeictic judgments are not in fact made with any peculiar certainty of conviction. 'There is a common idea that the apodeictic judgment stands for something higher than the assertorical. It is believed that if we start from the problematic judgment and ascend to the apodeictic we steadily increase the certainty of our knowledge, and add to the worth and dignity of our assertions. This idea must be relinquished. All mediate certainty must stand in the end on immediate knowledge; the ultimate premises of every proof cannot be proved. The usages of life stand in comic discrepancy with the emphasis we lay upon apodeictic certainty. The sayings "It must be so", "It must have so happened" are judgments apodeictic; but the confidence they express has most modest limits.'[1]

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[1] This must be read subject to reservation as regards the actual language in which abstract principles are expressed. It is a recognisable function of the body of knowledge, not a limited set of stereotyped ideas, which we may be justified in holding indispensably necessary to our reason. See below, Book II, chap. vii.

This is not the place to criticise the fundamental view which Sigwart here expresses, but to which he is, happily, not faithful throughout. In treating explicitly of the nature and bases of inference we shall see that the distinction between mediate and immediate knowledge coincides with the distinction between what is known and what is only on the way to be known. If Sigwart meant what he said in this place, he would have cut himself off from all possibility of believing in science. As to the examples which he adduces, they fall into their place, according to what has been said, as problematic judgments. 'It must have so happened' is an inference from reality under a condition, to reality without an expressed condition, and therefore is problematic.

The degrees of certainty belonging to the apodeictic judgment itself are as we saw of the same kind as those which characterise the problematic judgment. The whole of the reality on which the sequence is intended to be based may not have been brought to bear upon the sequence, and even if it has, its own relation to the reality which is the ideal of knowledge may be so disproportionately trifling as to make the judgment an especially inadequate embodiment of the ideal Reality which alone is certain.

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[1] Sigwart, Logik, i. 238, E. Tr. 183, quoted in Bradley's Logic, p. 186.

I may illustrate this conception by our present knowledge of Hypnotic and kindred phenomena.[1] The curative action of Hypnotism, or the beneficent anaesthesia which may be produced in some forms of the Hypnotic state, are now as it seems matters capable of embodiment in empirical generalisations. These rules or laws can be exhibited, like all rules or laws, in hypothetical or disjunctive form ; that is to say, as consequents following upon conditions, or as alternatives arising within a certain identical content. On a certain degree and kind of hypnotic trance a certain anaesthesia is consequent. A hypnotised subject is capable of some three or four recognisable alternative states. In certain forms of nervous derangement acquired, or, as it seems, congenital, repeated hypnotic treatment exercises a sanative influence.

Now although I personally entertain no doubt that there is, as the phrase goes, 'something in' all these conceptions, and though I am prepared to affirm them, i.e. to judge them true of reality, as embodiments of an experienced content which must be affirmed somehow and which I cannot affirm otherwise, yet so far as I understand myself I do not stake my intellectual existence upon them as I do on the existence of causation, or morality, or beauty. On what I *mean to say* in them, on the experienced content from which they are drawn, I do stake my intellectual existence ; but this content, apart from my explicit judgment, is to me an *x*, a thing-in-itself, a nothing. I must stand or fall by my judgment as it is, not merely by my ultimate intention to embody reality, which is the common and formal feature of all affirmation. And I know perfectly well that by my explicit judgment on such matters as these I am very likely to fall. The reason of this is not that the cases on which I rely are few in number, compared, e.g. with ordinary cases of the operation of digitalis on the heart or of mercury on the liver. One case is enough, as we all know, where the framework of knowledge stands ready to receive it with the grip of necessity. The reason is rather that the organised system of reality within which the sequences in question have their force, lies or has hitherto lain outside the great fabric even of biological science, not to speak either of deductive reasoning on the one hand or of philosophical construction on the other. The principles of abnormal psychical phenomena have not been as a whole identifiable either with the principles of physical causation or with those of normal psychical development. The judgment is therefore obstructed by the want of contact or necessary relation between the system of such abnormal phenomena, which it must in some way refer to reality, and the entire *cosmos* of normal evolution. Supposing, however (I speak merely by way of illustration), that the abnormal states in question, even those which present the apparent puzzle of a morbid origin combined with a curative effect, could be exhibited as cases under the known principles of evolution, the whole ground and certainty of the judgments relating to them would be put upon a new foundation. It is an old idea [1] that many states and susceptibilities of the soul, which are commonly treated as mere freaks of nature or capricious results of disease, may really have their place among the phenomena of evolution no less than sleep and waking, or the oneness, expressing itself through heredity, of parent and child. If peculiar forms of sensitiveness and peculiar 'morbid' states or transitions to states could be brought under such heads as survival, reversion, or analogous development, their underlying reality would be grafted on the main stem of the organised real world, and the necessity with which they were affirmed would become more determinate and more concrete.

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[1] I believe myself justified, when writing for purely logical purposes, in treating the actual phenomena known to science as freely as may be necessary to give them sharp outlines, and to avoid a mass of reservations and qualifications that would be only an encumbrance for my present purpose. I on my side hope to keep clear of wilful distortion, but the reader on his side must not accept my illustrations as citations from a biological textbook.

The Disjunctive Judgment.

ii. Disjunction by its form aims at the standard of a complete and therefore of a real system. 'Real' because it points to nothing beyond itself as an implied ground of truth. We have seen sufficiently above that no objection can be raised against the reality of the content of any judgment by reason of its being extended in space or time. No judgment confines its reference within an atomic 'now', and no reality can display itself as existent within an atomic now. It is grammatically possible however for a disjunctive proposition to express a judgment which is hypothetically disjunctive: 'A man who would act so must be a knave or a fool.' This shares the character of all hypothetical judgments in implying an unexpressed real system as the basis of its truth, and its certainty must be judged as the certainty of a hypothetical judgment. But the disjunction according to the ideal prescribed by its form is in itself an exposition of the reality that determines its parts, and therefore is not a sequence within a presupposed system, but is itself the content of a real system. It therefore properly ranks with the generic judgment, to which it is affiliated, as quasi-categorical; and has only the imperfection of certainty which arises from the comparatively minute range of reality that is comprehended in any such simple system.

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[1] See Hegel, Encyclopädie (Anthropologie), sect. 404.

It is obvious from what has been said above that the degrees of certainty here discussed are not numerically estimable, because they are not reducible to ratios of equal alternatives. It may therefore be justifiably complained that the phrase 'degree of certainty' is misplaced, and should be transformed into 'stage of logical necessity'. I have no objection to some such transformation provided that it is distinctly understood that modality affects the assertiveness of assertion, that this assertiveness is a matter of content and not of the formal copula or reference to self-feeling, but that if we extend the notion of the copula to include the material or logical grasp by which a complex content is fitted on to a complex Reality then we may say that Modality is a matter of the Copula. In any case, the progressive incorporation of a content with the understanding,[1] that is, with the organised ideal system by which the understanding permanently qualifies the Real, is the same thing as the progressive participation of that content in the certainty that could only be complete in a judgment that should exhaust Reality.

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[1] Cp. Whewell's account (see Mill's Logic, i, p. 279) of coming to perceive the necessity of a principle which he had before accepted as fact. This is merely the acquisition of a precise and coherent insight into its dependence on reality. Cp. Book II, chap. vii.

A NOTE ON ERROR.

In the first edition of this work there was no treatment of Error *eo nomine*. The reason of this was that it appeared to me to be substantially dealt with in the general account of the conditions of truth, in the theory of negation and possibility, and in the account of the rectification of judgment by scientific process.

It will be well, however, to sum up shortly the results of these views in their bearing on Error *eo nomine*. I acknowledge indebtedness to Professor Stout for the distinct form which he has given to the question (Aristotelian Proceedings, 1910–11, 'The Object of Thought and Real Being'), and for his precise statement, amplified in private correspondence with me, of the nature of 'relative possibility' as affecting conceivability. Yet I believe that I am advancing nothing which does not fairly follow from the theory of this work.

The fundamental character of Error is always the assertion of a merely possible alternative to be *the* alternative true of reality, i. e. the assertion of something conditionally true, without regard to its condition. (I should say 'unconditional assertion' were it not the case that in error there is rather a tendency to assertion under a false condition.)

I distinguish two main cases.

(1) There is what may be called 'Logical Error'. This is when the condition, under which alone the affirmed alternative is realised, is omitted or replaced by another, but not excluded by an incompatible condition. A simple case is when we are drawn on to affirm the consequent of a hypothetical judgment without thinking of its antecedent. 'If that one little link were made out, I should be of royal descent,' passes in cases of bias or failing mind into 'I am of royal descent'. Here there is not necessarily factual error. The alternative affirmed as actual is one relatively possible, i. e. conceivable from a certain point of view, and may be in addition actually true, i. e. conceivable from a complete point of view. But the limitation suggested by the incomplete point of view is not attended to and expressed, while, as in ordinary everyday judgment, other more or less indifferent qualifications may be expressed. Such judgments may be what we call true in fact (as the affirmed consequent of the hypothetical above *may* be true in fact), but as in every judgment there is a presumption of necessary connection (see vol. i, pp. 134–5), the indifferent qualifications produce a false nexus and therefore a Logical Error. All judgments except those, if any, which express absolute truth have some degree of 'logical error'. What they assert does not 'hang together'; but it may be a fact, and indeed all fact is asserted in this way.

(2) I call it 'Factual Error' when the necessary condition for the affirmed alternative being actual, not only is not expressed, but is replaced by a condition or character belonging to an actual alternative

which excludes the one affirmed. 'The lady in the garden, wearing a pink dress, is Mrs. A.' (when the wearer of the pink dress is Mrs. B., and Mrs. A. really has on a blue dress). For the judging mind, this case is much like the last. The status of the condition, which should attach the alternative to reality, is unnoticed. And in this case it is a status not merely of obscurity or irrelevance, but of hostility. But the point of view of partial truth is preserved, and what is asserted is a conditional element of reality taken as true without regard to its real condition. The alternative affirmed is conceivable or possible from a certain limited point of view (i. e. as 'lady in the garden'), but is affirmed as actual, i. e. as conceivable from a complete point of view, or, without allowance for the limitation suggested by the point of view from which it is possible. In factual error, the complete point of view involves a hostile or excluding condition, a condition peculiar to a different alternative, so that the alternative affirmed is no longer conceivable nor possible.

We may say, therefore, that the alternative affirmed in factual error 'belongs to reality' (Green, Prolegomena, sect. [illegible]3, above, i. 327), but not as affirmed. For the condition through which it is identified with actual reality is a condition which, S and P being fully determined, excludes it. Therefore from a complete point of view it is inconceivable, and does not belong to reality. The judging mind has not fully determined the content of its own affirmation, but when this is determined, the affirmation becomes a contradiction in terms.

END OF VOL.

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